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Fekete-Szegö Inequality For A Certain Class Of Analytic Function Associated With Convolution

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Abstract: *In this paper, a class of analytic functions associated with convolution is defined, and for this class we obtain Fekete-Szegö inequality, integral representation and structural formula for that class.*
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I. Introduction and Preliminaries

Let A_p denotes the class of functions of the form:

$$f(z) = z^p + \sum_{k=1}^{\infty} a_{p+k} z^{p+k} \ (p \in N = 1,2,3\ldots), \tag{1.1}$$

which are analytic and p-valent in the unit disk $\Delta = \{z \in C; |z| < 1\}$.

Let $g(z) \in A_p$ be of the form:

$$g(z) = z^p + \sum_{k=1}^{\infty} b_{p+k} z^{p+k}. \tag{1.2}$$

A convolution (Hadamard product) of $f(z) \in A_p$ of the form (1.1) with $g(z) \in A_p$ of the form (1.2) is defined by

$$(f * g)(z) = z^p + \sum_{k=1}^{\infty} a_{p+k} b_{p+k} z^{p+k} = (g * f)(z). \tag{1.3}$$

This convolution generalizes several convolution operators such as:

Dziok Srivastava operator [5], involving a generalized hypergeometric function ${}_qF_s$:

$${}_qH_s^p([\alpha_1])f(z) := z^p \, {}_qF_S(\alpha_1, \alpha_2, \ldots \alpha_q; \beta_1, \beta_2, \ldots \beta_s; z) * f(z),$$

If

$$b_{p+k} \equiv \frac{(\alpha_1)_k (\alpha_2)_k \ldots (\alpha_q)_k}{(\beta_1)_k (\beta_2)_k \ldots (\beta_s)_k} \frac{1}{k!}, \ q = s+1, \beta_i \neq 0, -1, -2, \ldots (i = 1,2,\ldots s), \tag{1.4}$$

which again generalizes Hohlov operator [7], involving Gaussian hypergeometric function ${}_2F_1$:

$${}_2H_1^p([\alpha_1])f(z) := z^p \, {}_2F_1(\alpha_1, \alpha_2; \beta_1; z) * f(z),$$

as well as Carlson and Shaffer operator [4], involving incomplete beta function:

$$L_p(\alpha_1, \beta_1) f(z) := z^p \, {}_2F_1(\alpha_1, 1; \beta_1; z) * f(z),$$

which further reduces to Ruschweyh derivative operator [12]:

$$D^{n+p-1} f(z) = \frac{z^p}{(1-z)^{n+p}} * f(z),$$

if $\alpha_1 = n + p > 0$, $\beta_1 = 1$ and $D^0 f(z) \equiv f(z)$.

In addition, the convolution (1.3) reduces to the Salagean operator [13], if

$$b_{p+k} = \left(\frac{p+k}{p}\right)^n, n = 0,1,2\ldots$$

and to a generalized Salagean operator [2], if

$$b_{p+k} = \left(\frac{p+\delta k}{p}\right)^n, \delta > 0, n = 0,1,2\ldots.$$

Further if

$$b_{p+k} = \left(\frac{p+k+\lambda}{p+\lambda}\right)^n, \left(\lambda \in C \setminus \{-p\}, n \in Z\right)$$

the convolution (1.3) reduces to the multiplier transformation, which is denoted as

$$l_p(n,\lambda)f(z) = z^p + \sum_{k=1}^{\infty}\left(\frac{p+k+\lambda}{p+\lambda}\right)^n a_{p+k} z^{p+k}.$$

The multiplier transformation has been studied by Aghalary et al. [1].
Further, the convolution (1.3) reduces to an integral operator involving fractional integral operator $D_z^{-\lambda} f(z)$, if

$$b_{p+k} = \frac{(p+1)_k}{(p+\lambda+1)_k}$$

and hence

$$(f * g)(z) = z^{-\lambda}\frac{\Gamma(p+\lambda+1)}{\Gamma(p+1)} D_z^{-\lambda} f$$

where

$$D_z^{-\lambda} z^{\rho} = \frac{\Gamma(\rho+1)}{\Gamma(\rho+\lambda+1)} z^{\rho+\lambda}, \quad (\lambda > 0).$$

Again, this convolution (1.3) reduces to the derivative operator involving fractional derivative operator D_z^{λ} if

$$b_{p+k} = \frac{(p+1)_k}{(p-\lambda+1)_k}$$

and hence,

$$(f * g)(z) = z^{\lambda}\frac{\Gamma(p-\lambda+1)}{\Gamma(p+1)} D_z^{\lambda} f,$$

where $$D_z^{\lambda} z^{\rho} = \frac{\Gamma(\rho+1)}{\Gamma(\rho-\lambda+1)} z^{\rho-\lambda}.$$

The fractional integral and fractional derivative operators of order λ is defined by Owa [9] and Srivastava and [14].
Recently, Patel and Mishra [10] defined a calculus operator $\Omega_z^{(\lambda,p)} : A_p \to A_p$ for a function $f \in A_p$ and for a real number $\lambda(-\infty < \lambda < p+1)$ by

$$\Omega_z^{(\lambda,p)} f(z) = \frac{\Gamma(p+1-\lambda)}{\Gamma(p+1)} z^{\lambda} D_z^{\lambda} f(z)$$

$$\Omega_z^{(\lambda,p)} f(z) = z^p + \sum_{k=1}^{\infty} \frac{\Gamma(p+1-\lambda)\Gamma(p+k+1)}{\Gamma(p+1)\Gamma(p+k-\lambda+1)} a_{p+k} z^{p+k}, z \in \Delta.$$

A function $f(z) \in A_p$ is said to be p-valently starlike of order α in Δ, if it satisfies the inequality

$$\operatorname{Re}\left\{ \frac{zf'(z)}{f(z)} \right\} > \alpha \left(z \in \Delta; 0 \le \alpha < p; p \in N \right).$$

The class of all p-valent starlike functions of order α is denoted by $S_p^*(\alpha)$ and write $S_1^*(\alpha) \equiv S^*(\alpha)$.
On the other hand, a function $f(z) \in A_p$ is said to be p-valently convex of order α in Δ, if it satisfies the inequality

$$\operatorname{Re}\left\{ 1 + \frac{zf''(z)}{f'(z)} \right\} > \alpha \left(z \in \Delta; 0 \le \alpha < p; p \in N \right).$$

The class of all p-valent convex functions of order α is denoted by $K_p(\alpha)$ and write $K_1(\alpha) \equiv K(\alpha)$.
Furthermore, a function $f(z) \in A_p$ is said to be p-valently close-to-convex of order α in Δ, if it satisfies the inequality

$$\operatorname{Re}\left\{ z^{1-p} f'(z) \right\} > \alpha \left(z \in \Delta; 0 \le \alpha < p; p \in N \right).$$

The class of all p-valent close-to-convex functions of order α is denoted by $CK_p(\alpha)$. $CK_p(0) \equiv CK_p$ and denote $CK_1(0) \equiv CK$.
A function $f \in A_p$ is said to be in the class $P(\alpha)$ if and only if

$$\operatorname{Re}\left\{ f'(z) \right\} > \alpha \left(z \in \Delta; 0 \le \alpha < p; p \in N \right).$$

For two functions f and g analytic in Δ, we say that the function f is subordinate to g in Δ, and we write

$$f(z) \prec g(z),$$

if there exists a Schwarz function w, which is analytic in Δ, with $w(0) = 0$ and $|w(z)| < 1$ for all $z \in \Delta$, such that

$$f(z) = g(w(z)), (z \in \Delta).$$

Let P be the class of the functions ψ with normalization $\psi(0) = 1$, which are convex and univalent in Δ and satisfy the condition $\operatorname{Re}[\psi(z)] > 0$ for $z \in \Delta$.

Definition 1.1 *A function $f \in A_p$ is said to be in the class $S_p(g,b,m;\psi)$, if and only if*

$$1 + \frac{1}{b}\left\{ \left(\frac{z(f*g)^{m+1}(z)}{(f*g)^{m}(z)} \right) - (p-m) \right\} \prec \psi(z), \tag{1.5}$$

with $p > m$, $m \in N_0 = \{0,1,2\ldots\}$, $b \in C \setminus \{0\}$, where $(f*g)^r(z)$ denotes the r^{th} derivative of $(f*g)$ and is given by

$$(f*g)^r(z)=\frac{p!}{(p-r)!}z^{p-r}+\sum_{k=1}^{\infty}\frac{(p+k)!}{(p+k-r)!}a_{p+k}b_{p+k}z^{p+k-r},\ r\in N_0. \tag{1.6}$$

It is observe that for $-1\le B<A\le 1$, $S_p(g,b,m;A,B):=S_p\left(g,b,m;\frac{1+Az}{1+Bz}\right)$

We note that $S_p\left(\frac{z^p}{1-z},1-\alpha,0;1,-1\right)=S_p^*(\alpha)$. $S_p\left(\frac{z^p}{1-z},-1,0;2\alpha-1,-1\right)$ is the class studied by Patil and Thakare [11].

The class $S_p\left(\frac{z^p}{1-z},b,m;1,-1\right)$ is introduced by Altintaş and Srivastava [3]. Also the class $S_p\left(\frac{z^p}{1-z},1,p-2;\left\{\left(2-\frac{\alpha}{p}\right)A-\left(1-\frac{\alpha}{p}\right)B\right\},B\right)$ is studied by Güney and Eker [6] for negative coefficients.

In this paper, we obtain Fekete-Szegö inequality, Integral representation and structural formula are also obtained for the classes $S_p(g,b,m;\psi)$ and $S_p(g,b,m;A,B)$.

II. Fekete-Szegö inequality for the class $S_p(g,b,m;A,B)$

Theorem 2.1 *Let* $g\in A_p$ *be of the form (1.2) with* $p>m$, $-1\le B<A\le 1$, $m\in N_0=\{0,1,2\ldots\}$, $b\in C\setminus\{0\}$, *if* $f(z)\in S_p(g,b,m;A,B)$, *then*

$$\left|a_{p+2}-\zeta a_{p+1}^2\right|\le\frac{(A-B)|b|(p-m+2)(p-m+1)}{2(p+1)(p+2)!\left|b_{p+2}\right|} \tag{2.1}$$

$$\max\left[1,\left|\frac{[(A-B)b-B](p-m+2)(p+1)b_{p+1}^2}{(p+1)b_{p+1}^2(p-m+2)}+\frac{2\zeta b(A-B)(p-m+1)(p+2)b_{p+2}}{(p+1)b_{p+1}^2(p-m+2)}\right|\right].$$

The estimate (21) is sharp.

Proof. Since $f(z)\in S_p(g,b,m;A,B)$, we have

$$1+\frac{1}{b}\left\{\left(\frac{z(f*g)^{m+1}(z)}{(f*g)^m(z)}\right)-(p-m)\right\}=\frac{1+Aw(z)}{1+Bw(z)}$$

where $w(z)=\sum_{k=1}^{\infty}w_kz^k$ is a bounded analytic function satisfying the condition $w(0)=0$ and $|z|<1$ for $z\in\Delta$, or

$$\begin{aligned}&\left\{Bz(f*g)^{m+1}(z)-[(A-B)b+B(p-m)](f*g)^m(z)\right\}w(z)\\&=(p-m)(f*g)^m(z)-z(f*g)^{m+1}(z).\end{aligned} \tag{2.2}$$

Writing corresponding series expansions in (2.2), we get

$$\left\{B\frac{p!z^{p-m}}{(p-m-1)!}+B\sum_{k=1}^{\infty}\frac{(p+k)!a_{p+k}b_{p+k}}{(p+k-m-1)!}z^{p+k-m}-[(A-B)b+B(p-m)]\cdot\right.$$
$$\left.\left\{\frac{p!}{(p-m)!}z^{p-m}+\sum_{k=1}^{\infty}\frac{(p+k)!a_{p+k}b_{p+k}}{(p+k-m)!}z^{p+k-m}\right\}\right\}(w_1z+w_2z^2+\ldots)$$
$$=\left\{\frac{p!z^{p-m}}{(p-m-1)!}+\sum_{k=1}^{\infty}\frac{(p-m)(p+k)!}{(p+k-m)!}a_{p+k}b_{p+k}z^{p+k-m}\right\}$$
$$-\left\{\frac{p!z^{p-m}}{(p-m-1)!}+\sum_{k=1}^{\infty}\frac{(p+k)!a_{p+k}b_{p+k}}{(p+k-m-1)!}z^{p+k-m}\right\}$$

or,

$$\left\{\frac{-(A-B)bp!}{(p-m)!}z^{p-m}+\sum_{k=1}^{\infty}\frac{(p+k)!a_{p+k}b_{p+k}}{(p+k-m)!}[Bk-(A-B)b]z^{p+k-m}\right\}(w_1z+w_2z^2+\ldots)$$
$$=-\sum_{k=1}^{\infty}k\frac{(p+k)!a_{p+k}b_{p+k}}{(p+k-m)!}z^{p+k-m}.$$

Equating the coefficient of z^{p-m+1} and z^{p-m+2} on both sides, we obtain

$$\frac{-(A-B)bp!}{(p-m)!}w_1=-a_{p+1}b_{p+1}\frac{(p+1)!}{(p+1-m)!}$$

or,

$$a_{p+1}=\frac{(A-B)b(p+1-m)}{(p+1)b_{p+1}}w_1 \tag{2.3}$$

and

$$\frac{-(A-B)bp!}{(p-m)!}w_2+\frac{(p+1)!a_{p+1}b_{p+1}}{(p+1-m)!}[B-(A-B)b]w_1=-\frac{2(p+2)!a_{p+2}b_{p+2}}{(p+2-m)!}$$

$$a_{p+2}=\frac{-(A-B)(p+2-m)(p+1-m)\{[B-(A-B)b]w_1^2-w_2\}}{2(p+2)(p+1)b_{p+2}}. \tag{2.4}$$

Now, for any complex number ζ, we write

$$\left|a_{p+2}-\zeta a_{p+1}^2\right|= \tag{2.5}$$
$$\left|\frac{-(A-B)(p+2-m)(p+1-m)\{[B-(A-B)b]w_1^2-w_2\}}{2(p+2)(p+1)b_{p+2}}-\zeta\left\{\frac{(A-B)b(p+1-m)}{(p+1)b_{p+1}}w_1\right\}^2\right|$$
$$=\frac{(A-B)|b|(p+2-m)(p+1-m)}{2(p+2)(p+1)|b_{p+2}|}\left|w_2-\xi w_1^2\right| \tag{2.6}$$

where

$$\xi=\frac{[(A-B)b-B](p-m+2)(p+1)b_{p+1}^2+2\zeta b(A-B)(p-m+1)(p+2)b_{p+2}}{(p+1)b_{p+1}^2(p-m+2)}. \tag{2.7}$$

From the result of Keogh and Merker [8], it is known that for any complex number ξ,

$$\left|w_2-\xi w_1^2\right|\le\max\{1,|\xi|\},$$

and the estimate is sharp for the functions $f_0(z)=z^p$ and $f_1(z)=z^{p+1}$ for $|\xi|\geq 1$ and $|\xi|<1$ respectively. From (2.5), it follows that

$$\left|a_{p+2}-\zeta a_{p+1}^2\right|\leq \frac{(A-B)|b|(p+2-m)(p+1-m)}{2(p+2)(p+1)|b_{p+2}|}\max\{1,|\xi|\},$$

where ξ is given by (2.6).

III. Integral Representation For The Classes $S_p(g,b,m;\psi)$ And $S_p(g,b,m;A,B)$

Theorem 3.1 *Let* $g(z)\in A_p$ *of the form (1.2) then a function* $f\in S_p$ *be in the class* $S_p(g,b,m;\psi)$ *if and only if there exist a Schwarz function* $w(z)$ *such that*

$$(f*g)^m(z)=z^{p-m}\exp\int_0^z \frac{b\{\psi(w(z))-1\}}{t}dt. \tag{3.1}$$

In particular, if $f\in S_p(g,b,m;A,B)$ *then*

$$(f*g)^m(z)=\exp\left((p-m)\int_0^z \frac{\left[1-\left\{B+\frac{(A-B)b}{(p-m)}\right\}Q(z)\right]}{t(1-BQ(t))}dt\right) \tag{3.2}$$

$$\textit{where } |Q(z)|<1 \textit{ and } (f*g)^m(z)=z^{p-m}\exp\int_x \log(1-Bxz)^{\frac{(A-B)b}{B}}d\mu(x), \tag{3.3}$$

where $\mu(x)$ *is the probability measure on* $X=\{x:|x|=1\}$.

Proof. Since $f\in A_p$ is said to be in the class $S_p(g,b,m;\psi)$, if and only if

$$1+\frac{1}{b}\left\{\left(\frac{z(f*g)^{m+1}(z)}{(f*g)^m(z)}\right)-(p-m)\right\}\prec\psi(z),$$

or,

$$\frac{(f*g)^{m+1}(z)}{(f*g)^m(z)}-\frac{(p-m)}{z}=\frac{b\{\psi(w(z))-1\}}{z}.$$

On integrating with respect to z, we get

$$(f*g)^m(z)=z^{p-m}\exp\int_0^z \frac{b\{\psi(w(z))-1\}}{t}dt.$$

Again, from the definition of the class $S_p(g,b,m;A,B)$

$$\left|\frac{w-1}{Bw-\left\{B+\frac{(A-B)b}{(p-m)}\right\}}\right|<1$$

where

$$w=\frac{1}{(p-m)}\left\{\frac{z(f*g)^{m+1}(z)}{(f*g)^m(z)}\right\}.$$

Let

$$\frac{w-1}{Bw-\left\{B+\frac{(A-B)b}{(p-m)}\right\}}=Q(z),\ \text{then}\ |Q(z)|<1.$$

Finally we can write

$$\frac{1}{(p-m)}\left\{\frac{z(f*g)^{m+1}(z)}{(f*g)^{m}(z)}\right\}=\frac{1-\left\{B+\frac{(A-B)b}{(p-m)}\right\}Q(z)}{(1-BQ(z))}$$

or,

$$\frac{(f*g)^{m+1}(z)}{(f*g)^{m}(z)}=\frac{(p-m)\left[1-\left\{B+\frac{(A-B)b}{(p-m)}\right\}Q(z)\right]}{z(1-BQ(z))}.$$

Integrating with respect to z , we get

$$\log\{(f*g)^{m}(z)\}=(p-m)\int_0^z\frac{\left[1-\left\{B+\frac{(A-B)b}{(p-m)}\right\}Q(t)\right]}{t(1-BQ(t))}dt$$

therefore, we get (3.2).

For obtaining the third representation let $X=\{x:|x|=1\}$ then, we have

$$\frac{w-1}{Bw-\left\{B+\frac{(A-B)b}{(p-m)}\right\}}=xz,\ x\in X,\ z\in\Delta$$

and then we conclude that

$$\frac{(f*g)^{m+1}(z)}{(f*g)^{m}(z)}=(p-m)\left\{\frac{1}{z}-\frac{\frac{(A-B)b}{(p-m)}x}{1-Bxz}\right\}.$$

Again integrating with respect to z , we get

$$\log\frac{(f*g)^{m}(z)}{z^{p-m}}=\frac{(A-B)b}{B}\log(1-Bxz)$$

or,

$$(f*g)^{m}(z)=z^{p-m}\exp\int_x\log(1-Bxz)^{\frac{(A-B)b}{B}}d\mu(x)$$

where $\mu(x)$ is the probability measure on $X=\{x:|x|=1\}$.

IV. Structural Formula For The Classes $S_p(g,b,m;\psi)$ And $S_p(g,b,m;A,B)$

Theorem 4.1 *Let $g(z)\in A_p$ of the form (1.2) then a function $f\in S_p(g,b,m;\psi)$ if and only if there exist a Schwarz function $w(z)$ such that*

$$f(z)*g(z)=\left[\sum_{k=0}^{\infty}\frac{(p+1-m)_k}{(p+1)_k}z^{p+k}\right]*\left[\frac{(p-m)!}{p!}z^{p}\exp\int_0^z\frac{b\{\psi(w(z))-1\}}{t}dt\right].\qquad(4.1)$$

Also let $f \in S_p(g,b,m;A,B)$ *then*

$$f(z)*g(z)=\left[\sum_{k=0}^{\infty}\frac{(p+1-m)_k}{(p+1)_k}z^{p+k}\right]*\left[\frac{(p-m)!}{p!}z^m\exp(p-m)\int_0^z\frac{\left[1-\left\{B+\frac{(A-B)b}{(p-m)}\right\}Q(z)\right]}{t(1-BQ(t))}dt\right] \tag{4.2}$$

where $p>m,-1\le B<A\le 1, m\in N_0=\{0,1,2\ldots\}, b\in C\setminus\{0\}, |Q(z)|<1$. *Also*

$$f(z)*g(z)=\left[\sum_{k=0}^{\infty}\frac{(p+1-m)_k}{(p+1)_k}z^{p+k}\right]*\left[\frac{(p-m)!}{p!}z^p\exp\int_x\log(1-Bxz)^{\frac{(A-B)b}{B}}d\mu(x)\right], \tag{4.3}$$

where $\mu(x)$ *is the probability measure on* $X=\{x:|x|=1\}$.

Proof. Let $f\in S_p(g,b,m;\psi)$. Then from the definition of the class $S_p(g,b,m;\psi)$ we have

$$1+\frac{1}{b}\left\{\left(\frac{z(f*g)^{m+1}(z)}{(f*g)^m(z)}\right)-(p-m)\right\}=\psi(w(z))$$

where $\psi\in P$ and $|w(z)|<1$ in Δ with $w(0)=0=\psi(0)-1$. Therefore

$$\frac{(f*g)^{m+1}(z)}{(f*g)^m(z)}-\frac{(p-m)}{z}=\frac{b\{\psi(w(z))-1\}}{z}.$$

Thus
$$\log\frac{(f*g)^m(z)}{z^{p-m}}=\int_0^z\frac{b\{\psi(w(z))-1\}}{t}dt$$

or,
$$\frac{(f*g)^m(z)}{z^{p-m}}=\exp\int_0^z\frac{b\{\psi(w(z))-1\}}{t}dt.$$

Therefore from (1.6) we obtain

$$f(z)*g(z)*\sum_{k=0}^{\infty}\frac{(p+1)_k}{(p+1-m)_k}z^{p+k}=\frac{(p-m)!}{p!}z^p\exp\int_0^z\frac{b\{\psi(w(z))-1\}}{t}dt$$

and our assertion (4.1) follows immediately.
Again, from (3.2) and (1.6) we obtain

$$f(z)*g(z)*\sum_{k=0}^{\infty}\frac{(p+1)_k}{(p+1-m)_k}z^{p+k}=\frac{(p-m)!}{p!}z^m\exp(p-m)\int_0^z\frac{\left[1-\left\{B+\frac{(A-B)b}{(p-m)}\right\}Q(z)\right]}{t(1-BQ(t))}dt$$

which gives assertion (4.2). Similarly

$$f(z)*g(z)*\sum_{k=0}^{\infty}\frac{(p+1)_k}{(p+1-m)_k}z^{p+k}=\frac{(p-m)!}{p!}z^p\exp\int_x\log(1-Bxz)^{\frac{(A-B)b}{B}}d\mu(x)$$

which gives assertion (4.3),where $\mu(x)$ is the probability measure on $X=\{x:|x|=1\}$.

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