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On The T Curvature Tensor of Generalized Sasakian Space Form

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Abstract: *The objects of the present paper is to characterize generalized Sasakian-space-form satisfying certain curvature conditions on T curavature tensor . In this paper we study T semisymmetric, T flat, T flat and T recurrent generalized Sasakian-space-forms. Generalized Sasakian-space-form satisfying T:S = 0 and T:R = 0 have also been studied.*
Keywords: *T curvature tensor, Generalized Sasakian manifold, T semisymmetric, T flat and T -flat .*

I. Introduction

Alegre [1] defined Generalized Sasakian-space-form as the almost contact metric manifold (M^{2n+1}; ; ; ; g) whose curvature tensor R is given by

$$R = f_1R_1 + f_2R_2 + f_3R_3; \tag{1.1}$$

where f_1; f_2; f_3 are some differentiable functions on M and

$R_1(X; Y)Z = g(Y; Z)X \; g(X; Z)Y;$
$R_2(X; Y)Z = g(X; Z) Y \; g(Y; Z) X + 2g(X; Y) Z;$ (1.2)
$R_3(X; Y)Z = (X) (Z)Y \; (Y) (Z)X + g(X; Z) (Y)$
$g(Y; Z) (X) ;$

for any vector fields X; Y; Z on M^{2n+1}. In such a case we denote the manifold as $M(f_1 ; f_2; f_3)$. This kind of manifold appears as a generalization of the well-known Sasakian-space-forms by taking $f_1 = (c + 3)=4$; $f_2 = f_3 =(c \; 1)=4$: It is known that any three-dimensional (;)-trans-Sasakian manifold with ; depending on is a generalized Sasakian-space-form. The author gave results about Chen inequality on submanifolds of generalized complex space-forms and generalized Sasakian-space-forms [2], [3] analyse the CR submanifolds of generalized Sasakian-space-forms [3] In [9] Kim studied conformally flat generalized Sasakian-space-forms and locally symmetric generalized Sasakian-space-forms. De and Sarkar [6] studied generalized Sasakian-space-forms regarding projective curvature tensor.

On the other hand, Tripathi and Gupta introduced the T-curvature tensor which in particular cases reduces to other some known curvature tensors [15] . The author studied T-curvature tensor in k-curvature and Sasakin and (N(k),) semi-Riemannian manifold respectively. In this paper, we study the T-curvature tensor in generalized Sasakian-space-forms.The present paper is organized as follows. In Sec-tion 2, some preliminary results are recalled. In Section 3. we gave the introduction of T-curvature tensor. In section 4, T-semisymmetric generalized Sasakian-space-forms is studied. Section 5 deals with T- flat generalized Sasakian-space-forms. -T flat of generalized Sasakian-space-forms are studied in Section 6 . Section 7 is devoted to the study generalized Sasakian-space form satisfying T:R = 0 . The last section contains the study of generalized Sasakian-space-forms satisfying T:S = 0:

II. Preliminaries

If, on an (2n+1) dimesional differentiable manifold M^{2n+1}, there exists a vector valued real linear function , a 1-form ;the associated vector field and Riemann-ian metric g satisfying

$^2X = X + (X) ;$
$( ) = 1; \quad g(X;$
$) = (X); \quad (X) = 0;$ (2.2)
$g(X; Y) = g(X; Y) \; (X) (Y);$

for arbitary vector fields X and Y on M^{2n+1} then (M^{2n+1}; g) is called an almost contact metric manifold [5] and the structure (; ; ; g) is called an almost contact

metric structure on M^{2n+1}: From(2.1),(2.2) and (2.3) we have

$g(X; Y) = g(X; Y); \quad g(X; X) = 0;$ (2.4)
$(r_X \;)(Y) = g(r_X ;Y):$

Again we know [1] that in a (2n+1)-dimensional generalized Sasakian-space-form

R(X; Y;)Z =f_1 fg(Y; Z)X g(X; Z)Y g
+f_2 fg(X; Z) Y g(Y; Z) X + 2g(X; Y) Zg (2.5)
+f_3f (X) (Z)Y (Y) (Z)X + g(X; Z) (Y)
g(Y; Z) (X) g:
S(X; Y) =(2nf_1 + 3f_2 f_3)g(X; Y) (3f_2 + (2n 1)f_3) (X) (Y); (2.6)
QX =(2nf_1 + 3f_2 f_3)X (3f_2 + (2n 1)f_3) (X) ;
r =2n(2n + 1)f_1 + 6nf_2 4nf_3: (2.8)

for all vector field X; Y; Z on M^{2n+1}, where R denotes the curvature tensor of $M^{2n+1}$and S ; r are Ricci tensor and Scalar curvature respectively and Q is Ricci operator, that is, g(QX; Y) = S(X; Y): On a generalized Sasakian-space-forms, the following equations hold:

R(X; Y) =(f_1 f_3) [(Y)X (X)Y] ; (2.9)
R(; X)Y = R(X;)Y = (f_1 f_3) [g(X; Y)(Y)X] ; (2.10)
(R(X; Y)Z) =(f_1 f_3) [(X)g(Y; Z) (Y)g(X; Z)] (2.11)
S(X;) =2n(f_1 f_3) (X); (2.12)
Q = 2n(f_1 f_3) ; (2.13)

A generalized Sasakian space-form is said to be -Einstein if its Ricci tensor S is of the form:

S(X; Y) =ag(X; Y) + b (X) (Y); (2.14)

for arbitary vector fields X and Y, where a and b are smooth functions on M^{2n+1}:

III. T- Curvature Tensor

Tripathi and Gupta [15] introduced a general curvature tensor called T -curvature tensor which in particular reduced to some known curvature tensors. For a (2n + 1) dimensional almost contact metric manifold the T curvature tensor is given by

T (X; Y)Z =a_0R(X; Y)Z + a_1S(Y; Z)X + a_2S(X; Z)Y (3.1)
+ a_3S(X; Y)Z + a_4g(Y; Z)QX + a_5g(X; Z)QY
+ a_6g(X; Y)QZ + a_7r(g(Y; Z)X g(X; Z)Y);

where R; S; Q; and r are the curvature tensor, the Ricci tensor, the Ricci op-erator and the scaler curvature respectively and a_0; a_1; :::::; a_7 are real numbers.

Equation (3.1) is equivalent to

T (X; Y; Z; U) =a_0R(X; Y; Z; U) + a_1S(Y; Z)g(X; U)
+ a_2S(X; Z)g(Y; U) + a_3S(X; Y)g(Z; U)
+ a_4g(Y; Z)S(X; U) + a_5g(X; Z)S(Y; U) (3.2)
+ a_6g(X; Y)S(Z; U) + a_7r(g(Y; Z;)g(X; U)
g(X; Z)g(Y; U));

where R denote the Riemannian curvature tensor of type (0,4) defined by R(X; Y; Z; U) = g (R(X; Y); Z; U) . Particularly ,the T -curvature tensor, defined by (3.1) is reduced to be

(1) the quasi-conformal curvature tensor C [17] if

a_1 = a_2 = a_4 = a_5; a_3 = a_6 = 0; a_7 = $\frac{1}{2n+1}$ ($\frac{a_0}{2}$ n + 2a_1),

(2) the conformal curvature tensor C [7] if

a_0 = 1; a_1 = a_2 = a_4 = a_5 = $\frac{1}{2n\ 1}$; a_3 = a_6 = 0; a_7 = $\frac{1}{2n(2n\ 1)}$,

(3) the conharmonic curvature tensor L [8] if

a_0 = 1; a_1 = a_2 = a_4 = a_5 = $\frac{1}{2n\ 1}$; a_3 = a_6 = 0; a_7 = 0;

(4) the concircular curvature tensor (Yano, K., 1940) if

a_0 = 1;
a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = 0; a_7 = $\frac{1}{2n(2n+1)}$;

(5) the pseudo-projective curvature tensor P [13]

a_1 = a_2; a_3 = a_4 = a_5 = a_6 = 0; a_7 = $\frac{1}{2n+1}$ ($\frac{a_0}{2n+1}$ a_1);

(6) the projective curvature tensor P [16] if
$a_0 = 1; a_1 = a_2 = \frac{1}{2n}; a_3 = a_4 = a_5 = a_6 = a_7 = 0;$
(7) the M projective curvature tensor [11] if
$a_0 = 1; a_1 = a_2 = a_4 = a_5 = \frac{1}{4n}, a_3 = a_6 = a_7 = 0;$
(8) the W_0-curvature tensor [16] if
$a_0 = 1; a_1 = a_5 = \frac{1}{2n}, a_2 = a_3 = a_4 = a_6 = a_7 = 0;$
(9) the W_0 curvature tensor [11] if
$a_0 = 1; a_1 = a_5 = \frac{1}{2n}; a_2 = a_3 = a_4 = a_6 = a_7 = 0;$
(10) the W_1-curvature tensor [11] if
$a_0 = 1; a_1 = a_2 = \frac{1}{2n}; a_3 = a_4 = a_5 = a_6 = a_7 = 0:$
(11) the W_1 -curvature tensor [11] if
$a_0 = 1; a_1 = a_2 = \frac{1}{2n}; , a_3 = a_4 = a_5 = a_6 = a_7 = 0;$
(12) the W_2 -curvature tensor (Pokhariyal G.P.and R.S.Mishra 1970) if
$a_0 = 1; a_4 = a_5 = \frac{1}{2n}, a_1 = a_2 = a_3 = a_6 = a_7 = 0;$
(13) the W_3 -curvature tensor [11] if
$a_0 = 1, a_2 = a_4 = \frac{1}{2n}; a_1 = a_3 = a_5 = a_6 = a_7 = 0;$
(14) the W_4-curvature tensor [11] if
$a_0 = 1; a_5 = a_6 = \frac{1}{2n}; a_1 = a_2 = a_3 = a_4 = a_7 = 0;$
(15) the W_5-curvature tensor [12] if
$a_0 = 1, a_2 = a_5 = \frac{1}{2n}; a_1 = a_3 = a_4 = a_6 = a_7 = 0;$
(16) the W_6-curvature tensor [12] if
$a_0 = 1, a_1 = a_6 = \frac{1}{2n}; a_2 = a_3 = a_4 = a_5 = a_7 = 0;$
(17) the W_7-curvature tensor [12] if
$a_0 = 1; a_1 = a_4 = \frac{1}{2n}; a_2 = a_3 = a_5 = a_6 = a_7 = 0;$
(18) the W_8-curvature tensor [12] if
$a = 1; a_1 = a_3 = \frac{1}{2n}; a_2 = a_4 = a_5 = a_6 = a_7 = 0,$
(19) the W_9-curvature tensor [12] if
$a_0 = 1; a_3 = a_4 = \frac{1}{2n}; a_1 = a_2 = a_5 = a_6 = a_7 = 0:$

The T -curvature tensor in a generalized Sasakian-space-form satisfies

$$T(X;Y) = (a_0(f_1 \quad f_3) + a_7 r)[(Y)X \quad (X)Y] + 2n(f_1 \quad f_3)[a_1 (Y)X + a_2 (X)Y] + a_3 S(X;Y) + a_4 (Y)QX + a_5 (X)QY + a_6 2n(f_1 \quad f_3) g(X;Y); \tag{3.3}$$

$$(T(X;Y)) = [a_1 + a_2 + a_4 + a_5]2n(f_1 \quad f_3)(X)(Y) + a_3 S(X;Y) + 2n(f_1 \quad f_3)a_6 g(X;Y); \tag{3.4}$$

$$T(;Y)Z = (a_0(f_1 \quad f_3) + a_7 r)[g(Y;Z)(Z)Y] + a_1 S(Y;Z) + 2n(f_1 \quad f_3)[a_2 (Z)Y + a_3 (Y)Z] + a_4 2n(f_1 \quad f_3) g(Y;Z) + a_5 (Z)QY + a_6 (Y)QZ; \tag{3.5}$$

$$(T(;Y)Z) = ((f_1 \quad f_3)a_0 + a_7 r)[g(Y;Z) \quad (Z)(Y)] + a_1 S(Y;Z) + 2n(f_1 \quad f_3)[a_2 + a_3 + a_5 + a_6](Y)(Z) + 2n(f_1 \quad f_3)a_4 g(Y;Z) \tag{3.6}$$

and

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$(T(X;Y)Z) = (a_0(f_1 \quad f_3) + a_7 r)\,(g(Y;Z)(X) \;\; g(X;Z)(Y))$
$+ a_1 S(Y;Z)(X) + a_2 S(X;Z)(Y) + a_3 S(X;Y)(Z)$ (3.7) $+ a_4 g(Y;Z)(X) + a_5 g(X;Z)(Y) + a_6 g(X;Y)(Z)$:

Now, we have the following definitions:

Definition(3.1) : A (2n+1)-dimensional generalized Sasakian-space-form is said to be T flat if T (X; Y) = 0 for all X; Y 2 TM:

Definition(3.2) : A (2n+1)-dimensional generalized Sasakian-space form is said to be T semisymmetric if it satisfies R:T = 0, where R is the Riemannian curvature tensor of the space-forms.

Definition (3.3) : A (2n+1)-dimensional generalized Sasakian-space form is said to be T flat if T = 0.

4. T Semisymmetric Generalized Sasakian-space-forms

Let us suppose that the generalized Sasakian Space-form $M(f_1; f_2; f_3)$ is T -semisymmetric. Then we can write

$R(; U):T(X;Y) = 0$: (4.1)

The above equation can be written as

$R(; U)\,T(X;Y) \quad T(R(; U)X;Y) \quad T(X; R(; U)Y)$ (4.2)
$T(X;Y)R(; U) = 0$:

In view of (2.10) the above equation reduces to

$(f_1 \quad f_3)[g(U; T(X;Y))(T(X;Y))U \quad g(X;U)T(;Y)$
$+ (X)T(U;Y) \quad g(Y;U)T(X;) + (Y)T(X;U)$ (4.3)
$(U)T(X;Y) + T(X;Y)U] = 0$:

Now, taking the inner product of above equation with and using equation (2.2) and (3.4) we get

$(f_1 \quad f_3)[\; g(X;U)(T(;Y)) + (X)(T(U;Y))$
$g(U;Y)(T(X;)) + (Y)(T(X;U))] = 0$: (4.4)

From the above equation, we have either $f_1 = f_3$ or

$(X)(T(U;Y)) \;\; g(X;U)(T(;Y)) \;\; g(U;Y)(T(X;))$ (4.5)
$+ (Y)(T(X;U)) = 0$:

which by using (3.1) and (3.3) gives

$[2n(f_1 \quad f_3)(a_1 + a_2 + a_3 + a_4 + a_5)$
$a_3(2nf_1 + 3f_2\, f_3)][g(Y;U)(X) + g(X;U)(Y)]$ (4.6) $+[4n(f_1\, f_3)(a_1 + a_2 + a_4 + a_5)$
$2a_3(3f_2 + (2n \;\; 1)f_3)]\,(X)(Y)(U) = 0$:

Hence we have the following theorem.

Theorem(3.2) : (2n+1)-dimensional generalized Sasakian-space-form is T - semisymmetric either $f_1 = f_3$ or equation (4.6) is satisfied.

5. T Flat Generalized Sasakian-space-forms

Theorem(5.1): A (2n+1)-dimensional generalized Sasakian-space-form is T flat if and only if

$$f_1 = \frac{3nf_2}{(n \;\; 1)} = f_3:$$

Proof: For a (2n+1) dimensional T flat generalised Sasakian-space-form, we have from (3.1)

$a_0 R(X;Y)Z = a_1 S(Y;Z)X + a_2 S(X;Z)Y + a_3 S(X;Y)Z$ (5.1)
$+ a_4 g(Y;Z)QX + a_5 g(X;Z)QY + a_6 g(X;Y)QZ$
$+ a_7 r(g(Y;Z)X \;\; g(X;Z)Y)$:

In view of (2.6) and (2.7) the above equation takes the form

$a_0 R(X;Y)Z = (2nf_1 + 3f_2 \quad f_3)[a_1 g(Y;Z)X + a_2 g(X;Z)Y$
$+ a_3 g(X;Y)Z] \;\; (3f_2 + (2n \;\; 1)f_3)[a_1 \;\; (Y)(Z)X$
$+a_2 \;\; (X)(Z)Y + a_3 \;\; (X)(Y)Z]$ (5.2)
$+ \quad (2nf_1 + 3f_2 \quad f_3)[a_4 g(Y;Z)X + a_5 g(X;Z)Y$
$+ \quad a_6 g(X;Y)Z] \;\; (3f_2 + (2n \;\; 1)f_3)[a_4 \;\; (X)g(Y;Z)$

$+\quad a_5 \ (Y)g(X;Z) + a_6 \ (Z)g(X;Y)]$
$+\quad a_7(2n(2n+1)f_1 + 6nf_2 \quad 4nf_3)\,[g(Y;Z)X \ \ g(X;Z)Y]:$
By virtue of (2.5) the above equation reduces to

$f_1 \, fg(Y;Z)X \ \ g(X;Z)Y \, g + f_2 \, fg(X; \ Z) \, Y \ \ g(Y; \ Z) \, X + 2g(X; \ Y) \, Zg$
(5.3)

$+f_3 f \ (X) \ (Z)Y \quad (Y) \ (Z)X + g(X;Z) \ (Y) \quad g(Y;Z) \ (X) \ g$
$= \quad (2nf_1 + 3f_2 \quad f_3)[ag(Y;Z)X$
$+ \quad a_2 g(X;Z)Y + a_3 g(X;Y)Z]$
$(3f_2 + (2n \ \ 1)f_3)[a_1 \ (Y) \ (Z)X$
$+ \quad a_2 \ (X) \ (Z)Y + a_3 \ (X) \ (Y)Z]$
$+ \quad (2nf_1 + 3f_2 \quad f_3)[a_4 g(Y;Z)X + a_5 g(X;Z)Y$
$+ \quad a_6 g(X;Y)Z] \ \ (3f_2 + (2n \ \ 1)f_3)[a_4 \ (X)g(Y;Z)$
$+ \quad a_5 \ (Y)g(X;Z) + a_6 \ (Z)g(X;Y)]$
$+ \quad a_7(2n(2n+1)f_1 + 6nf_2 \quad 4nf_3)[g(Y;Z)X \ \ g(X;Z)Y]:$

Now, replacing Z by Z in the above equation, we obtain

$2n(f_1 \quad f_3)((a_1 + 2a_4)g(Y; \ Z) \ (X) + (a_2 + 2a_5)g(X; \ Z) \ (Y))$
$+ (a_7(2n(2n+1)f_1 + 6nf_2 \quad 4nf_3) + a_0(f_1 \quad f_3)$ (5.4)
$(g(Y; \ Z) \ (X) \ \ g(X; \ Z) \ (Y)) = 0:$
Putting X= in the above equation, we get
$[2n(f_1 \quad f_3)(a_1 + 2a_4) + a_7 2n(2n+1)f_1$ (5.5)
$+6nf_2 \quad 4nf_3 + a_0(f_1 \quad f_3)]g(Y; \ Z) = 0:$
Since g(Y; Z)6= 0 in general we obtain
$[2n(a_1 + a_7) + 4n(a_4 + a_7) + a_0]f_1 + a_7 6nf_2$ (5.6)
$[2na_1 + 4n(a_4 + a_7) + a_0] \, f_3 = 0:$
Again replacing X by X in (5.3) and Y= we get
$[((2nf_1 + 3f_2 \quad f_3) \ \ (3f_2 + (2n \ \ 1)f_3))(a_2 + 2a_5)$
$a_7(2n(2n+1)f_1 + 6nf_2 \quad 4nf_3)$ (5.7)
$a_0 f_1 + a_0 f_3)]g(\ X;Z) = 0:$
Since g(X; Z) 6= 0, in general, we obtain
$[2n(a_2 \quad a_7(2n+1) + 4na_5 \quad a_0]f_1 \quad 6na_7 f_2$ (5.8)
$+[a_0 \quad 2na_2 + 4n(a_7 \quad a_5)]f_3 = 0:$
From (5.6) and (5.8) , we get
$f_1 = f_3:$ (5.9)
Using (5.9) in (5.6), we obtain

$$f_3 = \frac{3nf_2}{n \ \ 1}: \qquad (5.10)$$

Thus in view of (5.9) and (5.10) we have

$$f_1 = \frac{3nf_2}{n \ \ 1} = f_3: \qquad (5.11)$$

6. T flat generalized sasakian-space-forms

Theorem 6.1: If A (2n + 1)-dimensional generalized Sasakian-space- form is -T flat then it is -Einstein manifold, if a_4 6= 0:

Proof : Let us consider that a generalized Sasakian-space-form in -T flat, that is T (X; Y) = 0: Then in view of

(3.1) we have

$a_0R(X; Y\ ) = a_1S(Y; \ )X + a_2S(X; \ )Y$
$+a_3S(X;Y\ ) + a_4g(Y; \ )QX + a_5g(X; \ )QY$ (6.1)
$+a_6g(X; Y\ )Q + a_7r(g(Y; \ )X \ \ g(X; \ )Y\ )$:
By virtue of (2.9) and (2.12) the above equation reduces to
$(\ a_0(f_1 \ \ f_3) \ a_7r) ((Y\ ) X \ \ (X)Y\ ) = a_12n(f_1 \ \ f_3) (Y\ )X$
$+a_22n(f_1 \ \ f_3) (X)Y + a_3(2nf_1 + 3f_2 \ \ f_3)g(X; Y\ )$ (6.2)
$a_3(3f_2 + (2n \ 1)f_3) (X) (Y\ ) + a_4 \ (Y\ )QX$
$+a_5 \ (X)QY + a_62n(f_1 \ \ f_3)g(X; Y\ )$:
Which by putting Y = gives
$a_4QX =[\ a_0(f_1 \ \ f_3) \ a_12n(f_1 \ \ f_3) \ a_7r]X$
$[a_0(f_1 \ \ f_3) + a_7r \ a_22n(f_1 \ \ f_3) \ a_32n(f_1 \ \ f_3)$ (6.3)
$a_52n(f_1 \ \ f_3) \ a_62n(f_1 \ \ f_3)] (X)$:
Now, taking inner product of the above equation with U, we get
$a_4S(X; U) =[\ a_0(f_1 \ \ f_3) \ a_12n(f_1 \ \ f_3) \ a_7r]g(X; U)$
$[a_0(f_1 \ \ f_3) + a_7r \ a_22n(f_1 \ \ f_3) \ a_32n(f_1 \ \ f_3)$ (6.4)
$a_52n(f_1 \ \ f_3) \ a_62n(f_1 \ \ f_3)] (X) (U)$:
since a_4 6= 0, which shows that generalized Sasakian-space-form is an -Einstein manifold.

7. Generalized Sasakian-space-form Satisfying T:S = 0.

Theorem(7.1) : A (2n+1)-dimensional generalized Sasakian-space-form satisfying T:S = 0 is an -Einstein manifold if

$(a_0(f_1 \ \ f_3) + a_7r) \ a_12n(f_1 \ \ f_3) \ a_22n(f_1 \ \ f_3)$ 6= 0:

Proof : Let us consider generalized Sasakian-space-form M^{2n+1} satisfying T (; X):S = 0 In this case we can write

$$S (T (; X)Y; Z) + S (Y; T (; X)Z) = 0: \qquad (7.1)$$

In view of (3.5) the above equation reduces to
$(a_0 (f_1 \ \ f_3) + a_7r)[g(X; Y\ )S(; Z) \ \ (Y\ )S(X; Z)$
$+g(X; Z)S(; Y\ ) \ \ (Z)S(X; Y\ )]$ (7.2)
$+a_1[S(X; Y\ )S(; Z) + S(X; Z)S(; Y\ )]$
$+a_22n(f_1 \ \ f_3) [(Y\ )S(X; Z) + \ (Z)S(X; Y\ )]$
$+a_32n(f_1 \ \ f_3) [S(Y; Z) (X) + S(Z; Y\ ) (X)]$
$+a_42n(f_1 \ \ f_3) [g(X; Y\ )S(; Z) + g(X; Z)S(; Y\ )]$
$+a_5[(Y\ )S(QX; Z) + \ (Z)S(QX; Y\ )]$
$+a_6 \ (X)[S(QY\ ; Z) + S(QZ; Y\ ] = 0$:
Now, putting Z = in the above equation, we get
$[a_0 (f_1 \ \ f_3) + a_7r \ a_12n(f_1 \ \ f_3) \ a_22n(f_1 \ \ f_3)]S(X; Y\ )$
$= [2n(f_1 \ \ f_3)(a_0(f_1 \ \ f_3) + a_7r) + a_44n^2(f_1 \ \ f_3)^2]g(X; Y\ )$ (7.3)
$+[a_14n^2(f_1 \ \ f_3)^2 + a_24n^2(f_1 \ \ f_3)^2 + a_38n^2(f_1 \ \ f_3)^2$
$+a_44n^2(f_1 \ \ f_3)^2 + a_5(2n(f_1 \ \ f_3)$
$+f_3(1 \ 2n) \ 3f_2)2n(f_1 \ \ f_3)$
$+a_68n^2(f_1 \ \ f_3)^2] (X) (Y\ )$:
In view of (2.6) the above equation takes the form
1

$S(X; Y\ ) =$ $\overline{K}$ $[(2n(f_1 \ \ f_3)(a_0(f_1 \ \ f_3) + a_7r)$
$+ a_44n^2(f_1 \ \ f_3)^2]g(X; Y\ ) + [a_14n^2(f_1 \ \ f_3)^2$ (7.4)
$+ a_24n^2(f_1 \ \ f_3)^2 + a_38n^2(f_1 \ \ f_3)^2 + a_44n^2(f_1 \ \ f_3)^2$
$+ a_5(2n(f_1 f_3) + f_3(1 \ 2n) \ 3f_2) + a_68n^2(f_1 f_3)^2)] (X) (Y\ )$;
where $K= (a_0(f_1 f_3)+a_7r) \ a_12n(f_1 f_3) \ a_22n(f_1 f_3)$ 6= 0: This completes the proof.

8. Generalised ssakian-space form satisfying T.R=0.

Theorem (8.1): A (2n + 1)-dimensional generalized Sasakian-space-form sat-isfying T.R=0 is an -Einstein manifold , if ((2na_1 + a_5)f_3 (a_1 + a_5)f_1) 6= 0: Proof : Let us consider a generalized Sasakian-space-form satisfying

T (; X):R(Y; Z)U = 0: (8.1)

This can be written as

T (; X)R(Y; Z)U R(T (; X)Y; Z)U R(Y; T (; X)Z)U (8.2)
R(Y; Z)T (; X)U = 0:

which on using (3.5) takes the following form

[$f_1$fg(Z; U)T (; X)Y g(Y; U)T (; X)Z)g + $f_2$fg(Y; U)T (; X) Z
g(Z; U)T (; X) Y + 2g(Y; Z)T (; X) Ug
+ f_3f (Y) (U)(T (; X)Z)
(Z) (U)(T (; X)Y) + g(Y; U) (Z)T (; X)
g(Z; U) (Y)T (; X) g]
[(a_0(f_1 f_3) + a_7r)(g(X; Y)R(; Z)U
(Y)(R(X; Z)U) + a_1S(X; Y)(R(; Z)U) + 2n(f_1 f_3)a_2 (Y)(R(X; Z)U)
+ 2n(f_1 f_3)a_3 (X)(R(Y; Z)U) + a_4g(X; Y)2n(f_1 f_3)(R(; Z)U)
+ a_5 (Y)(R(QX; Z)U) + a_6 (X)R(QY; Z)U)] (8.3)
+ [a_0(f_1 f_3) + a_7r)(g(X; Z)(R(; Y)
(Z)(R(X; Y)U) + a_1S(X; Z)(R(; Y)U) + 2n(f_1 f_3)a_2 (Z)(R(X; Y)U)
+ 2n(f_1 f_3)a_3 (X)(R(Z; Y)U) + a_4g(X; Z)2n(f_1 f_3)(R(; Y)U)
+ a_5 (Z)(R(QX; Y)U) + a_6 (X)(R(QZ; Y)U)]
[(a_0(f_1 f_3) + a_7r)(g(X; U)(R(Y; Z))
(U)(R(Y; Z)X) + a_1S(X; U)R(Y; Z)
+$a_2$2n(f_1 f_3) (U)(R(Y; Z)X)
+2n(f_1 f_3)a_3 (X)(R(Y; Z)U)
+a_4g(X; U)2n(f_1 f_3)(R(Y; Z)) + a_5 (U)(R(Y; Z)QX
+a_6 (X)(RY; Z)QU)] = 0:

Now taking the inner product of the above equation with , we get

[$f_1$fg(Z; U) (T (; X)Y) g(Y; U) (T (; X)Z)g + $f_2$fg(Y; U) (T (; X) Z)
g(Z; U) (T (; X) Y) + 2g(Y; Z) (T (; X) U)g
+ f_3f (Y) (U) (T (; X)Z) (Z) (U) (T (; X)Y)
+ g(Y; U) (Z) (T (; X)) g(Z; U) (Y) (T (; X))g] (8.4)
+ [(a_0(f_1 f_3) + a_7r)((g(X; Y) (R(; Z)U)
(Y) (R(X; Z)U)) + (g(X; Z) (R(; Y)U)
(Z) (R(X; Y)U) (g(X; U) (R(Y; Z))
(U) (R(Y; Z)X))] + a_1(S(X; Y) (R(; Z)U) + S(X; Z) (R(; Y)U) S(X; U) (R(Y; Z)
)]
+2n(f_1 f_3)a_2[(Y) (R(X; Z)U) + (Z) (R(X; Y)U) (U) (R(Y; Z)X)] + $a_3$2n(f_1 f_3)[(R(Y; Z)U)
+(R(Z; Y)U) (R(Y; Z)U)] (X)
+$a_4$2n(f_1 f_3)[g(X; Y) (R(; Z)U)
+g(X; Z) (R(; Y)U) g(X; U) (R(Y; Z))]
+a_5[(Y) (R(QX; Z)U) + (Z) (R(QX; Y)U)
(U) (R(Y; Z)QX)] + a_6 (X)[(R(QY; Z)U) + (R(QZ; Y)U) (R(Y; Z)QU)] = 0:

Let fe_1; e_2; ::::::::; e_{2n+1}) be an orthonormal basis of TM. Putting Z = U = e_i in above equation, taking summation over i, 1 i 2n + 1 and noting that

2n+1
(e_i)T (; X; e_i) = [(a_0(f_1 f_3) + a_7r) + 2n(f_1 f_3)(a_1 + a_3 + a_4 + a_6)
i=1
a_5(3f_2 + (2n 1)f_3)] (Y) + [((a_0(f_1 f_3) + a_7r)) +2n(f_1 f_3)a_2 + a_5(2nf_1 + 3f_2 f_3)]Y;

we obtain

(2na_1 + a_5)f_3 (a_1 + a_5)f_1)S(X; Y)
= (f_1:f_3 f_3)^2[2n(a_0 + 2na_4)g(X; Y) 4n((a_0 + a_1 + a_4) (X) (Y)]; (8.5)

since ((2na_1 + a_5)f_3 (a_1 + a_5)f_1) 6= 0;which shows that $M^{2n+1}$is an -Einstein manifold. This completes the proof.

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