IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 3 Ver. I (May - Jun. 2015), PP 27-31
www.iosrjournals.org

Parameterization of design variables for tyre profile optimization

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Abstract: *The aim of the present paper is to find the design variables which influence the tyre profile design. The scope of the present work is to parameterize these design variables so that we can easily optimize tyre profile. The relation between derivative of an arc length and radius of curvature is derived to find arc length of tyre and radius of curvatures were compared.*
Keywords: *Derivative of arc length, radius of curvature, tyre*

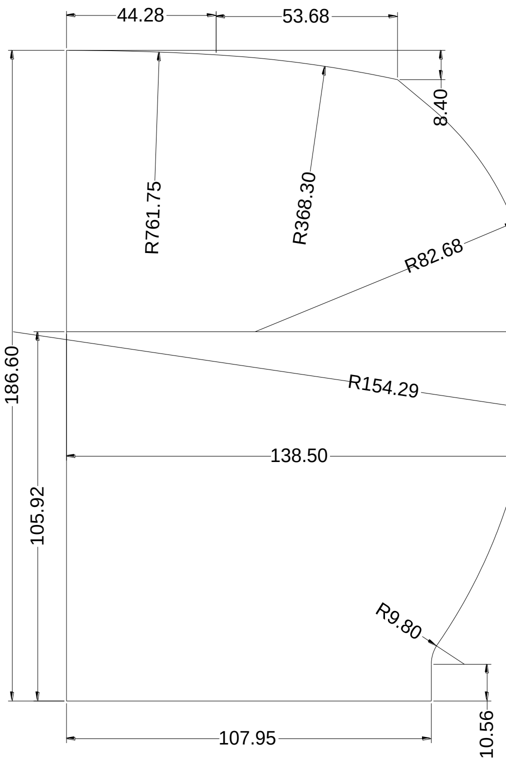
I. Introduction:

A tyre is a composite structure, i.e. an assembly of interdependent materials with very diverse properties whose construction requires a high degree of accuracy.
It is composed of the following materials:

- An airtight, synthetic rubber liner
- The carcass ply
- The filler
- Beads
- Crown plies
- The tread

In this paper, the relation between derivative of an arc length and radius of curvature is derived to find an arc length of tyre and radius of curvatures. Radius of curvature is calculated by using trial and error method,which helps to improve strength and life of tyre in tyre manufacture industries.

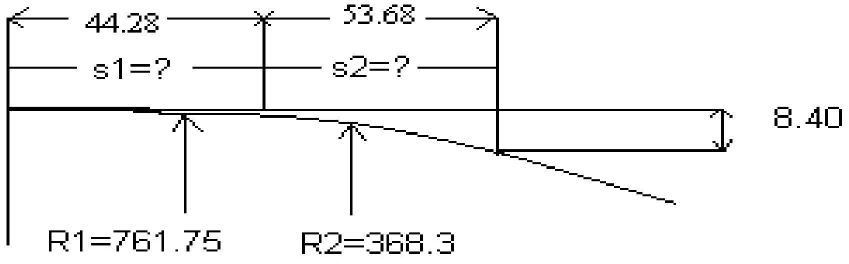
Tyre profile:



[The given data are in millimeter.]

II. Methods:

For finding arc length:



1. Equation of Derivative of an Arc length:-

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \qquad \text{……………} (1)$$

2. Equation of Radius of Curvature:-

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} \qquad \text{………………} (2)$$

Where R is radius of curvature
S is an arc length
X is a linear length

III. Results:

From eqn.1

$$\left(\frac{ds}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{\left(\frac{ds}{dx}\right)^2 - 1}$$

From eqn.2

$$R = \frac{\left\{\left(\frac{ds}{dx}\right)^2 - 1\right\}^{1/2} \left(\frac{ds}{dx}\right)^2}{\frac{d^2s}{dx^2}}$$

$$\left(\frac{d^2s}{dx^2}\right)^2 = \frac{1}{R^2}\left[\left(\frac{ds}{dx}\right)^6 - \left(\frac{ds}{dx}\right)^4\right]$$

By putting $\dfrac{ds}{dx} = t$ & $\dfrac{d^2s}{dx^2} = \dfrac{dt}{dx}$ we get,

$$\left(\frac{dt}{dx}\right)^2 = \frac{1}{R^2}\left[t^6 - t^4\right]$$

$$\left(\frac{dt}{dx}\right) = \frac{t^2}{R}\sqrt{\left[t^2 - 1\right]}$$

Integrating we get,

$$\frac{\sqrt{t^2 - 1}}{t} = \frac{x}{R} - c$$

Where c is an integral constant.

But $t = \dfrac{ds}{dx}$

Therefore we get,

$$\frac{\sqrt{\left(\frac{ds}{dx}\right)^2 - 1}}{\frac{ds}{dx}} = \frac{x}{R} - c$$

Simplifying above eqn. we get,

$$\left(\frac{ds}{dx}\right) = \frac{1}{\sqrt{\left[1 - \left(\frac{x}{R} - c\right)^2\right]}}$$

Integrating both sides we get,

$$S = R\, sin^{-1}\left(\frac{x}{R} - c\right) + c1$$

Now imposing boundary condition

$$S(0) = 0 \ \& \ S'(0) = 1$$

We get, $S = R\, sin^{-1}\left(\frac{x}{R}\right)$

(i) When x=44.28 & R=761.75

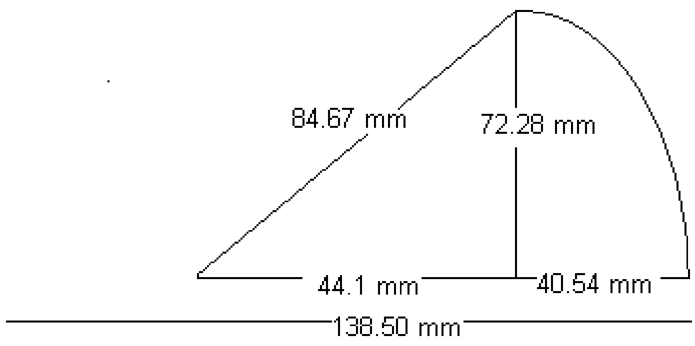
We get, **S_1=44.30**

(ii) When x=53.68 & R=368.30

We get, **S_2=53.87**

IV. Finding radius of curvature:

For curve 1:



Using trial & error method,

$$x^2 + a^2 = (y+a)^2$$

$$x^2 + a^2 = y^2 + 2ay + a^2$$

Where $x = 72.28$ and $y = 40.54$

$$\frac{x^2 - y^2}{2y} = a$$

$$x^2 + y^2 = 5224.39 + 1944.81$$

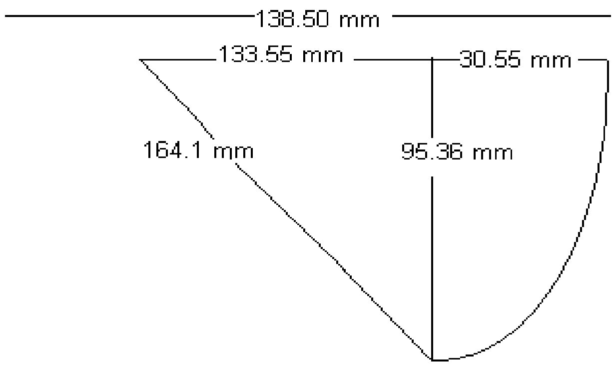
$a = 44.1$

$$\sqrt{(x^2 + a^2)} = 84.67$$

$$y + a = 40.54 + 44.1 = 84.64$$

Therefore, $x^2 + a^2 \approx (y+a)^2$

For curve2:



Where $x = 95.36$

$y = 30.55$

$a = 133.50$

$$x^2 + a^2 = 9093.52 + 17822.25$$

$$\sqrt{(x^2 + a^2)} = 164.06$$

$$y + a = 30.55 + 133.50 = 164.05$$

Therefore, $x^2 + a^2 \approx (y+a)^2$

V. Conclusion:

Tyre profile is used to improve strength and life of tyre. Here one tyre profile is taken and arc length and radius of curvature is calculated and compared with given data. The future work is planned to parameterize these equations & further developing this into a code. Extension of this approach to other parameters also.

Acknowledgement:

I want to express my profound gratitude to Dr. P.S. Kantawala and Dr.DhaneshP.Patel,Head of Applied Mathematics Department, Faculty of Technology and Engineering, M. S. University of Barodafor providing me necessary facilities during this research work.I am grateful to Mr. S.K.P. Amarnath,and Mr.P.Sankarganeshfrom **APOLLO TYRES Ltd**. (R and D Department)for their constant encouragement and support for this work. I extend my gratitude to Dr. A. Mohanraj for helping me in understanding the tyre concepts. Lastly I want to thank my colleague Dhara Mehta for her kind support.

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