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Probabilistic Analysis of General Manpower – SCBZ Machine System with Exponential Production, General Sales and General Recruitment

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Abstract: *Two Manpower planning models with different recruitment patterns are studied. During the operation time a machine produces random number of products. After the operation time, the sale time starts, and it has one among two distinct distributions depending on the magnitude of production is within or exceeding a random threshold magnitude. In this paper, the operation times have SCBZ property and the man power system, sales and recruitment have general distribution. Joint transforms of the variables, their means and covariance operation time and recruitment time with numerical results are presented.*

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I. Introduction

Nowadays it is found that labor has become a buyer market as well as seller market. Any company normally runs on commercial basis wishes to keep only the optimum level of any resources needed to meet company's requirement at any time during the course of the business and manpower is not an exception. In an organization, the total flow out of the Manpower System (MPS) is termed as shortage, The flow out of the MPS of an organization happens due to resignation, dismissal and death. The shortages that have occurred due to the outflow of manpower should be compensated by recruitment. But recruitment cannot be made frequently since it involves cost. Therefore, the MPS is allowed to undergo Cumulative Shortage Process (CSP). The breakdown point can be interpreted as that point at which immediate recruitment is required.

The shortage of MPS depends on individual propensity to leave the organization, which in turn depends on various factors as discussed before. Manpower Planning models have been studied by Grinold and Marshall [2]. For statistical approach one may refer to Bartholomew [1]. Lesson [6] has given methods to compute shortages (Resignations, Dismissals, Deaths). Markovian models are designed for shortage and promotion in MPS by Vassiliou [11]. Subramanian. [10] has made an attempt to provide optimal policy for recruitment, training, promotion and shortages in manpower planning models with special provisions such as time bound promotions, cost of training and voluntary retirement schemes. For other manpower models one may refer Setlhare [9]. For three characteristics system in manpower models one may refer to Mohan and Ramanarayanan [8].

Esary et al. [3] have discussed cumulative damage processes. Stochastic analysis of manpower levels affecting business with varying recruitment rate are presented by Hari Kumar, Sekar and Ramanarayanan [4]. Manpower System with Erlang departure and one by one recruitment is discussed by Hari Kumar [5]. For the study of Semi Markov Models in Manpower planning one may refer Meclean [7].

In this paper, the operation times have SCBZ property and all other distributions are general. Two models are treated. In model 1, the recruitments are done one by one. In model 2, they are done together when the operation time is more than a threshold or they are one by one when the operation time is less than the threshold. The joint Laplace transforms, the means, covariance of production and sale times and numerical results are presented.

II. Model 1

2.1 Assumptions:

1. Inter departure times of employees are independent and identically distributed (i.i.d) random variables with Cdf $F(x)$ and pdf $f(x)$. The manpower collapses with probability p when an employee leaves and with probability q it survives and continues operation where $p + q = 1$.
2. The machine attended by manpower has SCBZ failure property. In phase I it has exponential failure time distribution with parameter 'a'. If it does not fail in an exponential time with parameter 'c' it moves to phase 2 and the parameter 'a' changes to 'b'.

3. The man power machine system fails when one of them fails.
4. When the system fails, the vacancies caused by employees are filled up one by one with recruitment time V whose Cdf is V(y) and pdf is v(y).
5. When the machine fails with parameter 'a' before the change of parameter 'c' its repair time Cdf is $R_1(z)$ and pdf is $r_1(z)$. When it fails with parameter b, its repair time Cdf is $R_2(z)$ and pdf is $r_2(z)$.When the manpower machine system fails due to manpower failure, the machine is provided preventive maintenance which is a random variable R_3 with Cdf $R_3(z)$ and pdf $r_3(z)$ when the machine is in phase 2(after change of parameter) and no repair or maintenance is done when it is in phase 1(when no change of parameter has occurred).
6. When the man power machine system is in operation, products are produced for sales one by one. The inter production times of products have exponential distribution with parameter μ.
7. The sale times of products are i.i.d random variables G with Cdf G(w) and pdf g(w).
8. When the man power machine system fails, sales time, recruitment time and repair or maintenance begins.

2.2 Analysis:

To study the above model the probability density function and the distribution of SCBZ machine life time are required. Since the parameter 'a' changes to 'b' in an exponential time with parameter 'c' if the machine does not fail in phase 1, the life time pdf of machine satisfies the following equation.

$$h(x) = ae^{-ax}e^{-cx} + \int_0^x ce^{-cu}e^{-au}be^{-b(x-u)}du \qquad (1)$$

The first term of (1) of R.H.S is the pdf part that the machine fails in phase 1 before the change in parameter. The second term is the pdf part that the machine moves to phase2 at time u, no failure occurs in phase 1 and the machine fails at x in phase 2.

On simplification, the pdf of failure time of the machine is

$$h(x) = ae^{-x(a+c)} + \frac{cb}{c+a-b}(e^{-bx} - e^{-(c+a)x}) \qquad (2)$$

The first term of (2) is phase 1 failure density and the second term of (2) is phase 2 failure density. The survival probability function s(x) of the machine that it does not fail satisfies the following equation

$$s(x) = e^{-x(a+c)} + \int_0^x e^{-au}ce^{-cu}e^{-b(x-u)}du \qquad (3)$$

The first term of right side of equation (3) is the probability that the machine survives in phase 1 and second term is the probability that the machine survives in phase 2. On simplification it may be obtained as follows.

$$s(x) = e^{-x(a+c)} + \frac{c}{c+a-b}(e^{-bx} - e^{-(a+c)x}) \qquad (4)$$

Simplifying equations (2) and (4), it can be seen as

$$h(x) = \alpha(c+a)e^{-x(a+c)} + \beta be^{-bx} \qquad (5)$$

$$and\ \ s(x) = \alpha e^{-x(a+c)} + \beta e^{-bx} \qquad (6)$$

$$Here\ \alpha = \frac{a-b}{c+a-b}, \beta = \frac{c}{c+a-b}\ \ and\ \ \alpha+\beta = 1 \qquad (7)$$

The equations (2) and (4) present the failure or survival function in phase 1 and phase 2 explicitly for repair and maintenance study.

The joint probability density function of four variables, namely $(X,\widehat{V},\widehat{R},\widehat{S})$ is required for the study of the models

Here (i) X is the manpower-machine system operation time

(ii) $\widehat{V}$, the total recruitment time of employees

(iii) $\widehat{R}$ is the repair time or the maintenance time of the machine and

(iv) $\widehat{S}$ is the total sales time of the products.

When k_1 employees have left and k_2 products produced then

$$\hat{V} = V_1 + V_2 + \dots V_{k_1} \quad and \quad \hat{S} = G_1 + G_2 + \dots G_{k_2}$$

The repair time $\hat{R}$ is either R_1 or R_2 or R_3 or no repair time according as the machine fails in phase 1 or fails in phase 2 or is in phase 2 when the system fails or is in phase 1 when the system fails respectively. Here X is the minimum of the life times of both manpower and machine systems.

We find the pdf f(x,y,z,w) of $(X,\widehat{V},\widehat{R},\widehat{S})$ as follows.

$$f(x,y,z,w) = \left\{ \begin{array}{l} \left[ae^{-x(a+c)} r_1(z) + \dfrac{cb}{c+a-b}(e^{-bx} - e^{-(c+a)x}) r_2(z) \right] \left[\sum_{n=0}^{\infty} (F_n(x) - F_{n+1}(x)) q^n v_n(y) \right] + \\ \left[e^{-x(a+c)} + \dfrac{c}{c+a-b}(e^{-bx} - e^{-(c+a)x}) r_3(z) \right] \left[\sum_{n=1}^{\infty} f_n(x) q^{n-1} p v_n(y) \right] \end{array} \right\} \left[\sum_{k=0}^{\infty} e^{-\mu x} \frac{(\mu x)^k}{k!} g_k(w) \right] \quad (8)$$

Here $v_n(y)$ and $g_k(w)$ are n fold convolution of v(y) and k fold convolution of g(w) with itself. $F_n(x)$ the k-fold Stiltjes convolution of Cdf F(x) with itself. To write down the equation (8) the two cases namely (i) the machine fails when the manpower system is working and (ii) the manpower system fails when the machine is working are considered. They are given as two terms inside the flower bracket. The first term has two square brackets. The first bracket is presented using the terms of equation (2) multiplied by phase 1 and phase 2 failure repair densities when the machine fails. The second square bracket is the case presenting n departure of employees the manpower system survives and recruitments of them done one by one. The second term has two square brackets of which the first one is presented using the two terms of equation (4) when the machine survives calling only maintenance on phase 2 case. The second square bracket is presented considering the manpower system fails on the departure of the n th employee and n recruitments are done one by one. The last square bracket considers the case of k productions during the operation time and one by one sale of the products.

The quadruple Laplace transform of the joint pdf of $(X,\widehat{V},\widehat{R},\widehat{S})$ is

$$f^*(\xi,\eta,\varepsilon,\delta) = \int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty e^{-\xi x - \eta y - \varepsilon z - \delta w} f(x,y,z,w)\,dxdydzdw \quad (9)$$

Here * indicates Laplace transform. Because of the structure of the pdf given by (8), equation (9) reduces to single integral as follows.

$$f^*(\xi,\eta,\varepsilon,\delta) = \int_0^\infty e^{-\xi x} e^{-\mu x(1-g^*(\delta))} \left\{ \begin{array}{l} [ae^{-x(a+c)} r_1^*(\varepsilon) + \dfrac{cb}{c+a-b}(e^{-bx} - e^{-(c+a)x}) r_2^*(\varepsilon)] \\ [\sum_{n=0}^{\infty} (F_n(x) - F_{n+1}(x)) q^n v_n^*(\eta)] + \\ [e^{-x(a+c)} + \dfrac{c}{c+a-b}(e^{-bx} - e^{-(c+a)x}) r_3^*(\varepsilon)] \\ [\sum_{n=1}^{\infty} f_n(x) q^{n-1} p v_n^*(\eta)] \end{array} \right\} dx \tag{10}$$

This becomes

$$f^*(\xi,\eta,\varepsilon,\delta) = \frac{1}{(1-qv^*(\eta) f^*(\chi_1))} \left[\begin{array}{l} \dfrac{(1-f^*(\chi_1))}{\chi_1}(ar_1^*(\varepsilon) - \dfrac{cb}{c+a-b} r_2^*(\varepsilon)) \\ + pv^*(\eta) f^*(\chi_1)(1 - \dfrac{c}{c+a-b} r_3^*(\varepsilon)) \end{array} \right] + \frac{1}{(1-qv^*(\eta) f^*(\chi_2))} \left[\begin{array}{l} \dfrac{(1-f^*(\chi_2))}{\chi_2}(\dfrac{cb}{c+a-b}) r_2^*(\varepsilon) \\ + pv^*(\eta) f^*(\chi_2)(\dfrac{c}{c+a-b} r_3^*(\varepsilon)) \end{array} \right] \tag{11}$$

Here

$$\chi_1 = \xi + \mu(1-g^*(\delta)) + a + c \ \ and \ \ \chi_2 = \xi + \mu(1-g^*(\delta)) + b \tag{12}$$

$$Now \ \ f^*(\xi,0,0,0) = \frac{1}{(1-qf^*(\chi_3))} \left[\frac{(1-f^*(\chi_3))}{\chi_3} \alpha(a+c) + p\alpha f^*(\chi_3) \right] + \frac{1}{(1-qf^*(\chi_4))} \left[\frac{(1-f^*(\chi_4))}{\chi_4} \beta b + p\beta f^*(\chi_4) \right] \tag{13}$$

$$\chi_3 = \xi + a + c \ \ and \ \ \chi_4 = \xi + b \tag{14}$$

α and β are given in equation (7). This is the Laplace transform of operation time of the manpower –machine system. Now its mean is

$$E(X) = -\frac{\partial}{\partial \xi} f^*(\xi,0,0,0) \, at \xi = 0 \tag{15}$$

$$which \ is \ as \ seen \ as, E(X) = \frac{\alpha}{(a+c)} \frac{(1-f^*(a+c))}{(1-qf^*(a+c))} + \frac{\beta}{b} \frac{(1-f^*(b))}{(1-qf^*(b))} \tag{16}$$

Now the Laplace transform of repair time pdf $\hat{R}$ is given by

$$f^*(0,0,\varepsilon,0)=\frac{1}{(1-qf^*(a+c))}\frac{(1-f^*(a+c))}{(a+c)}(ar_1^*(\varepsilon)-\frac{cb}{c+a-b}r_2^*(\varepsilon))$$
$$+\frac{pf^*(a+c)}{(1-qf^*(a+c))}(1-\frac{c}{c+a-b}r_3^*(\varepsilon))+\frac{(1-f^*(b)}{(1-qf^*(b))}\frac{c}{c+a-b}r_2^*(\varepsilon)$$
$$+\frac{pf^*(b)}{(1-qf^*(b))}(\frac{c}{c+a-b})r_3^*(\varepsilon) \qquad (17)$$

$$E(\hat{R})=-\frac{\partial}{\partial\varepsilon}f^*(0,0,\varepsilon,0)\, at\, \varepsilon=0, is\, given\, by$$

$$E(\hat{R})=\frac{(1-f^*(a+c))}{(a+c)(1-qf^*(a+c))}(aE(R_1)-\beta bE(R_2))$$
$$-\frac{pf^*(a+c)}{(1-qf^*(a+c))}\beta E(R_3)+\frac{(1-f^*(b)}{(1-qf^*(b))}\beta E(R_2)+\frac{pf^*(b)}{(1-qf^*(b))}\beta E(R_3) \qquad (18)$$

Now the Laplace transform of $\hat{S}$ is given by

$$f^*(0,0,0,\delta)=\frac{1}{(1-qf^*(\chi_5))}\left[\frac{(1-f^*(\chi_5))}{\chi_5}\alpha(a+c)+p\alpha f^*(\chi_5)\right]+$$
$$\frac{1}{(1-qf^*(\chi_6))}\left[\frac{(1-f^*(\chi_6))}{\chi_6}\beta b+p\beta f^*(\chi_6)\right] \qquad (19)$$

$$Here\, \chi_5=\mu(1-g^*(\delta))+a+c \quad and \quad \chi_6=\mu(1-g^*(\delta))+b \qquad (20)$$

$$E(\hat{S})=-\frac{\partial}{\partial\delta}f^*(0,0,0,\delta)\,|\,\delta=0, gives$$

$$E(\hat{S})=\frac{\alpha\mu}{(a+c)}E(G)\frac{(1-f^*(a+c))}{(1-qf^*(a+c))}+\frac{\beta\mu}{b}E(G)\frac{(1-f^*(b))}{(1-qf^*(b))} \qquad (21)$$

The Laplace transform joint pdf of $(X\hat{V})$ can be seen as

$$f^*(\xi,\eta,0,0)=\frac{1}{(1-qv^*(\eta)f^*(\chi_3))}\left[\frac{(1-f^*(\chi_3))}{\chi_3}\alpha(a+c)+p\alpha v^*(\eta)f^*(\chi_3)\right]+$$
$$\frac{1}{(1-qv^*(\eta)f^*(\chi_4))}\left[\frac{(1-f^*(\chi_4))}{\chi_4}\beta b+p\beta v^*(\eta)f^*(\chi_4)\right] \qquad (22)$$

Here χ_3 and χ_4 are as given as in equation (15).

$$E(\hat{V})=-\frac{\partial}{\partial\eta}f^*(\xi,\eta,0,0)\,|\,\xi=\eta=0.$$

$$This\, gives\, E(\hat{V})=E(V)\left[\frac{\alpha f^*(a+c)}{(1-qf^*(a+c))}+\frac{\beta f^*(b)}{(1-qf^*(b))}\right] \qquad (23)$$

The product moment $E(X\hat{V})$ is given by $E(X\hat{V})=\dfrac{\partial^2}{\partial\xi\partial\eta}f^*(\xi,\eta,0,0)\ at\xi=\eta=0$

This gives after simplification

$$E(X\hat{V}) = E(V)\left[\begin{array}{l}\dfrac{\alpha q f^*(a+c)}{(a+c)}\dfrac{(1-f^*(a+c)) - \alpha(a+c) f^{*'}(a+c)}{(1-qf^*(a+c))^2} + \\ \dfrac{\beta q f^*(b)(1-f^*(b)) - b\beta f^{*'}(b)}{b(1-qf^*(b))^2}\end{array}\right] \tag{24}$$

Now $Cov(X\hat{V}) = E(X\hat{V}) - E(X)E(\hat{V})$ can be written using the equations (24), (23) & (16).

III. Model 2

In this section the previous model 1 with all assumptions (1),(2),(3),(5),(6),(7) and (8) except the assumptions (4) for manpower recruitment pattern is treated.

3.1 Assumptions For Manpower Recruitment

(4.1) When the operation time X is more than a threshold time U, the recruitments are done all together. It is assigned to an agent whose service time V_1 to fill up all vacancies has Cdf $V_1(y)$ and pdf $v_1(y)$.
(4,2) When the operation time X is less than a threshold time U, the recruitments are done one by one and recruitment time V_2 for each has Cdf $V_2(y)$ and pdf $v_2(y)$.
(4.3) The threshold U has exponential distribution with parameter θ.

3.2 Analysis:

Using the arguments given for model 1 the joint pdf of $(X,\hat{V},\hat{R},\hat{S})$ (operation time, recruitment time of the employees, repair time of the machine, sales time) may be obtained as follows.

$$f(x,y,z,w) = \left\{\begin{array}{l}[ae^{-x(a+c)}r_1(z) + \dfrac{cb}{c+a-b}(e^{-bx} - e^{-(c+a)x})r_2(z)] \\ [\sum\limits_{n=0}^{\infty}(F_n(x) - F_{n+1}(x))q^n((1-e^{-\theta x})v_1(y) + e^{-\theta x}v_{2,h}(y))] + \\ [e^{-x(a+c)} + \dfrac{c}{c+a-b}(e^{-bx} - e^{-(c+a)x})r_3(z)] \\ [\sum\limits_{n=1}^{\infty} f_n(x)q^{n-1}p((1-e^{-\theta x})v_1(y) + e^{-\theta x}v_{2,h}(y))]\end{array}\right\}$$

$$[\sum_{k=0}^{\infty} e^{-\mu x}\frac{(\mu x)^k}{k!}g_k(w)] \tag{25}$$

Same arguments given for model 1 may be used to write down for all terms except the second square appearing in the first term and the second square bracket appearing in the second term inside the flower bracket. The pdf $v_n(y)$ appearing in two places in the equation (8) is replaced by $(1-e^{-\theta x})v_1(y)+e^{-\theta x}v_{2,h}(y)$ indicating the cases namely the operation time is greater than the threshold X>U and the operation time is less than the threshold X<U. The function $v_{2,h}(y)$ is the h fold convolution of $v_2(y)$ with itself. The Laplace transform of the pdf of four variables $(X,\hat{V},\hat{R},\hat{S})$ is given by

$$f^*(\xi,\eta,\varepsilon,\delta) = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-\xi x - \eta y - \varepsilon z - \delta w} f(x,y,z,w)\,dxdydzdw \tag{26}$$

Equation (26) using equation (25) reduces to a single integral given in equation (10) with the replacement of $v^{*n}(\eta)$ by $(1-e^{-\theta x})v_1^*(\eta)+e^{-\theta x}v_2^*(\eta)$ in two places. This gives

$$f^*(\xi,\eta,\varepsilon,\delta)$$

$$=\int_0^\infty e^{-\xi x}e^{-\mu x(1-g^*(\delta))}\left\{\begin{array}{l}[ae^{-x(a+c)}r_1^*(\varepsilon)+\dfrac{cb}{c+a-b}(e^{-bx}-e^{-(c+a)x})r_2^*(\varepsilon)]\\ [\sum_{n=0}^{\infty}(F_n(x)-F_{n+1}(x))q^n((1-e^{-\theta x})v_1^*(\eta)+e^{-\theta x}v_2^{*n}(\eta))]+\\ [e^{-x(a+c)}+\dfrac{c}{c+a-b}(e^{-bx}-e^{-(c+a)x})r_3^*(\varepsilon)]\\ [\sum_{n=1}^{\infty}f_n(x)q^{n-1}p((1-e^{-\theta x})v_1^*(\eta)+e^{-\theta x}v_2^{*n}(\eta))]\end{array}\right\}dx \qquad (27)$$

$$f^*(\xi,\eta,\varepsilon,\delta)=\left(ar_1^*(\varepsilon)-\frac{cb}{c+a-b}r_2^*(\varepsilon)\right)\left[\begin{array}{l}\dfrac{(1-f^*(\chi_1))v_1^*(\eta)}{\chi_1(1-qf^*(\chi_1))}-\dfrac{(1-f^*(\chi_1+\theta))v_1^*(\eta)}{(\chi_1+\theta)(1-qf^*(\chi_1+\theta))}\\ +\dfrac{(1-f^*(\chi_1+\theta))}{(\chi_1+\theta)(1-qv_2^*(\eta)f^*(\chi_1+\theta))}\end{array}\right]$$

$$+p\left(1-\frac{c}{c+a-b}r_3^*(\varepsilon)\right)\left[\frac{f^*(\chi_1)v_1^*(\eta)}{(1-qf^*(\chi_1))}-\frac{f^*(\chi_1+\theta)v_1^*(\eta)}{(1-qf^*(\chi_1+\theta))}+\frac{v_2^*(\eta)f^*(\chi_1+\theta)}{(1-qv_2^*(\eta)f^*(\chi_1+\theta))}\right]$$

$$+\frac{cb}{(c+a-b)}r_2^*(\varepsilon)\left[\frac{(1-f^*(\chi_2))v_1^*(\eta)}{\chi_2(1-qf^*(\chi_2))}-\frac{(1-f^*(\chi_2+\theta))v_1^*(\eta)}{(\chi_2+\theta)(1-qf^*(\chi_2+\theta))}+\frac{(1-f^*(\chi_2+\theta))v_2^*(\eta)}{(\chi_2+\theta)(1-qv_2^*(\eta)f^*(\chi_2+\theta))}\right]$$

$$+p(\frac{c}{c+a-b})r_3^*(\varepsilon)\left[\frac{f^*(\chi_2)v_1^*(\eta)}{(1-qf^*(\chi_2))}-\frac{f^*(\chi_2+\theta)v_1^*(\eta)}{(1-qf^*(\chi_2+\theta))}+\frac{f^*(\chi_2+\theta)v_2^*(\eta)}{(1-qv_2^*(\eta)f^*(\chi_2+\theta))}\right] \qquad (28)$$

Since there is only change in the manpower recruitment pattern, to fill up the manpower loss, $E(X), E(\hat{R}) and E(\hat{S})$ *remain the same as that of model 1. The Laplace transform of* $(X,\hat{V})$ *is*

$$f^*(\xi,\eta,0,0)=$$

$$\alpha(a+c)\left[\frac{(1-f^*(\chi_3))v_1^*(\eta)}{\chi_3(1-qf^*(\chi_3))}-\frac{(1-f^*(\chi_3+\theta))v_1^*(\eta)}{(\chi_3+\theta)(1-qf^*(\chi_3+\theta))}+\frac{(1-f^*(\chi_3+\theta))}{(\chi_3+\theta)(1-qv_2^*(\eta)f^*(\chi_3+\theta))}\right]$$

$$+p\alpha\left[\frac{f^*(\chi_3)v_1^*(\eta)}{(1-qf^*(\chi_3))}-\frac{f^*(\chi_3+\theta)v_1^*(\eta)}{(1-qf^*(\chi_3+\theta))}+\frac{v_2^*(\eta)f^*(\chi_3+\theta)}{(1-qv_2^*(\eta)f^*(\chi_3+\theta))}\right]$$

$$+\beta b\left[\frac{(1-f^*(\chi_4))v_1^*(\eta)}{\chi_4(1-qf^*(\chi_4))}-\frac{(1-f^*(\chi_4+\theta))v_1^*(\eta)}{(\chi_4+\theta)(1-qf^*(\chi_4+\theta))}+\frac{(1-f^*(\chi_4+\theta))}{(\chi_4+\theta)(1-qv_2^*(\eta)f^*(\chi_4+\theta))}\right]$$

$$+p\beta\left[\frac{f^*(\chi_4)v_1^*(\eta)}{(1-qf^*(\chi_4))}-\frac{f^*(\chi_4+\theta)v_1^*(\eta)}{(1-qf^*(\chi_4+\theta))}+\frac{f^*(\chi_4+\theta)v_2^*(\eta)}{(1-qv_2^*(\eta)f^*(\chi_4+\theta))}\right] \qquad (29)$$

Here χ_3 and χ4 are as given as in equation (15). Using equation (29) we find the expected recruitment time $E(\hat{V})$ *as*

$$E(\hat{V})=-\frac{\partial}{\partial\eta}f^*(\xi,\eta,0,0)\,at\,\xi=\eta=0,$$

$$E(\hat{V}) =$$
$$\alpha E(V_1)\left[\frac{(1-f^*(a+c))}{(1-qf^*(a+c))} - (\frac{a+c}{a+c+\theta})\frac{(1-f^*(a+c+\theta))}{(1-qf^*(a+c+\theta))} + \frac{pf^*(a+c)}{(1-qf^*(a+c))} - \frac{pf^*(a+c+\theta)}{1-qf^*(a+c+\theta)}\right]$$
$$+\alpha E(V_2)\left[\frac{q(a+c)}{(a+c+\theta)}\frac{(1-f^*(a+c+\theta))f^*(a+c+\theta)}{(1-qf^*(a+c+\theta))^2} + \frac{pf^*(a+c+\theta)}{(1-qf^*(a+c+\theta))^2}\right]$$
$$+\beta E(V_1)\left[\frac{(1-f^*(b))}{(1-qf^*(b))} - \frac{b}{b+\theta}\frac{(1-f^*(b+\theta))}{(1-qf^*(b+\theta))} + \frac{pf^*(b)}{(1-qf^*(b))} - \frac{pf^*(b+\theta)}{(1-qf^*(b+\theta))}\right]$$
$$+\beta E(V_2)\left[q(\frac{b}{b+\theta})\frac{(1-f^*(b+\theta))f^*(b+\theta)}{(1-qf^*(b+\theta))^2} + \frac{pf^*(b+\theta)}{(1-qf^*(b+\theta))^2}\right] \quad (30)$$

Now the product moment $E(X,\hat{V}) is given by$

$$E(X\hat{V}) = \frac{\partial^2}{\partial\xi\partial\eta} f^*(\xi,\eta,0,0) \ at\ \xi=\eta=0\ and$$

$$E(X\hat{V}) = \alpha E(V_1)[\frac{(1-f^*(a+c))}{(1-qf^*(a+c))} - \frac{(a+c)}{(a+c+\theta)^2}\frac{(1-f^*(a+c+\theta))}{(1-qf^*(a+c+\theta))}$$
$$+\frac{\theta pf^{*'}(a+c+\theta)}{(a+c+\theta)(1-qf^*(a+c+\theta))^2}] + \alpha E(V_2)[\frac{q(a+c)}{(a+c+\theta)^2}\frac{(1-f^*(a+c+\theta))f^*(a+c+\theta)}{(1-qf^*(a+c+\theta))^2}$$
$$-\frac{f^{*'}(a+c+\theta)[(a+c)(1-qf^*(a+c+\theta))+\theta p(1+qf^*(a+c+\theta))]}{(a+c+\theta)(1-qf^*(a+c+\theta))^3}]$$
$$+\beta E(V_1)[\frac{(1-f^*(b)}{(1-qf^*(b))} - \frac{b}{(b+\theta)^2}\frac{(1-f^*(b+\theta)}{(1-qf^*(b+\theta))} + \frac{\theta pf^{*'}(b+\theta)}{(b+\theta)(1-qf^*(b+\theta))^2}]$$
$$+\beta E(V_2)[\frac{qb}{(b+\theta)^2}\frac{(1-f^*(b+\theta))f^*(b+\theta)}{(1-qf^*(b+\theta))^2} - \frac{f^{*'}(b+\theta)[b(1-qf^*(b+\theta))+\theta p(1+qf^*(b+\theta))]}{(b+\theta)(1-qf^*(b+\theta))^3}] \quad (31)$$

Since $Cov(X\hat{V}) = E(X\hat{V}) - E(X)E(\hat{V})$ Co-variance may be written using equations (30), (31) and (16).

IV. Numerical Examples For Model 1 & 2

To enlighten the usefulness of the results obtained so far numerical examples are presented. Here the two models I and II are treated together. Since there is only change in the manpower recruitment pattern, to fill up the manpower loss, the expected operation time E(X), expected repair time and expected sales time coincide in the two models.

a=15, b=10, c=5, p=0.4, q=0.6, $E(R_1)=15$, $E(R_2)=10$, $E(R_3)=10$, $E(V_1)=4$, $E(V_2)=8$, E(G)=12
$\theta=1$, $\mu=5$, 10,15,20,25 and $\delta=2$, 4, 6,8,10.

Table 1: The effect of variation of μ and δ on E(X)

μ / δ	2	4	6	8	10
5	0.105587	0.094555	0.085816	0.078706	0.072797
10	0.105587	0.094555	0.085816	0.078706	0.072797
15	0.105587	0.094555	0.085816	0.078706	0.072797
20	0.105587	0.094555	0.085816	0.078706	0.072797
25	0.105587	0.094555	0.085816	0.078706	0.072797

Table 2: The effect of variation of μ and δ on $E(\hat{R})$

μ / δ	2	4	6	8	10
5	12.5969	12.21805	11.86131	11.52482	11.2069
10	12.5969	12.21805	11.86131	11.52482	11.2069
15	12.5969	12.21805	11.86131	11.52482	11.2069
20	12.5969	12.21805	11.86131	11.52482	11.2069
25	12.5969	12.21805	11.86131	11.52482	11.2069

Table 3: The effect of variation of μ and δ on $E(\hat{S})$

μ / δ	2	4	6	8	10
5	6.335204	5.673274	5.148945	4.722366	4.367816
10	12.67041	11.34655	10.29789	9.444733	8.735632
15	19.00561	17.01982	15.44683	14.16710	13.10345
20	25.34082	22.69310	20.59578	18.88947	17.47126
25	31.67602	28.36637	25.74472	23.61183	21.83908

Table 4: The effect of variation of μ and δ on $E(\hat{V})$

μ / δ	2	4	6	8	10
5	1.733233	2.841788	3.788796	4.610674	5.333333
10	1.733233	2.841788	3.788796	4.610674	5.333333
15	1.733233	2.841788	3.788796	4.610674	5.333333
20	1.733233	2.841788	3.788796	4.610674	5.333333
25	1.733233	2.841788	3.788796	4.610674	5.333333

Fig 1: The effect of variation of μ and δ on E(X)

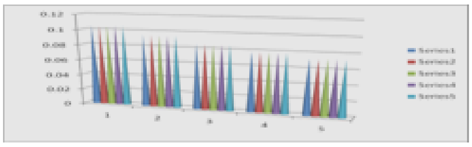


Figure 2: The effect of variation of μ and δ on $E(\hat{R})$

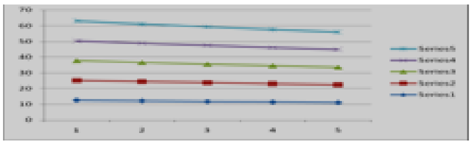


Figure 3: The effect of variation of μ and δ on $E(\hat{S})$

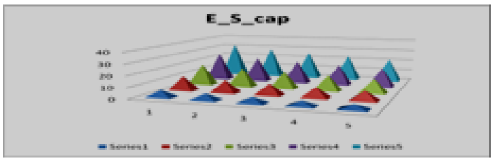


Figure 4: The effect of variation of μ and δ on $E(\hat{V})$

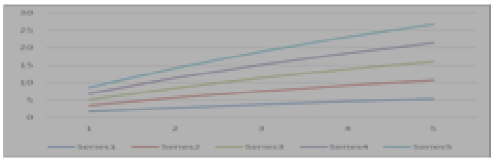


Table 5: The effect of variation of μ and δ on $E(\hat{XV})$

μ / δ	2	4	6	8	10
5	3.603023	3.528649	3.440455	3.34676	3.252138
10	3.603023	3.528649	3.440455	3.34676	3.252138
15	3.603023	3.528649	3.440455	3.34676	3.252138
20	3.603023	3.528649	3.440455	3.34676	3.252138
25	3.603023	3.528649	3.440455	3.34676	3.252138

Table 6: The effect of variation of μ and δ on $COV(\hat{XV})$

μ / δ	2	4	6	8	10
5	3.420017	3.259945	3.115316	2.983872	2.863888
10	3.420017	3.259945	3.115316	2.983872	2.863888
15	3.420017	3.259945	3.115316	2.983872	2.863888
20	3.420017	3.259945	3.115316	2.983872	2.863888
25	3.420017	3.259945	3.115316	2.983872	2.863888

Figure 5: The effect of variation of μ and δ on $E(\hat{XV})$

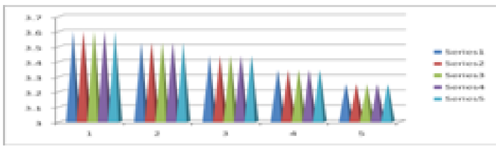
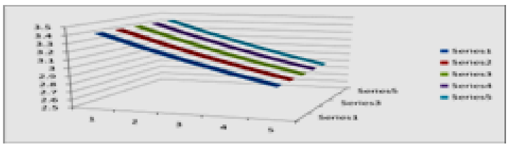


Figure 6: The effect of variation of μ and δ on $COV(\hat{XV})$



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