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Unsteady Magnetohydrodynamic Stokes Flow of Viscous Fluid with Radiative Heat Transfer

K.Sharmilaa[*1], C.Gayathri[2]
[*1]*Department of Mathematics, P.S.G.R Krishnammal College for Women, Coimbatore, Tamilnadu, India*
[2]*Research Scholar, P.S.G.R Krishnammal College for Women, Coimbatore, Tamilnadu, India*

Abstract: *The unsteady magnetohydrodynamic flow of an electrically conducting viscous, incompressible fluid between two parallel porous plates of a channel in the presence of a transverse magnetic field when the fluid is being withdrawn through both the walls of the channel at the same rate is discussed. An exact solution is obtained for all values of R (Suction Reynolds number) and M (Hartmann number). Expressions for the velocity components and the pressure are obtained. The graphs of axial and radial velocity profiles have been drawn for different values of M.*

Key words: *Unsteady flow, parallel porous plates, transverse magnetic field, suction Reynolds number, Hartmann number, pressure drop.*

I. Introduction

The unsteady magnetohydrodynamic flow in a channel is a classical problem whose solution has many applications in magnetohydrodynamic power generators, cooling system, aerodynamics heating, polymer technology, petroleum industry, centrifugal separation of matter from fluid, purification of crude oil and fluid droplets sprays. Hassanien and Mansour (1990) discussed the Unsteady magnetic flow through a porous medium between two infinite parallel plates. Bagchi (1996) studied the problem of Unsteady flow of viscoelastic Maxwell fluid with transient pressure gradient through a rectangular channel.Attia and Kotb (1996) studied the Steady fully developed MHD flow and heat transfer between two parallel plates with temperature dependent viscosity. Attia (1999) extended the problem to the transient state. Ezzat et al. (1999) studied the problem of micropolar Magnetohydrodynamic boundary layer flow. Aboul-Hassan and Attia (2002) discussed the Flow of a conducting viscoelastic fluid between two horizontal porous plates in the presence of a transverse magnetic field. Nabil et al. (2003) studied the MHD flow of non-newtonian visco-elastic fluid through a porous medium near an accelerated plate. Attia (2004) has considered the Unsteady Hartmann flow with heat transfer of a visco-elastic fluid considering the Hall effect. Hayat et al. (2004) studied the Hall effects on the unsteady hydromagnetic oscillatory flow of a second grade fluid. Krishnambal and Ganesh (2004) discussed the Unsteady stokes flow of viscous fluid between two parallel porous plates. Attia (2005) studied the Unsteady laminar flow of an incompressible viscous fluid and heat transfer between two parallel plates in the presence of a uniform suction and injection considering variable properties. Attia (2005) studied the Unsteady flow of a dusty conducting fluid between parallel porous plates with temperature dependent viscosity. Krishnambal and Ganesh (2007) discussed the Unsteady magnetohydrodynamic stokes flow of viscous fluid between two parallel porous plates. Sharma and singh(2009) is investigated the effect of variable thermal conductivity and heat source/sink on flow of a viscous incompressible electrically conducting fluid in the presence of uniform transverse magnetic fluid and variable free stream near a stagnation point on a non conducting stretching sheet. Malashetty and Umavathi (1997) studied two-phase MHD flow and heat transfer in an inclined channel in the presence of buoyancy effects for the situation where only one of the phases is electrically conducting. Malashetty et al. (2000, 2001) analyzed the problem of fully developed two fluid magnetohydrodynamic flows with and without applied electric field in an inclined channel. Felix Ilesanmi Alao1, Samson Babatunde Folarin(2013) investigated the similarity solution of the influence of the thermal radiation and heat transfer on steady compressible boundary layer flow. The objective of this study is to analyse the Unsteady Magnetohydrodynamic Stokes flow of viscous fluid between two parallel porous plates when the fluid is being withdrawn through both the walls of the channel at the same rate. The problem is reduced to a third order nonlinear differential equation which depends on a Suction. Reynolds number R and a Hartmann number M for which an exact solution is obtained.

II. Mathematical Formulation

The unsteady laminar flow of an incompressible viscous fluid in a channel is considered in the presence of a transverse magnetic field H_0 applied perpendicular to the walls. Also the effect of heat transfer is considered. The origin is taken at the centre of the channel. Let x and y be the coordinate axes parallel and perpendicular to the channel walls. Let u and v be the velocity components in x and y directions respectively.

The equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

The equations of momentum are

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]-\sigma_e B_0^2 u \quad (2)$$

$$\rho\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial y}+\mu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right] \quad (3)$$

$$\rho C_p\frac{\partial T}{\partial t}=K\frac{\partial^2 T}{\partial y^2}-\frac{\partial q}{\partial y} \quad (4)$$

The boundary conditions are

$$y=0: \quad u=0, v=v_0, T=T_w$$

$$y\to\infty: \quad u=0, v=v_0, T=T_0+(T_w-T_0)\cos\omega t \quad (5)$$

Method of solution

Let

$$u=u(x,y)e^{i\omega t},$$

$$v=v(x,y)e^{i\omega t}, \quad (6)$$

$$p=p(x,y,)e^{i\omega t}$$

The radiative heat flux is given by

$$\frac{\partial q}{\partial y}=4\beta^2(T_o-T)$$

The equations (2)-(4) becomes,

$$\rho i\omega u=-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]-\sigma_e B_0^2 u \quad (7)$$

$$\rho i\omega v=-\frac{\partial p}{\partial y}+\mu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right] \quad (8)$$

$$\rho C_p\frac{\partial T}{\partial t}=K\frac{\partial^2 T}{\partial y^2}-4\beta^2(T_o-T) \quad (9)$$

Let ψ be the stream function such that

$$u=\frac{\partial\psi}{\partial y} \qquad v=-\frac{\partial\psi}{\partial x}$$

The equation of continuity can be satisfied by a stream function of the form

$$\psi=[u(0)+v_0 x]f(y)$$

The velocity components can be written as

$$u=[u(0)-v_0 x]f'(y) \quad v=v_0 f(y)$$

Equations (7) and (8) are reduced as

$$-\frac{\partial p}{\partial x}=[u(0)-v_0 x][\rho i\omega f'(y)-\mu f'''(y)+\sigma_e B_0^2 f'(y)] \quad (10)$$

$$-\frac{\partial p}{\partial y}=v_0[\rho i\omega f(y)-\mu f''(y)] \quad (11)$$

To eliminate p, differentiate (10) and (11) with respect to y and x respectively,

$$\frac{\partial^2 p}{\partial x\partial y}=0 \quad (12)$$

$$-\frac{\partial^2 p}{\partial x \partial y} = [u(0) - v_0 x]\frac{d}{dy}\left[i\omega\rho f'(y) - \mu f'''(y) + \sigma_e B_0^2 f'(y)\right] = 0 \tag{13}$$

From (12) and (13),

$$\frac{d}{dy}\left[i\omega\rho f'(y) - \mu f'''(y) + \sigma_e B_0^2 f'(y)\right] = 0 \tag{14}$$

Integrating (14) with respect to y,

$$i\omega\rho f'(y) - \mu f'''(y) + \sigma_e B_0^2 f'(y) = const$$

$$f'''(y) - \frac{i\omega\rho}{\mu} f'(y) - \frac{\sigma_e B_0^2}{\mu} f'(y) = const(K) \tag{15}$$

Now introducing dimensionless variables and parameters,

$$\alpha^2 = \frac{i\omega\rho}{\mu}; \theta = \frac{T - T_0}{T_W - T_0}; M = B_0\left(\frac{\sigma_e}{\mu}\right)^{\frac{1}{2}}; Pe = \frac{\rho C_p U a}{K}; N^2 = \frac{4\alpha^2 a^2}{K}$$

Equation (15) becomes,

$$f'''(y) - (\alpha^2 + M^2) f'(y) = K \tag{16}$$

Solving(16)

$$f(y) = A + Be^{\sqrt{\alpha^2+M^2}y} + Ce^{-\sqrt{\alpha^2+M^2}y} - \frac{Ky}{\alpha^2 + M^2} \tag{17}$$

$$f'(y) = B\sqrt{\alpha^2 + M^2}e^{\sqrt{\alpha^2+M^2}y} - C\sqrt{\alpha^2 + M^2}e^{-\sqrt{\alpha^2+M^2}y} - \frac{K}{\alpha^2 + M^2} \tag{18}$$

With boundary conditions

$$f(0) = 1, f(\infty) = 0; f'(0) = -1, f'(\infty) = 0$$

Solving(17) and(18) we get,

$$A = y - \frac{1}{\sqrt{\alpha^2 + M^2}}; \quad B = \frac{e^{-\sqrt{\alpha^2+M^2}y}}{\sqrt{\alpha^2 + M^2}};$$

$$C = \frac{\sqrt{\alpha^2 + M^2} - y\sqrt{\alpha^2 + M^2} - e^{-\sqrt{\alpha^2+M^2}y} + 1}{\sqrt{\alpha^2 + M^2}}; \quad K = \alpha^2 + M^2$$

Equations (17) and (18) becomes,

$$f(y) = \left[1 - e^{-\sqrt{\alpha^2+M^2}y} - (\alpha^2 + M^2)\right]y + \left[1 + \frac{1}{\sqrt{\alpha^2 + M^2}}\right]e^{-\sqrt{\alpha^2+M^2}y} \tag{19}$$

$$f'(y) = e^{-\sqrt{\alpha^2+M^2}y}\left[y\sqrt{\alpha^2 + M^2} - \sqrt{\alpha^2 + M^2} - 2\right] - (\alpha^2 + M^2) + 1 \tag{20}$$

Using dimensionless variable θ,

$$\frac{\partial T}{\partial t} = (T_W - T_0)\frac{\partial \theta}{\partial t}$$

$$\frac{\partial^2 T}{\partial y^2} = (T_W - T_0)\frac{\partial^2 \theta}{\partial y^2}$$

Equation (9) is reduced as

$$Pe\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2\theta \tag{21}$$

with boundary conditions,

$$y = 0 : u = 0, v = v_0, \theta = 1$$

$$y \to \infty : u = 0, v = 0, \theta = \cos \omega t$$

To solve (19), let

$$\theta = \theta_0(x, y)e^{i\omega t} + \theta_1(x, y)e^{-i\omega t}$$

Equation (19) becomes,

$$Pe\left[i\omega\theta_0(x,y)e^{i\omega t} - i\omega\theta_1(x,y)e^{-i\omega t}\right] = e^{i\omega t}\frac{\partial^2\theta_0}{\partial y^2} + e^{-i\omega t}\frac{\partial^2\theta_1}{\partial y^2} + N^2\left[\theta_0 e^{i\omega t} + \theta_1 e^{-i\omega t}\right] \tag{22}$$

Equating $e^{i\omega t}$ & $e^{-i\omega t}$,

$$\frac{\partial^2\theta_0}{\partial y^2} + N_1^2\theta_0 = 0 \quad where \quad N_1^2 = N^2 - Pei\omega \tag{23}$$

$$\frac{\partial^2\theta_1}{\partial y^2} + N_2^2\theta_1 = 0 \quad where \quad N_2^2 = N^2 + Pei\omega \tag{24}$$

with boundary conditions,

$$y = 0 : \theta_0 = \theta_1 = 0$$

$$y \to \infty : \theta_0 = \theta_1 = 1/2$$

Solving (21) and (22) we get,

$$\theta_0 = \frac{\sin N_1 y}{2\sin N_1} \qquad \theta_1 = \frac{\sin N_2 y}{2\sin N_2}$$

Finally the temperature distribution is,

$$\theta = \frac{1}{2}\left[\frac{\sin N_1 y}{\sin N_1}e^{i\omega t} + \frac{\sin N_2 y}{\sin N_2}e^{-i\omega t}\right] \tag{25}$$

From equation (6), (19) and (20) the velocity components can be obtained

$$u = [u(0) - v_0 x]\left[e^{-\sqrt{\alpha^2+M^2}\,y}\left\{y\sqrt{\alpha^2+M^2} - \sqrt{\alpha^2+M^2} - 2\right\} - \left(\alpha^2 + M^2\right) + 1\right]e^{i\omega t} \tag{26}$$

$$v = v_0\left\{y\left[1 - e^{-\sqrt{\alpha^2+M^2}\,y} - \left(\alpha^2 + M^2\right)\right] + e^{-\sqrt{\alpha^2+M^2}\,y}\left[1 + \frac{1}{\sqrt{\alpha^2+M^2}}\right]\right\}e^{i\omega t} \tag{27}$$

From (10), (11) and (15),

$$\frac{\partial p}{\partial x} = K\mu[u(0) - v_0 x]$$

$$\frac{\partial p}{\partial y} = v_0\left[\mu f''(y) - i\omega\rho f(y)\right]$$

But $dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy$

$$dp = \{K\mu[u(0) - v_0 x]\}dx + \{v_0[\mu f''(y) - i\omega\rho f(y)]\}dy$$

Integrating,

$$p(x,y) = K\mu\left[u(0)x - \frac{v_0}{2}x^2\right] + v_0 f'(y)\mu - i\omega\rho v_0\int_0^\infty f(y)dy + K_1$$

$$p(0,0) = v_0\mu f'(0) + K_1$$

Hence the pressure drop is given by,

$$p(x,y) - p(0,0) = K\mu\left[u(0)x - \frac{v_0}{2}x^2\right] - i\omega\rho v_0\int_0^\infty f(y)dy + v_0\mu\left[f'(y) - f'(0)\right] \tag{28}$$

III. Result And Discussion

The effect of Peclet number on temperature distribution is shown in figures 1-3 respectively. It is observed that the temperature increase with increasing Peclet number for N=1. But, it is observed that the temperature decrease and started increasing at a certain point with increasing Peclet number for N=2. And, it is observed that the temperature increase and started decreasing at a certain point with increasing Peclet number for N=3. The effect of Nusselt number on temperature distribution is shown in figures 4-6 respectively. It is observed that the temperature increase with increasing Nusselt number for Pe=1 and Pe=3 and decreases for Pe=2.

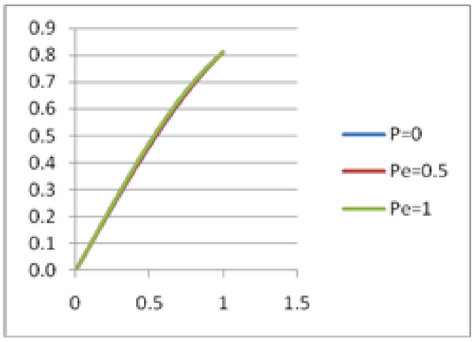

Fig. 1 Effect of Peclet Number on Temperature Distribution when N=1, ωt=1

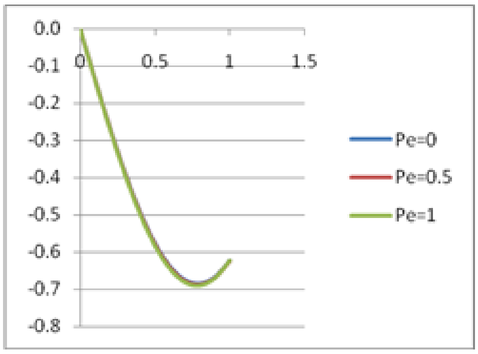

Fig. 2 Effect of Peclet Number on Temperature Distribution when N=2, ωt=2

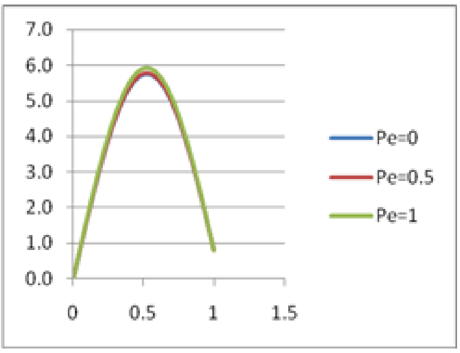

Fig. 3 Effect of Peclet Number on Temperature Distribution when N=3, ωt=1

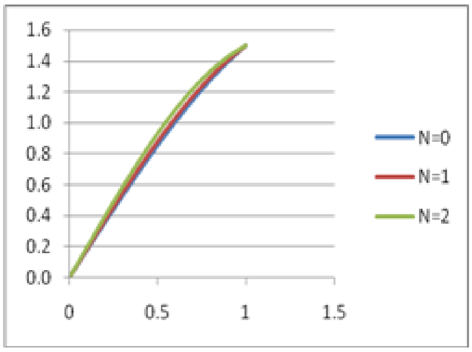

Fig. 4 Effect of Nusselt number on Temperature Distribution when Pe=1, ωt=1

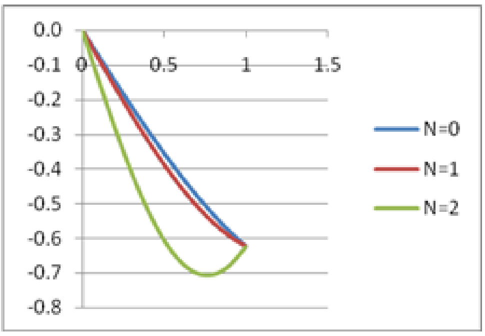

Fig. 5 Effect of Nusselt number on Temperature Distribution when Pe=2, ωt=2

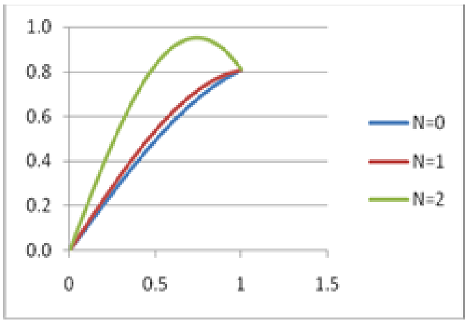

Fig. 6 Effect of Nusselt number on Temperature Distribution when Pe=3, ωt=1

IV. Conclusion

In the above analysis a class of solutions of the effect of radiative heat transfer on unsteady magnetohydrodynamic stokes flow when the fluid is being withdrawn through both the walls of the channel. Approximate solutions have been obtained for velocity profiles, temperature and pressure distribution. The conclusion of the study is as follows. It is observed that the temperature increase with increasing Peclet number for N=1. But, it is observed that the temperature decrease and started increasing at a certain point with increasing Peclet number for N=2. And, it is observed that the temperature increase and started decreasing at a certain point with increasing Peclet number for N=3. It is observed that the temperature increase with increasing Nusselt number for Pe=1 and Pe=2.

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