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Elzaki Transform and a Bulge Function on Volterra Integral Equations of the Second Kind

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Abstract: *The aim of this article is to find the solution of the Volterra integral equations of the second kind with a Bulge function by using Elzaki transform, inverse Elzaki transform and the convolution theorem..*
Keywords: *Elzaki transform, Volterra integral equations, Convolution.*

I. Introduction

ELzaki transform [12-15], whose fundamental properties are presented in this paper, is still not widely known, nor used, ELzaki transform may be used to solve problems without resorting to a new frequency domain. Integral equations [2] are significant in numerous applications. Problems in which integral equations are faced include radiative energy transfer and the oscillation of a string, membrane, or axle. Oscillation problems may also be solved as differential equations. Y. F. Mirzaee [1] introduced a numerical method for solving linear Volterra integral equations of the second kind based on the adaptive Simpson's quadrature method. M. M. Rahman et al, [3] solved numerically Volterra integral equations of second kind with regular kernels by well known Galerkin weighted residual method. They also derived a simple and efficient matrix formulation using Chebyshev polynomials as trial functions .

In the resent years, . P. Haarsa and S. Pothat [4] introduce a solution of A Bulge Function on Volterra Integral Equations of the Second Kind by Using the Laplace Transform

In this paper, we study Volterra integral equations of second kind with a bulge function. The solution is derived by using Elzaki transform, inverse Elzaki transform, the convolution theorem and the Taylor series expansion. The numerical solution is obtained by the modified Simpson's method. This method is illustrated by giving example of various types already described in [4].

Definition 1.
Elzaki Transform of a Given a function $f(t)$ defined for all $t \geq 0$, by,

$$E[f(t), v] = T(v) = v\int_0^t f(t)e^{-\frac{t}{v}}dt \quad , \quad v \in (k_1, k_2) \tag{1}$$

for all values of v for which the improper integral converges

Theorem 1. The Convolution Theorem
Let $f(t)$ and $g(t)$ be defined in A . having Elzaki transform $M(v)$ and $N(v)$ then the Elzaki transform of the convolution of $f(t)$ and $g(t)$ is,

$$E[(f*g)(t)] = \frac{1}{v} M(v)N(v) \tag{2}$$

Where

$$(f*g)(t) = \int_0^t f(x-\tau)g(\tau)d\tau$$

whenever the integral is defined.
The Volterra integral equations are a special kind of integral equations.
One type has the form

$$y(t) = f(t) + \int_0^t k(t-\tau)y(\tau)d\tau \tag{3}$$

where f and K are known and y is to be determined. For this paper, we study the case that f(t) is a bulge function [4]which is given by $e^{-\frac{(t-l)^2}{2}}$ where l is a positive constant.
The modified Simpson's method for solving integral $\int_{x_i}^{x_{i+2}} f(x)\,dx$ is as follows:

$$\int_{x_i}^{x_{i+2}} f(x)\,dx = \frac{h}{3}[f_i + 4f_{i+1} + f_{i+2}] + \frac{h^4}{180}[f_i''' - f_{i+2}'''] - \frac{h^7}{120}f^{(6)}(\varepsilon_i) \ ,$$
$$\varepsilon_i \in [x_i, x_{i+2}] \tag{4}$$

II. The Solution Of Volterra Integral Equation Of The Second Kind By Using Elzaki Transform

Theorem 2.

Elzaki transform of the bulge function $e^{-\frac{(t-l)^2}{2}}$ is expressed by.

$$E\left\{e^{-\frac{(t-l)^2}{2}}\right\} = e^{-\frac{l^2}{2}}[v^2 + lv^3 + (-1 + l^2)v^4 + (-3l + l^3)v^5] \quad (5)$$

Proof.

The Taylor series expansion e^x is of the form

$$e^{-\frac{(t-l)^2}{2}} = \sum_{n=0}^{\infty} \frac{x_n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \quad (6)$$

Therefore, by substituting equation (6) with $x = -\frac{(t-l)^2}{2}$, we obtain

$$e^{-\frac{(t-l)^2}{2}} = e^{-\frac{l^2}{2}} + e^{-\frac{l^2}{2}}lt + e^{-\frac{l^2}{2}}\left(-\frac{1}{2} + \frac{l^2}{2}\right)t^2 + e^{-\frac{l^2}{2}}\left(-\frac{l}{2} + \frac{l^3}{6}\right)t^3 \quad (7)$$

By taking Elzaki transform to equation (7) and using the fact that the Elzaki transform is linear, we derived,

$$E\left\{e^{-\frac{(t-l)^2}{2}}\right\} = e^{-\frac{l^2}{2}}[v^2 + lv^3 + (-1 + l^2)v^4 + (-3l + l^3)v^5] \quad (8)$$

Theorem 3.

The solution of the Volterra integral equation of the second kind

$$y(t) - \int_0^t y(\tau)\sin(t-\tau)\,d\tau = e^{-\frac{(t-l)^2}{2}}$$

is expressed as

$$y(t) = \frac{e^{-\frac{l^2}{2}}}{120}[120 + 120lt + 60l^2t^2 + 20(-2l + l^3)t^3 + 5(-l + l^2)t^4 + (-3l + l^2)t^5] \quad (9)$$

Proof.

By taking the Elzaki transform to the above equation, we have,

$$E\{y(t)\} - E\left\{\int_0^t y(\tau)\sin(t-\tau)\,d\tau\right\} = E\left\{e^{-\frac{(t-l)^2}{2}}\right\} \quad (10)$$

Applying the convolution theorem, it yields

$$E\{y(t)\} - E\{y(t) * \sin t\} = E\left\{e^{-\frac{(t-l)^2}{2}}\right\} \quad (11)$$

And again by applying the convolution theorem and theorem 2 to equation (11), we obtain:

$$E\{y(t)\}\left[1 - \frac{1}{v}\frac{v^3}{1+v^3}\right] = e^{-\frac{l^2}{2}}[v^2 + lv^3 + (-1 + l^2)v^4 + (-3l + l^3)v^5] \quad (12)$$

Or

$$E\{y(t)\} = e^{-\frac{l^2}{2}}[v^2 + lv^3 + (-1 + l^2)v^4 + (-3l + l^3)v^5] \times (1 + v^2) \quad (13)$$

We can next use the partial fraction method to equation (13), we have,

$$E\{y(t)\} = e^{-\frac{l^2}{2}}\begin{bmatrix} v^2 + lv^3 + l^2v^4 + (-2l + l^3)v^5 + (-1 + l^2)v^6 \\ +(-3l + l^3)v^7 \end{bmatrix} \quad (14)$$

Then, the inverse Elzaki transform can be employed to equation (14) to obtain,

$$y(t) = \frac{e^{-\frac{l^2}{2}}}{120}[120 + 120lt + 60l^2t^2 + 20(-2l + l^3)t^3 + 5(-l + l^2)t^4 + (-3l + l^2)t^5] \quad (15)$$

In theorem 3 we but $l = 1$ and $h = 0.1$ in the modified Simpson's method, we compare the exact solution as in equation (15) and the approximate solution as shown in table below.

Taple of exact and approximation solution

t	Approx solution	Exact solution
0	0.60653065971263	0.60653065972112
0.1	0.67011518945073	0.67011518946742
0.2	0.73915546247294	0.73915546247344
0.3	0.81302978485307	0.81302978485311
0.4	0.89108220143851	0.89108220143900
0.5	0.97266036523708	0.97266036523713
0.6	1.05700340680385	1.05700340681135
0.7	1.14332980362787	1.14332980362720
0.8	1.23077524951906	1.23077524951910
0.9	1.31839052399497	1.31839052399591
1	1.40512936166760	1.40512936167723

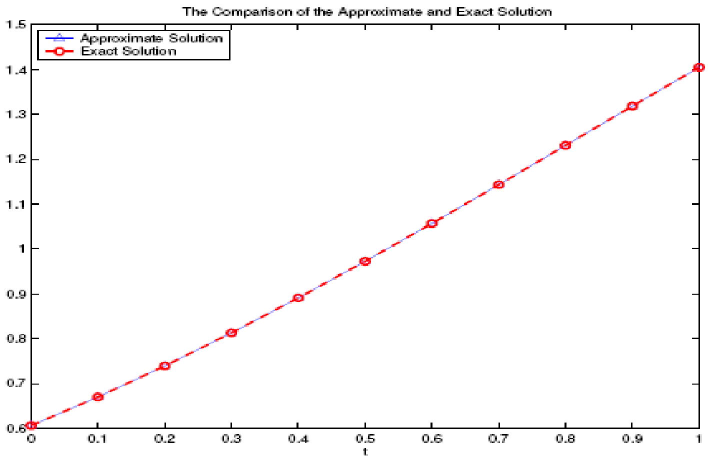

Figure 1: Exact solution and numerical solution of example for l = 1 and h = 0:1.

III. Conclusion

In this work, we studied the Volterra integral equations of the second kind with a Bulge function. To approach the exact solution, we used Elzaki transform, the inverse Elzaki transform, the Taylor series expansion and the convolution theorem. The modified Simpson'd method was employed for the numerical solution. Then, we compare the exact and numerical solutions in our example.

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