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On Steady State Response of a Magnetoelastic Half-Space to a Moving Normal Load

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Abstract: *A study has been made of the disturbances produced by a normal line load moving along the boundary surface of a perfectly conducting magnetoelastic semi-space. The displacement on the boundary has been obtained. The stress distribution shows how both the normal stress and tangential stress varies with depth, and with the increase in the magnetic field intensity.*
Key words: *Magnetoelastic, Moving Load, Perfect Conductor.*

I. Introduction

The response of solid materials to moving loads and sources are of practical interest in the fields of geophysics, seismology and also engineering. The problems of steady state or transient responses of a half-space to a moving point or line load have been discussed by many authors. Sneddon [1] investigated the problem of a line load moving with a constant speed on the boundary of an elastic half-space. This problem was later expanded by Chakraborty [2], to include the case of transverse isotropy. Mitra [3] studied the disturbance produced in an elastic half-space by a transient pressure applied on a part of the boundary surface.

Problems of loads moving subsonic, transonic and supersonic velocity in a half-space have been discussed among others, by Cole and Huth [4], Fung [5] and Fryba [6].

Payton[7] has considered the transient motion of an elastic half-space due to a moving line load, and also the problem of steady-state stresses in a transversely isotropic solid by a moving dislocation[8]. A study of steady-state response to moving loads in an elastic half-space with an overlying layer in infinite time was done by Nath and Sengupta [9].

The dynamical responses of perfectly conducting elastic media under the influence of a bias magnetic field has assumed interest in view of possible applications in geophysics, especially seismological wave motion in the earth's mantle and core. A study of steady-state response to moving loads in a magneto-elastic initially stressed conducting medium was done by Roy and Sengupta [10]. Chattopadhyay and Maugin [11] have discussed the propagation of surface SAWS due to a momentary point source in a magnetoelastic half-space. Effect of point source on horizontally polarised shear waves in a self-reinforced magnetoelastic layer over a self reinforced heterogeneous half-space was studied by Chattopadhyay, Gupta, Sing and Sahu [12]. Recently, Singh, Kumar and Chattopadhyay have also studied the effect of a smooth moving punch in an initially stressed monoclinic magnetoelastic strip[13].

In the present paper, the authors have formulated a problem of plane strain for a normal line load moving with constant velocity on the surface of a semi-infinite perfectly conducting elastic medium, under the influence of a bias magnetic field perpendicular to the plane of motion. Following the method illustrated by Sneddon [1],the problem has been solved for the stresses produced in the medium for case of subsonic velocity of load. Displacements on the surface has been found. Numerical study of the stress distribution with depth has been done for different intensity of the magnetic fields.

II. Formulation of the Problem

We consider a moving normal line load on the surface z = 0 of a perfectly conducting semi-infinite elastic medium occupying the regions ($-\infty< x <\infty, -\infty< y <\infty, z \geq 0$). The problem is formulated as one of plane strain, where the displacement components are taken as $\mathbf{u}=(u(x,z,t), 0, w(x,z,t))$. We assume the existence of a bias magnetic field $\mathbf{H} = (0, H_2, 0)$, where H_2 is a constant.

The basic equations of for magnetoelastic disturbances are given by

(I). Maxwell's equations of the electromagnetic field are

$$\nabla . \mathbf{D} = 0 , \nabla . \mathbf{B} = 0;$$
$$\nabla \times \mathbf{H} = \mathbf{J},\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}; \tag{1.1}$$

(displacement current is neglected)

where **D** is the electric displacement, **B** is the magnetic induction, **H** is the magnetic field, **E** is electric field and **J** is current density.

(II). The equations of motion are

$$\tau_{ij,j} + [\mathbf{J} \times \mathbf{B}]_i = \rho \frac{\partial^2 u_i}{\partial t^2},\quad i, j = 1, 2, 3. \tag{1.2}$$

where **J**×**B** is the Lorenz force due to the electromagnetic field and τ_{ij} being the usual elastic stress tensor in the medium .

(III). The constitutive equations are

$$\mathbf{B}=\mu\mathbf{H}; \quad \mathbf{D}=\varepsilon\mathbf{E}, \tag{1.3}$$

with Ohm's law as

$$\mathbf{J}=\sigma(\mathbf{E}+\dot{\mathbf{u}}\times\mathbf{B}), \qquad (1.4)$$

where σ is the electric conductivity, μ is the magnetic permeability, $\dot{\mathbf{u}}$ the particle velocity and ε the permittivity.

(IV). Elastic Stress-Strain relations

$$\tau_{ij} = \lambda\, u_{k,k}\, \delta_{i,j} + 2G\, e_{ij} \qquad (1.5)$$

where e_{ij} are the strain components and

$$2e_{ij} = u_{i,j} + u_{j,i}\ , \ i, j = 1, 2, 3. \qquad (1.6)$$

u_i being particle displacement and λ, G are Lame constants.

(V). The electromagnetic boundary conditions

$$\mathbf{n}.|\mathbf{B}|=0, \quad \mathbf{n}\times|\mathbf{H}|=0, \quad \mathbf{n}\cdot|\mathbf{E}+\dot{\mathbf{u}}\times\mathbf{B}|=0. \qquad (1.7)$$

where **n** is normal to the interfaces or surface z = constant and |.| discontinuity jump across the boundary of the vector within.

(VI). Stress continuity conditions across the boundaries z = constant are

$$\left(\tau_{3j} + \tau^{E}{}_{3j}\right)_{+} - \left(\tau_{3j} + \tau^{E}{}_{3j}\right)_{-} = 0,\ j = 1,2,3. \qquad (1.8)$$

where $\tau^{E}{}_{3j}$ are Maxwell's stress tensor components due to a magnetic field given by $\tau^{E}{}_{3j} = \mu(H_i h_j + H_j h_i - H_k h_k \delta_{i,j})$.

Due to motion there will be a perturbation in the magnetic field **H** which is taken correct to first order in small quantities h_i, as $\mathbf{H}+\mathbf{h}=(h_1, H_2+h_2, h_3)$.

For a perfectly conducting medium σ→∞, we have

$$\mathbf{E}+\dot{\mathbf{u}}\times\mathbf{B},$$

which leads to

$$\mathbf{h} = \left(0\,, -H_2\frac{\partial w}{\partial z} - H_2\frac{\partial u}{\partial x}\,, 0\right) \qquad (1.9)$$

The components of Lorenz force are

$$\left(-\mu H_2 h_{2_x}\,, 0\,, -\mu H_2 h_{2_z}\right) \qquad (1.10)$$

correct to first order in small quantities.

The equations of motion hence reduce to

$$(\alpha^2 + a^2)u_{xx} + b^2 u_{zz} + (\alpha^2 - b^2 + a^2)w_{xz} = u_{tt} \qquad (1.11)$$

$$(\alpha^2 - b^2 + a^2)u_{xz} + b^2 w_{xx} + (\alpha^2 + a^2)w_{zz} = w_{tt} \qquad (1.12)$$

where $$a^2 = \frac{\mu H_2{}^2}{\rho}\ ;\ b^2 = \frac{G}{\rho}\ ;\ \alpha^2 = \frac{\lambda+2G}{\rho} \qquad (1.13)$$

a is Alfven wave velocity, α is P-wave velocity and b is S-wave velocity.

2.1.Stress-boundary conditions

For a normal moving line load the boundary conditions are

$$\tau_{31} = 0 \quad \text{at } z = 0 \qquad (2.1)$$

$$\tau_{33} = -P\delta(x - ct) \quad \text{at } z = 0 \qquad (2.2)$$

where δ(x) stands for Dirac delta function of the argument x.

III. Steady-State Solution

The solution of the equations of motion are assumed in the form

$$u(x,z,t) = \int_0^\infty A\, e^{-kqz} \sin k(x - ct)dk \qquad (3.1)$$

$$w(x,z,t) = \int_0^\infty B\, e^{-kqz} \cos k(x - ct)dk \qquad (3.2)$$

where q is independent of k.

Substituting (3.1) and (3.2) into (1.11) and (1.12) we get the two following equations for A and B

$$A[c^2 - (\alpha^2 + a^2) + q^2 b^2] + B\left[\alpha^2 - b^2 + a^2\right]q = 0 \qquad (3.3)$$

$$-A\left[\alpha^2 - b^2 + a^2\right]q + B\left[c^2 - b^2 + (\alpha^2 + a^2)\, q^2\right] = 0 \qquad (3.4)$$

For a non-trivial solution for A and B, we must have

$$q^4 - \left(1 - \frac{c^2}{b^2} + 1 - \frac{c^2}{\alpha^2 + a^2}\right)q^2 + \left(1 - \frac{c^2}{b^2}\right)\left(1 - \frac{c^2}{\alpha^2 + a^2}\right) = 0 \qquad (3.5)$$

which is quadratic equation in q^2, with roots $q_1{}^2 = 1 - \frac{c^2}{b^2}$ and $q_2{}^2 = 1 - \frac{c^2}{\alpha^2 + a^2}$.

We consider the subsonic case only i.e. $q_1^2>0$ and $q_2^2>0$

In this case, q_1 and q_2 are real. So u(x,z,t) and w(x,z,t) can be written as

$$u(x,z,t) = \int_0^{\infty} (A_1\, e^{-kq_1 z} + A_2\, e^{-kq_2 z}) \sin k\,(x-ct) dk \qquad (3.6)$$

and

$$w(x,z,t) = \int_0^{\infty} (A_1 m_1\, e^{-kq_1 z} + A_2 m_2\, e^{-kq_2 z}) \cos k\,(x-ct) dk \qquad (3.7)$$

where $$m_{1,2} = -\frac{c^2-(\alpha^2+a^2)+q_{1,2}{}^2 b^2}{(\alpha^2-b^2+a^2)q_{1,2}} \qquad (3.8)$$

Writing $\delta(x-ct) = \frac{1}{\pi}\int_0^{\infty} \cos k\,(x-ct) dk$ and applying the boundary conditions (2.1)and (2.2) on (3.6) and (3.7) we have,

$$A_1(m_1 + q_1) + A_2(m_2 + q_2) = 0 \qquad (3.9)$$

$$A_1\left(\alpha^2 - 2b^2 - m_1 q_1 \alpha^2\right) + A_2\left(\alpha^2 - 2b^2 - m_2 q_2 \alpha^2\right) = -\frac{P}{\pi\rho k} \qquad (3.10)$$

Solving for A_1 and A_2 we get form (3.9) and (3.10)

$$A_1 = \frac{D}{\rho k} \qquad (3.11)$$

$$A_2 = -\frac{(m_1+q_1)}{(m_2+q_2)}.\frac{D}{\rho k} \qquad (3.12)$$

where $$D = -\frac{P}{\pi\left[(\alpha^2-2b^2-m_1 q_1 \alpha^2) - \frac{(m_1+q_1)}{(m_2+q_2)}.(\alpha^2-2b^2-m_2 q_2 \alpha^2)\right.} \qquad (3.13)$$

Substituting the values of A_1 and A_2 from (3.11) and (3.12) into (3.6) and (3.7) we have the displacement components

$$u(x,z,t) = \int_0^{\infty} \frac{D}{\rho}\left[\frac{e^{-kq_1 z}}{k} - \frac{(m_1+q_1)}{(m_2+q_2)}.\frac{e^{-kq_2 z}}{k}\right] \sin k\,(x-ct) dk \qquad (3.14)$$

and

$$w(x,z,t) = \int_0^{\infty} \frac{D}{\rho}\left[m_1.\frac{e^{-kq_1 z}}{k} - m_2.\frac{(m_1+q_1)}{(m_2+q_2)}.\frac{e^{-kq_2 z}}{k}\right] \cos k\,(x-ct) dk \qquad (3.15)$$

Here q_1, q_2, m_1, m_2 do not depend on k.

Using the value of u(x,z,t) and w(x,z,t), we have stress components at any point of the medium are given by

$$\tau_{31} = Db^2 \frac{(m_1+q_1)(x-ct)(q_1{}^2-q_2{}^2)z^2}{\{q_1{}^2 z^2+(x-ct)^2\}\{q_2{}^2 z^2+(x-ct)^2\}} \qquad (3.16)$$

$$\tau_{33} = Dz\left[\frac{(\alpha^2-2b^2-m_1 q_1 \alpha^2)q_1}{q_1{}^2 z^2+(x-ct)^2} - \frac{(m_1+q_1)}{(m_2+q_2)}.\frac{(\alpha^2-2b^2-m_2 q_2 \alpha^2)q_2}{q_2{}^2 z^2+(x-ct)^2}\right. \qquad (3.17)$$

In the particular case, when $H_2 = 0$ i.e. for a = 0 this result agree with Sneddon [1]. The expression for τ_{33} shows that in any plane parallel to the boundary surface the maximum normal stress happen along x = ct i.e. directly below the load. So the point of maximum normal stress moves with velocity c along any plane z .$= z_0\ (\neq 0)$. This also agree with the observation Chakraborty [2].

At z .$= z_0\ (\neq 0)$, x = ct

$$[\tau_{33}]_{z.=z_0\ (\neq 0), x=ct} = \frac{D}{z_0}\left[\frac{(\alpha^2-2b^2-m_1 q_1 \alpha^2)}{q_1} - \frac{(m_1+q_1)}{(m_2+q_2)}.\frac{(\alpha^2-2b^2-m_2 q_2 \alpha^2)}{q_2}\right] \qquad (3.18)$$

Hence the maximum normal stress in any plane parallel to the boundary varies inversely as the depth.

IV. Numerical Result and Discussion

Numerical study of the normal and shear stresses have been done for Poisson's material only, with different physical parameters. It is seen that for a fixed intensity of the magnetic field, the maximum of the normal stress decrease with depth, and decreases with increase in the velocity load (Fig 1 and 2). However, the Alfven-wave velocity is increased, which implies an increase in the intensity of the bias field, the maximum normal stress increase with depth, although the tendency of the stresses is to ultimately decrease with increasing field at every surface z = constant(Fig 3).

The normal stresses has also been calculated ahead of the source, i.e. for x - ct > 0, and we note that after some initial variations, all these stresses gradually approach a steady-state as z increase (Fig 4). We note that the maximum normal stress decreases in every layer if the velocity of the source is gradually increased (Fig 5) with the magnetic field, whereas, for a variable magnetic field, the normal stress ahead of the load on every surface z = constant increases and then approaches a steady-state (Fig 6).

The shearing stresses are clearly symmetric about x = ct, and as once again approaches a steady value with increase in depth ahead of the load (Fig 7). However the shearing stress shows an oscillatory pattern on either side of the load line (Fig 8) when the magnetic field is kept fixed. If the field is varied (Fig 9) we find that these stresses will decrease ahead of the load, but increase behind the load. The situation is shown also in Fig 10, with a fixed depth,(at x - ct = 0, these stresses vanish).

V. Conclusion

It is seen that the effect of the magnetic field is to decrease both normal and shear stresses (ahead of the load).

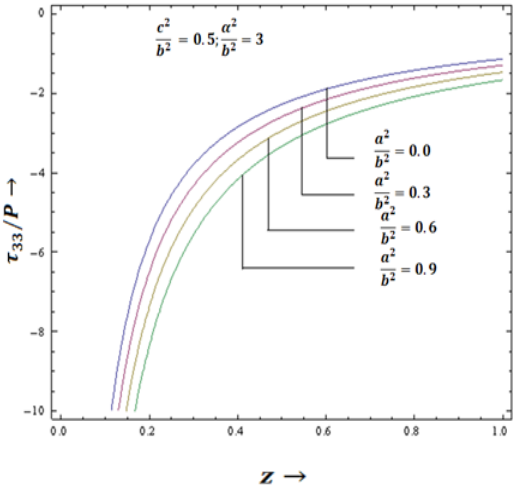

Fig 1: Variation of maximum normal stress with depth (velocity of load fixed).

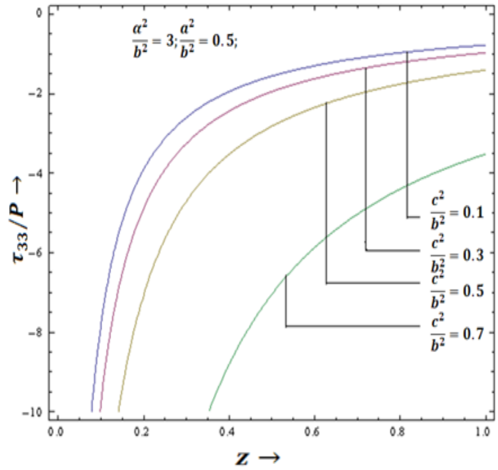

Fig 2: Variation of maximum normal stress with depth (Alfven wave velocity fixed).

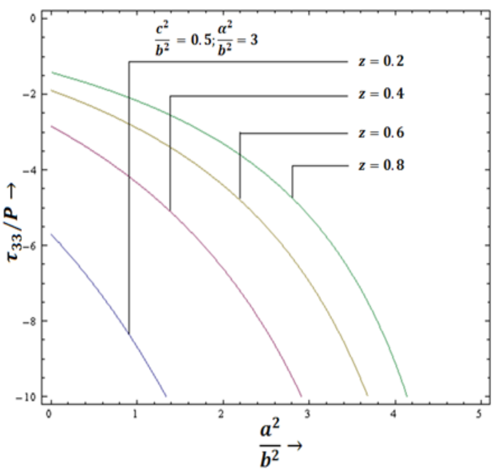

Fig3: Variation of maximum normal stress with increasing Alfven wave velocity along fixed planes parallel to the boundary (velocity of load fixed).

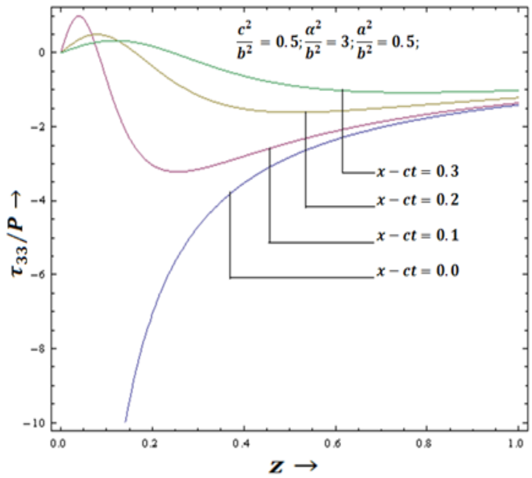

Fig 4: Variation of maximum normal stress with depth ahead of the load(Alfven wave velocity and velocity of load are fixed).

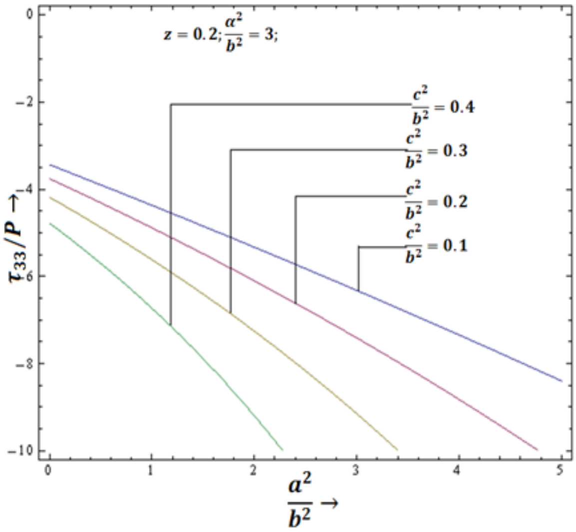

Fig5: Variation of maximum normal stress with increasing Alfven wave velocity (fixed depth).

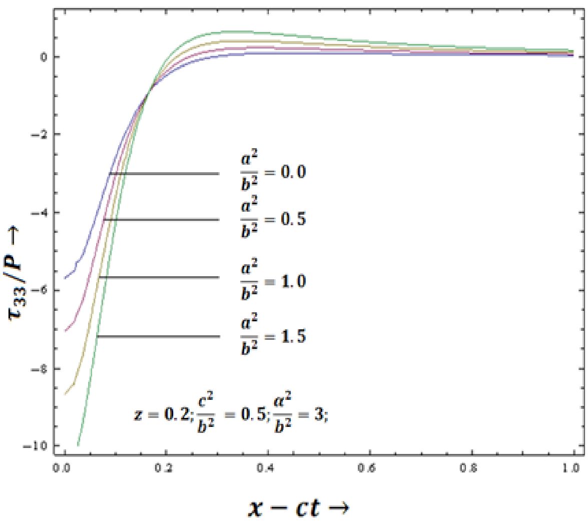

Fig6: Variation of maximum normal stress ahead of the load (fixed depth and velocity of load fixed).

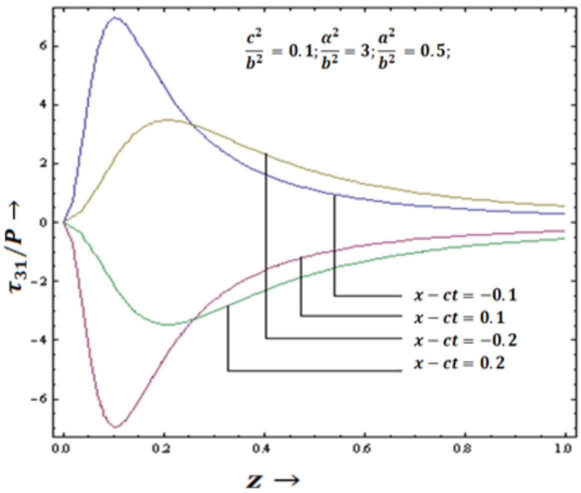

Fig7: Variation of maximum shear stress with depth ahead of the load (Alfven wave velocity and velocity of load are fixed).

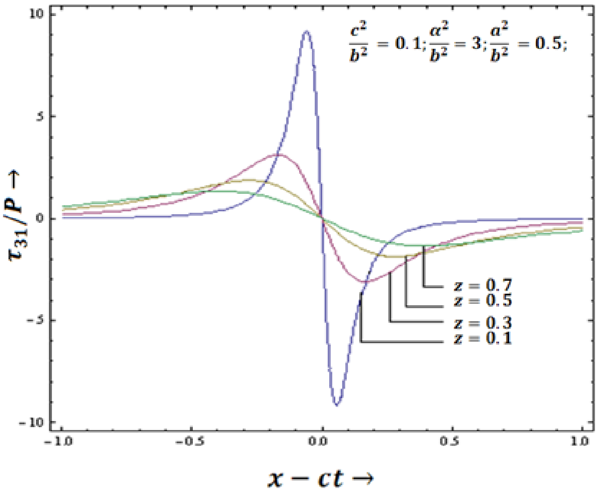

Fig8: Variation of maximum shear stress ahead of the load (Alfven wave velocity and velocity of load are fixed).

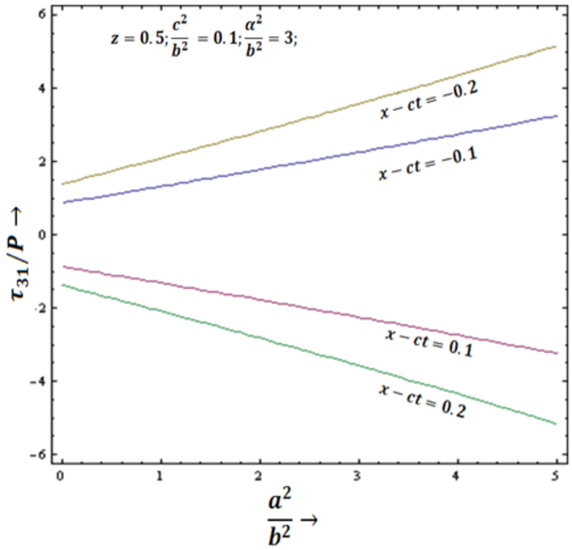

Fig9: Variation of maximum shear stress with increasing Alfven wave velocity ahead of the load (fixed depth and velocity of load).

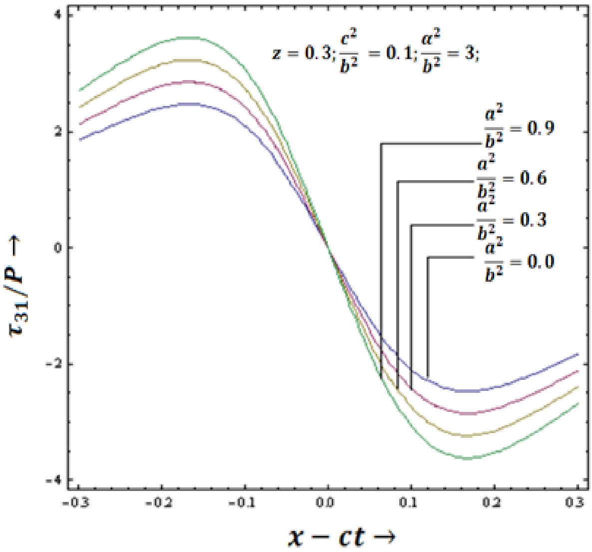

Fig10: Variation of maximum shear stress ahead of the load (fixed depth and velocity of load).

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