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Perfectly Alpha Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

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Abstract: *In this paper I introduce intuitionistic fuzzy perfectly alpha continuous mappings and their properties are studied.*
Key words and phrases: *Intuitionistic fuzzy topology, intuitionistic fuzzy alpha generalized closed set, intuitionistic fuzzy alpha generalized continuous mappings and intuitionistic fuzzy perfectly alpha continuous mappings.*

I. Introduction

The concept of fuzzy sets was introduced by Zadeh [12] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. Coker [4] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper I introduce intuitionistic fuzzy perfectly alpha continuous mappings and studied some of their properties. Also I provide some characterizations of intuitionistic fuzzy perfectly alpha continuous mappings.

II. Preliminaries

Definition 2.1: [1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$$

where the functions $\mu_A(x): X \to [0, 1]$ and $\nu_A(x): X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non -membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form
$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
(d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$
(e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \{ \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle \}$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_\sim = \{\langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $0_\sim, 1_\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(iii) $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4:[3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by
$\mathrm{int}(A) = \cup \{ G / G \text{ is an IFOS in X and } G \subseteq A \}$,

$cl(A) = \cap \{ K / K$ is an IFCS in X and $A \subseteq K \}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = [int(A)]^c$ and $int(A^c) = [cl(A)]^c$.

Definition 2.5:[6] An IFS $A = \{ \langle x, \mu_A, \nu_A \rangle \}$ in an IFTS (X, τ) is said to be an
(i) *intuitionistic fuzzy semi open set* (IFSOS in short) if $A \subseteq cl(int(A))$,
(ii) *intuitionistic fuzzy α-open set* (IFαOS in short) if $A \subseteq int(cl(int(A)))$,

Definition 2.6:[6] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an
(i) *intuitionistic fuzzy semi closed set* (IFSCS in short) if $int(cl(A)) \subseteq A$,
(ii) *intuitionistic fuzzy α-closed set* (IFαCS in short) if $cl(int(cl(A)) \subseteq A$,

The family of all IFCS (respectively IFSCS, IFαCS) of an IFTS (X, τ) is denoted by IFC(X) (respectively IFSC(X), IFαC(X)).

Definition 2.7:[12] Let A be an IFS in an IFTS (X, τ). Then
$\alpha int(A) = \cup \{ G / G$ is an IFαOS in X and $G \subseteq A \}$,
$\alpha cl(A) = \cap \{ K / K$ is an IFαCS in X and $A \subseteq K \}$.

Definition 2.8: An IFS A in an IFTS (X, τ) is an
(i) intuitionistic fuzzy generalized closed set (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X[11].
(ii) intuitionistic fuzzy generalized semi-pre closed set (IFGSPCS in short) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X[9].

Definition 2.9:[10] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ).

Definition 2.10:[8] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy alpha generalized closed set (IFαGCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ).

Result 2.11:[8] Every IFCS is an IFGCS, IFαCS and IFαGCS but the converses may not be true in general.

Definition 2.12:[7] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy weekly generalized closed set (IFWGCS in short) if $cl(int(A))) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ).

Definition 2.13:[8] An IFS A is said to be an intuitionistic fuzzy alpha generalized open set (IFαGOS in short) in X if the complement A^c is an IFαGCS in X.

Definition 2.14:[4] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy continuous (IF continuous in short) if $f^{-1}(B) \in IFO(X)$ for every $B \in \sigma$.

Definition 2.15:[6] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be
(i) intuitionistic fuzzy semi continuous (IFS continuous in short) if $f^{-1}(B) \in IFSO(X)$ for every $B \in \sigma$.
(ii) intuitionistic fuzzy α- continuous (IFα continuous in short) if $f^{-1}(B) \in IF\alpha O(X)$ for every $B \in \sigma$.
(iii) intuitionistic fuzzy pre continuous (IFP continuous in short) if $f^{-1}(B) \in IFPO(X)$ for every $B \in \sigma$.

Definition 2.16:[5] A mapping $f: (X, \tau) \to (Y,\sigma)$ is called an *intuitionistic fuzzy γ continuous* (IFγ continuous in short) if $f^{-1}(B)$ is an IFγOS in (X, τ) for every $B \in \sigma$.

Definition 2.17:[11] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy generalized continuous (IFG continuous in short) if $f^{-1}(B) \in IFGCS(X)$ for every IFCS B in Y.

Definition 2.18:[10] A mapping f: $(X, \tau) \to (Y,\sigma)$ is called an *intuitionistic fuzzy generalized semi continuous* (IFGS continuous in short) if $f^{-1}(B)$ is an IFGSCS in (X, τ) for every IFCS B of (Y, σ).

Definition 2.19:[7] A mapping f: $(X, \tau) \to (Y,\sigma)$ is called an *intuitionistic fuzzy weekly generalized continuous* (IFWG continuous in short) if $f^{-1}(B)$ is an IFWGCS in (X, τ) for every IFCS B of (Y, σ).

Definition 2.20:[7] A mapping f: $(X, \tau) \to (Y,\sigma)$ is called an *intuitionistic fuzzy perfectly weekly generalized continuous mapping* (IF perfectly WG continuous in short) if $f^{-1}(B)$ is clopen in (X, τ) for every IFWGCS B of (Y, σ).

Definition 2.21:[8] A mapping f: $(X, \tau) \to (Y,\sigma)$ is called an *intuitionistic fuzzy alpha generalized continuous* (IFαG continuous in short) if $f^{-1}(B)$ is an IFαGCS in (X, τ) for every IFCS B of (Y, σ).

III. Intuitionistic Fuzzy Perfectly Alpha Continuous Mappings

In this section I introduce intuitionistic fuzzy perfectly alpha continuous mapping and studied some of its properties.

Definition 3.1: A mapping f: $(X, \tau) \to (Y,\sigma)$ is called an *intuitionistic fuzzy perfectly alpha continuous* (IFpα continuous in short) if $f^{-1}(B)$ is clopen in (X, τ) for every IFαCS B of (Y, σ).

Theorem 3.2: Every IFpα continuous mapping is an IF continuous mapping but not conversely.
Proof: Let us consider a mapping f: $(X, \tau) \to (Y, \sigma)$ be an IFpα continuous mapping. Let B be an IFCS in Y. Clearly then B is an IFαCS in Y. Since f is an IFpα continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen in X. That is $f^{-1}(B)$ is an IFCS in X. Hence f is an IF continuous mapping.

Example 3.3: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.1, 0.2), (0.9, 0.8) \rangle$, $G_2 = \langle y, (0.1, 0.2), (0.9, 0.8) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. The IFS $B = \langle y, (0.9, 0.8), (0.1, 0.2) \rangle$ is IFCS in Y. Then $f^{-1}(B)$ is also IFCS in X but not IFOS in X. Therefore f is an IF continuous mapping but not an IFpα continuous mapping.

Theorem 3.4: Every IFpα continuous mapping is an IFα continuous mapping but not conversely.
Proof: Let f: $(X, \tau) \to (Y,\sigma)$ be an IFpα continuous mapping. Let B be an IFCS in Y. Clearly then B is an IFαCS in Y. Then by hypothesis $f^{-1}(B)$ is an IFCS in X. Since every IFCS is an IFαCS, $f^{-1}(B)$ is an IFαCS in X. Hence f is an IFα continuous mapping.
Example 3.5: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.4, 0.4), (0.4, 0.2) \rangle$ and $G_2 = \langle y, (0.8, 0.8), (0.2, 0.2) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Hence the mapping f an is IFα continuous mapping. But f is not an IFpα continuous mapping since an IFS $B = \langle y, (0.1, 0.2), (0.9, 0.8) \rangle$ is an IFαCS in Y and an IFS B is not an intuitionistic fuzzy clopen in X.

Theorem 3.6: Every IFpα continuous mapping is an IFG continuous mapping but not conversely.
Proof: Let f: $(X, \tau) \to (Y,\sigma)$ be an IFpα continuous mapping. Let B be an IFCS in Y. Clearly then B is an IFαCS in Y. Then by hypothesis $f^{-1}(B)$ is an IFCS in X. Since every IFCS is an IFGCS, $f^{-1}(B)$ is an IFGCS in X. Hence f is an IFG continuous mapping.

Example 3.7: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.1, 0.3), (0.4, 0.5) \rangle$, $G_2 = \langle y, (0.7, 0.7), (0.2, 0.3) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Hence the mapping f is IFG continuous mapping. But f is not an IFpα continuous mapping since an IFS $B = \langle y, (0.2, 0.2), (0.7, 0.8) \rangle$ is an IFαCS in Y and an IFS B is not an intuitionistic fuzzy clopen in X.

Theorem 3.8: Every IFpα continuous mapping is an IFGS continuous mapping but not conversely.
Proof: Let f: $(X, \tau) \to (Y,\sigma)$ be an IFpα continuous mapping. Let B be an IFCS in Y. Clearly then B is an IFαCS in Y. Then by hypothesis $f^{-1}(B)$ is an IFCS in X. Since every IFCS is an IFGSCS, $f^{-1}(B)$ is an IFGSCS in X. Hence f is an IFGS continuous mapping.

Example 3.9: Let X = { a, b }, Y = { u, v } and $G_1 = \langle$ x, (0.1, 0.2), (0.5, 0.5) $\rangle$, $G_2 = \langle$ y, (0.6, 0.7), (0.2, 0.3) $\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{ 0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Hence the mapping f an is IFGS continuous mapping. But f is not an IFpα continuous mapping since an IFS B = $\langle$ y, (0.2, 0.2), (0.7, 0.7) $\rangle$ is an IFαCS in Y and an IFS B is not an intuitionistic fuzzy clopen in X.

Theorem 3.10: Every IFpα continuous mapping is an IFαG continuous mapping but not conversely.
Proof: Let f: $(X, \tau) \rightarrow (Y,\sigma)$ be an IFpα continuous mapping. Let B be an IFCS in Y. Then B is an IFαCS in Y. Then by hypothesis $f^{-1}(B)$ is an IFCS in X. Since every IFCS is an IFαGCS, $f^{-1}(B)$ is an IFαGCS in X. Hence f is an IFαG continuous mapping.

Example 3.11: Let X = { a, b }, Y = { u, v } and $G_1 = \langle$ x, (0.2, 0.2), (0.5, 0.5) $\rangle$, $G_2 = \langle$ y, (0.6, 0.6), (0.2, 0.3) $\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{ 0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Hence the mapping f is an IFαG continuous mapping. But f is not an IFpα continuous mapping since an IFS B = $\langle$ y, (0.2, 0.2), (0.7, 0.8) $\rangle$ is an IFαCS in Y and it is not an intuitionistic fuzzy clopen in X.

Theorem 3.12: Every IFpα continuous mapping is an IFWG continuous mapping but not conversely.
Proof: Let f: $(X, \tau) \rightarrow (Y,\sigma)$ be an IFpα continuous mapping. Let B be an IFCS in Y. Then B is an IFαCS in Y. Then by hypothesis $f^{-1}(B)$ is an IFCS in X. Since every IFCS is an IFWGCS, $f^{-1}(B)$ is an IFWGCS in X. Hence f is an IFWG continuous mapping.

Example 3.13: Let X = { a, b }, Y = { u, v } and $G_1 = \langle$ x, (0.4, 0.5), (0.6, 0.5) $\rangle$, $G_2 = \langle$ y, (0.7, 0.7), (0.2, 0.3) $\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{ 0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Hence f is an IFWG continuous mapping. But f is not an IFpα continuous mapping since an IFS B = $\langle$ y, (0.2, 0.2), (0.7, 0.8) $\rangle$ is an IFαCS in Y and an IFS B is not an intuitionistic fuzzy clopen in X.

Theorem 3.14: Every IFpα continuous mapping is an IFP continuous mapping but not conversely.
Proof: Let us consider a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ be an IFpα continuous mapping. Let B be an IFCS in Y. Clearly then B is an IFαCS in Y. Since f is IFpα continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen in X. That is $f^{-1}(B)$ is an IFCS in X. Since every IFCS is an IFPCS, $f^{-1}(B)$ is an IFPCS in X. Hence f is an IFP continuous mapping.

Example 3.15: Let X = {a, b}, Y = {u, v} and $G_1 = \langle$ x, (0.5, 0.2), (0.4, 0.4) $\rangle$, $G_2 = \langle$ y, (0.1, 0.6), (0.3, 0.4) $\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{ 0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFP continuous mapping. But f is not an IFpα continuous mapping since an IFS B = $\langle$ y, (0.2, 0.2), (0.7, 0.8) $\rangle$ is an IFαCS in Y and it is not an intuitionistic fuzzy clopen in X.

Theorem 3.16: Every IFpα continuous mapping is an IFγ continuous mapping but not conversely.
Proof: Let a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ be an IFpα continuous mapping. Let B be an IFCS in Y. Clearly then B is an IFαCS in Y. Since f is IFpα continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen in X. That is $f^{-1}(B)$ is an IFCS in X. Since every IFCS is an IFγCS, $f^{-1}(B)$ is an IFγCS in X. Hence f is an IFγ continuous mapping.

Example 3.17: Let X = {a, b}, Y = {u, v} and $G_1 = \langle$ x, (0.5, 0.2), (0.4, 0.4) $\rangle$, $G_2 = \langle$ y, (0.2, 0.6), (0.3, 0.4) $\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{ 0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFγ continuous mapping. But f is not an IFpα continuous mapping since an IFS B = $\langle$ y, (0.2, 0.2), (0.7, 0.8) $\rangle$ is an IFαCS in Y and an IFS B is not an intuitionistic fuzzy clopen in X.

The relations between various types of intuitionistic fuzzy continuity are given in the following diagram. In this diagram 'cts.' means continuous.

$$\begin{tikzcd}
 & \text{IFG cts.} \arrow[dr] & \\
\text{IF cts.} \arrow[ur] \arrow[dr] & \text{IFp}\alpha\text{ cts.} \arrow[l] \arrow[u] \arrow[r] \arrow[d] & \text{IFGS cts.} \\
 & \text{IF}\alpha\text{G cts.} \arrow[ur] &
\end{tikzcd}$$

The reverse implications are not true in general.

Theorem 3.18: Every IFpα continuous mapping is an IFGSP continuous mapping but not conversely.
Proof: Let f: $(X, \tau) \to (Y,\sigma)$ be an IFpα continuous mapping. Let B be an IFCS in Y. Then B is an IFαCS in Y. Then by hypothesis $f^{-1}(B)$ is an IFCS in X. Since every IFCS is an IFGSPCS, $f^{-1}(B)$ is an IFGSPCS in X. Hence f is an IFGSP continuous mapping.

Example 3.19: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6) \rangle$, $G_2 = \langle y, (0.2, 0.1), (0.8, 0.9) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{ 0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Hence the mapping f is an IFGSP continuous mapping. But f is not an IFpα continuous mapping since an IFS B = $\langle y, (0.8, 0.9), (0.2, 0.1) \rangle$ is an IFαCS in Y and it is not an intuitionistic fuzzy clopen in X.

Theorem 3.20: Every IF perfectly WG continuous mapping is an IFpα continuous mapping.
Proof: Let f: $(X, \tau) \to (Y,\sigma)$ be an IF perfectly WG continuous mapping. Let B be an IFWCS in Y. Then by hypothesis, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X. Clearly from the definition of an IFαCS, every IFαCS is an IFWGCS. Hence the mapping f is an IFpα continuous mapping.

Theorem 3.21: A mapping f: $X \to Y$ is an IFpα continuous if and only if the inverse image of each IFαOS in Y is an intuitionistic fuzzy clopen in X.
Proof: Necessary Part: Let a mapping f: $X \to Y$ is IFpα continuous mapping. A be an IFαOS in Y. This implies A^c is IFαCS in Y. Since f is an IFpα continuous, $f^{-1}(A^c)$ is an intuitionistic fuzzy clopen in X. Hence $f^{-1}(A)$ is an intuitionistic fuzzy clopen in X.
Sufficient Part: Let B is an IFαCS in Y. Then B^c is an IFαOS in Y. By hypothesis $f^{-1}(B^c)$ is intuitionistic fuzzy clopen in X. This implies $f^{-1}(B)$ is an intuitionistic fuzzy clopen in X. Hence a mapping f is an IFpα continuous mapping.

Theorem 3.22: Let f : $(X, \tau) \to (Y, \sigma)$ be an IFpα continuous mapping and g : $(Y, \sigma) \to (Z, \delta)$ is IFpα continuous mapping, then g o f : $(X, \tau) \to (Z, \delta)$ is an IFpα continuous mapping.
Proof: Let A be an IFαCS in Z. Then $g^{-1}(A)$ is an intuitionistic fuzzy clopen in Y, by hypothesis. This implies $g^{-1}(A)$ is IFCS in Y. Since every IFCS is an IFαCS, $g^{-1}(A)$ is an IFαCS in Y. Since f is an IFpα continuous mapping, $f^{-1}(g^{-1}(A))$ is an intuitionistic fuzzy clopen in X. Hence g o f is an IFpα continuous mapping.

Theorem 3.23: Let f : $(X, \tau) \to (Y, \sigma)$ be an IF continuous mapping and g : $(Y, \sigma) \to (Z, \delta)$ is IFpα continuous mapping, then g o f : $(X, \tau) \to (Z, \delta)$ is an IF continuous mapping.
Proof: Let A be an IFCS in Z. This implies A is an IFαCS in Z. Since g is an IFpα continuous mapping, $g^{-1}(A)$ is an intuitionistic fuzzy clopen in Y. Thus $g^{-1}(A)$ is IFCS in Y. Since f is an IF continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFCS in X. Hence g o f is an IF continuous mapping.

Theorem 3.24: Let f : $(X, \tau) \to (Y, \sigma)$ be an IFα continuous mapping and g : $(Y, \sigma) \to (Z, \delta)$ is IFpα continuous mapping, then g o f : $(X, \tau) \to (Z, \delta)$ is an IFα continuous mapping.

Proof: Let A be an IFCS in Z. This implies A is an IFαCS in Z. Then $g^{-1}(A)$ is an intuitionistic fuzzy clopen in Y, by hypothesis. Thus $g^{-1}(A)$ is IFCS in Y. Since f is an IFα continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFαCS in X. Hence $g \circ f$ is an IFα continuous mapping.

Theorem 3.25: If a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ is an IFpα continuous mapping then $cl(int(cl(f^{-1}(int(A))))) \subseteq f^{-1}(\alpha cl(A))$ for every IFS B in Y.

Proof: Let B be an IFS in Y. Then $\alpha cl(B)$ is an IFαCS in Y. By hypothesis, $f^{-1}(\alpha cl(B))$ is an intuitionistic fuzzy clopen in X. This implies $f^{-1}(\alpha cl(B))$ is an IFCS in X. Therefore $cl(f^{-1}(\alpha cl(B)) = f^{-1}(\alpha cl(B))$. Now $cl(int(cl(f^{-1}(int(B))))) \subseteq cl(int(cl(f^{-1}(\alpha cl(B))))) \subseteq cl(f^{-1}(\alpha cl(B))) = f^{-1}(\alpha cl(B))$. Hence $cl(int(cl(f^{-1}(int(B))))) \subseteq f^{-1}(\alpha cl(B))$ for every IFS B in Y.

Theorem 3.26: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent.

(i) f is an IFpα continuous mapping.

(ii) If A is an IFαCS in Y then $f^{-1}(A)$ is an intuitionistic fuzzy clopen in X.

(iii) $int(f^{-1}(\alpha cl(B))) = f^{-1}(\alpha cl(B)) = cl(f^{-1}(\alpha cl(B)))$ for every B in Y.

Proof:

(i) $\Rightarrow$ (ii): is obviously true.

(ii) $\Rightarrow$ (iii): Let B be any IFS in Y. Then $\alpha cl(B)$ is an IFαCS in Y. By hypothesis, $f^{-1}(\alpha cl(B))$ is an intuitionistic fuzzy clopen in X. Thus $f^{-1}(\alpha cl(B))$ is IFCS in X. Hence $f^{-1}(\alpha cl(B))$ is an intuitionistic fuzzy open set in X. That is $int(f^{-1}(\alpha cl(B)) = f^{-1}(\alpha cl(B))$. Also $f^{-1}(\alpha cl(B))$ is an intuitionistic fuzzy closed set in X. That is $cl(f^{-1}(\alpha cl(B)) = f^{-1}(\alpha cl(B))$. Hence $int(f^{-1}(\alpha cl(B))) = f^{-1}(\alpha cl(B)) = cl(f^{-1}(\alpha cl(B)))$ for every B in Y.

(iii) $\Rightarrow$ (i): Let B be an IFS in Y. Then $\alpha cl(B)$ is an IFαCS in Y. By hypothesis, $int(f^{-1}(\alpha cl(B)) = f^{-1}(\alpha cl(B))= cl(f^{-1}(\alpha cl(B)))$. This implies $f^{-1}(\alpha cl(B))$ is IFOS in X and also an IFCS in X. That is $f^{-1}(\alpha cl(B))$ is an intuitionistic fuzzy clopen in X. Hence f is an IFpα continuous mapping.

Theorem 3.27: If a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ is an IFpα continuous mapping then $f^{-1}(\alpha cl(B)) \subseteq int(cl(int(f^{-1}(cl(B)))$ for every IFS B in Y.

Proof: Let B be an IFS in Y. Then $\alpha cl(B)$ is an IFαCS in Y. By hypothesis, $f^{-1}(\alpha cl(B))$ is an intuitionistic fuzzy clopen in X. This implies $f^{-1}(\alpha cl(B))$ is an IFOS in X. Therefore $int(f^{-1}(\alpha cl(B)) = f^{-1}(\alpha cl(A))$. Now $int(f^{-1}(\alpha cl(B))))) \subseteq int(cl(int(f^{-1}(\alpha cl(B))))) \subseteq int(cl(int(f^{-1}(cl(B)))))$. This implies $f^{-1}(\alpha cl(B)) \subseteq int(cl(int(f^{-1}(cl(B)))$ for every IFS B in Y.

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