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On The Elzaki Transform of Heaviside Step Function with a Bulge Function

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Abstract: *The aim of this paper, is study the non-homogeneous second order differential equation of the Heaviside step function with a bulge function. Elzaki transform, inverse Elzaki transform and Power series expansion are mentioned to obtain the solution of differential equation of the Heaviside step function with a bulge function.*
Keywords: *Elzaki transform, Heaviside step function, Bulge function.*

I. Introduction

Elzaki transform can be employed in not only solving the linear ordinary differential equations with constant coefficient but also can be used with ordinary differential equations with variable coefficients. In addition, Elzaki transforms of derivatives have been studied in numerous approaches to solve the ODEs. Ig. Cho and Hj. Kim [12] showed that the laplace transform of derivative can be expressed by an infinite series or Heaviside function. T. Lee and H. Kim [13] found the representation of energy equation by laplace transform. In this study, ELzaki transform is applied to the non-homogeneous second order differential equation with a bulge function involved the Heaviside step function.

The technique that we used is ELzaki transform method which is based on Fourier transform, it introduced by Tarig Elzaki (2011) see [1, 2, 3, 4, 11]. Solution of these equations have a major role in the fields of science and engineering.

Definition 1.
Elzaki Transform [2]. Given a function $f(t)$ defined for all $t \geq 0$, as follow:

$$E[f(t),v] = T(v) = v\int_0^t f(t)e^{-\frac{t}{v}}dt \quad , \quad v \in (k_1, k_2) \tag{1}$$

for all values of s, for which the improper integral converges
We have non-homogeneous differential equation with constant coefficients An equation of the form,

$$a_n \frac{d^n y}{dx^n} + a_{n-1}\frac{d^{n-1}y}{dx^{n-1}} + a_{n-2}\frac{d^{n-2}y}{dx^{n-2}} + \cdots + a_1\frac{dy}{dx} + a_0 y = f(x) \tag{2}$$

In this work, we study the non-homogeneous second order differential equation with a bulge function in the form $y'' + w^2 y = e^{-\frac{(t-l)^2}{2}}$.

Theorem 1: [1]

Let $T(u)$ be ELzaki transform of $f(t)$ $[E(f(t)) = T(u)]$ then:

(i) $E[f'(t)] = \frac{T(u)}{u} - uf(0)$ (ii) $E[f''(t)] = \frac{T(u)}{u^2} - f(0) - uf'(0)$

Theorem 2:

Elzaki transform of the bulge function $e^{-\frac{(t-l)^2}{2}}$ is expressed by.

$$E\left\{e^{-\frac{(t-l)^2}{2}}\right\} = e^{-\frac{l^2}{2}}[v^2 + lv^3 + (-1+l^2)v^4 + (-3l+l^3)v^5] \tag{3}$$

Proof.
The Taylor series expansion e^x is of the form

$$e^{-\frac{(t-l)^2}{2}} = \sum_{n=0}^{\infty}\frac{x_n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \tag{4}$$

Therefore, by substituting equation (4) with $x = -\frac{(t-l)^2}{2}$, we obtain

$$e^{-\frac{(t-l)^2}{2}} = e^{-\frac{l^2}{2}} + e^{-\frac{l^2}{2}} lt + e^{-\frac{l^2}{2}}\left(-\frac{1}{2}+\frac{l^2}{2}\right)t^2 + e^{-\frac{l^2}{2}}\left(-\frac{l}{2}+\frac{l^3}{6}\right)t^3 \quad (5)$$

By taking Elzaki transform of equation (5) and using the fact that the Elzaki transform is linear, we find,

$$E\left\{e^{-\frac{(t-l)^2}{2}}\right\} = e^{-\frac{l^2}{2}}[v^2 + lv^3 + (-1+l^2)v^4 + (-3l+l^3)v^5] \quad (6)$$

Heaviside step function of a bulge function of a piecewise continuous function is defined as:

$$f(t) = \begin{cases} e^{-\frac{(t-l)^2}{2}} & , \quad 0 < t < \delta \\ a & , \quad t > \delta \end{cases}$$

Is expressed by

$$f(t) = e^{-\frac{(t-l)^2}{2}} + au(t-\delta) - e^{-\frac{(t-l)^2}{2}} u(t-\delta) \quad (7)$$

where a , δ are constants.

Theorem 3:

Elzaki transform of $e^{-\frac{(t-l)^2}{2}} u(t-\delta)$ is expressed by

$$E\left\{e^{-\frac{(t-l)^2}{2}} u(t-\delta)\right\} = Ae^{\frac{-\delta}{V}}v^2 + Ale^{\frac{-\delta}{V}}[v^3+\delta v^2] + ABe^{\frac{-\delta}{V}}[2v^4+2\delta v^3+\delta^2 v^2] + ACe^{\frac{-\delta}{V}}[6v^5+6\delta v^4 +$$

3δ2v3+δ3v2 (8)

Where $A = e^{-\frac{l^2}{2}}$, $B = \left(-\frac{1}{2}+\frac{l^2}{2}\right)$, $C = \left(-\frac{1}{2}+\frac{l^3}{6}\right)$.

Proof

From equation (4) and the Heaviside step function, we have

$$e^{-\frac{(t-l)^2}{2}} u(t-\delta) = e^{-\frac{l^2}{2}} u(t-\delta) + e^{-\frac{l^2}{2}} ltu(t-\delta) + e^{-\frac{l^2}{2}}\left(-\frac{1}{2}+\frac{l^2}{2}\right)t^2 u(t-\delta) + e^{-\frac{l^2}{2}}\left(-\frac{1}{2}+\frac{l^3}{6}\right)t^3 u(t-\delta) \quad (9)$$

Therefore, by taking Elzaki transform to equation (9), we get,

$$E\left\{e^{-\frac{(t-l)^2}{2}} u(t-\delta)\right\} = AE\{u(t-\delta)\} + AlE\{tu(t-\delta)\} + ABE\{t^2u(t-\delta)\} + ACE\{t^3u(t-\delta)\}$$

$$= Ae^{\frac{-\delta}{V}}v^2 + Ale^{\frac{-\delta}{V}}[v^3+\delta v^2] + ABe^{\frac{-\delta}{V}}[2v^4+2\delta v^3+\delta^2 v^2] + ACe^{\frac{-\delta}{V}}[6v^5+6\delta v^4+3\delta^2 v^3 +$$

δ3v2 (=k) (10)

II. Main Result

Theorem 4:

Elzaki transform of Heaviside step function of a bulge function of a piecewise continuous function ,

$$f(t) = \begin{cases} e^{-\frac{(t-l)^2}{2}} & , \quad 0 < t < \delta \\ a & , \quad t > \delta \end{cases}$$

can be expressed by.

$$M + ae^{\frac{-\delta}{V}}v^2 + K \quad (11)$$

where a , δ are constants.

Proof.

By taking the Elzaki transform to equation (7) and **theorem** 2 and 3, we have,

$$E\{f(t)\} = E\left\{e^{-\frac{(t-l)^2}{2}}\right\} + E\left\{\left[a - e^{-\frac{(t-l)^2}{2}}\right]u(t-\delta)\right\}$$

$$= E\left\{e^{-\frac{(t-l)^2}{2}}\right\} + E\{au(t-\delta)\} - E\left\{e^{-\frac{(t-l)^2}{2}} u(t-\delta)\right\}$$

$$= e^{-\frac{l^2}{2}}[v^2 + lv^3 + (-1+l^2)v^4 + (-3l+l^3)v^5] + ae^{\frac{-\delta}{V}}v^2 - Ae^{\frac{-\delta}{V}}v^2 - Ale^{\frac{-\delta}{V}}[v^3+\delta v^2] - ABe^{\frac{-\delta}{V}}[2v^4 +$$

2δv3+δ2v2−ACe−δV6v5+6δv4+3δ2v3+δ3v2 =M+ae−δVv2+K

(12)

Where $M = e^{-\frac{l^2}{2}}[v^2 + lv^3 + (-1 + l^2)v^4 + (-3l + l^3)v^5]$,
$K = Ae^{\frac{-\delta}{V}}v^2 + Ale^{\frac{-\delta}{V}}[v^3 + \delta v^2] + ABe^{\frac{-\delta}{V}}[2v^4 + 2\delta v^3 + \delta^2 v^2] + ACe^{\frac{-\delta}{V}}[6v^5 + 6\delta v^4 + 3\delta^2 v^3 + \delta^3 v^2]$.

Theorem 5:
The solution of the non-homogeneous differential equation with constant coefficients, where,

$$f(t) = \begin{cases} e^{-\frac{(t-l)^2}{2}} & , \quad 0 < t < \delta \\ a & , \quad t > \delta \end{cases}$$

And $y(0) = w_0$, $y'(0) = w_1$, is expressed by:

$$y(t) = w_0 \cos wt + \frac{w_1}{w}\sin wt + e^{\frac{-l^2}{2}}\left[1 + lt - \frac{t^2}{2} + \frac{t^2 l^2}{2} - \frac{lt^3}{3} + \frac{l^3 t^3}{6}\right] + a \times Heaviside\ (t - \delta) + L^{-1}\{k\} \tag{13}$$

Where a , w , w_0 and w_1 are constant .

Proof.
By taking Elzaki transform of the non-homogeneous differential equation with constant coefficients and the Heaviside step function and **theorem** 3, we obtain:

$$\frac{E\{y\ (t)\}}{v^2} - w_0 - vw_1 + w^2 E\{y(t)\} = E\{Heaviside\ step\ fuction\ of\ f(t)\}$$

Or

$$E\{y(t)\} = \frac{w_0 v^2}{1 + w^2 v^2} + \frac{w_1 v^3}{1 + w^2 v^3} + M + \frac{a e^{-\delta s}}{s} + K \tag{14}$$

By taking the inverse Elzaki transform to equation (14) we obtain the solution of the non-homogeneous differential equation with the Heaviside step function as:

$$y(t) = w_0 \cos wt + \frac{w_1}{w}\sin wt + e^{\frac{-l^2}{2}}\left[1 + lt - \frac{t^2}{2} + \frac{t^2 l^2}{2} - \frac{lt^3}{3} + \frac{l^3 t^3}{6}\right] + a \times Heaviside\ (t - \delta) + L^{-1}\{k\} \tag{15}$$

III. Conclusion

In this work, we solve the non-homogeneous second order differential equation of the Heaviside step function with a bulge function by elzaki transform and in this work, we found that This technique is useful to solve the non-homogeneous second order differential equation with a bulge function involved the Heaviside step function.

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