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Intuitionistic Fuzzy Perfectly Regular Weakly Generalized Continuous Mappings

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Abstract: *The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy perfectly regular weakly generalized continuous mappings and intuitionistic fuzzy perfectly regular weakly generalized open mappings in intuitionistic fuzzy topological space. Some of their properties are explored.*
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I. Introduction

Fuzzy set (FS) as proposed by Zadeh [19] in 1965, is a framework to encounter uncertainty, vagueness and partial truth and it represents a degree of membership for each member of the universe of discourse to a subset of it. After the introduction of fuzzy topology by Chang [2] in 1968, there have been several generalizations of notions of fuzzy sets and fuzzy topology.

By adding the degree of non-membership to FS, Atanassov [1] proposed intuitionistic fuzzy set (IFS) in 1986 which looks more accurate to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. In 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological space.

In this paper, we introduce the notion of intuitionistic fuzzy perfectly weakly generalized continuous mappings and intuitionistic fuzzy perfectly weakly generalized open mappings in intuitionistic fuzzy topological space and study some of their properties. We provide some characterizations of intuitionistic fuzzy perfectly regular weakly generalized continuous mappings and establish the relationships with other classes of early defined forms of intuitionistic fuzzy mappings.

II. Preliminaries

Definition 2.1: [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ where the functions $\mu_A(x) : X \to [0, 1]$ and $\nu_A(x) : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$.

Definition 2.2: [1] Let A and B be IFSs of the forms $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ and

$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$. Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$,

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$,

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$.

For the sake of simplicity, the notation $A = \langle x, \mu_A, \nu_A\rangle$ shall be used instead of the longer $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle \mid x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle \mid x \in X\}$ are the empty set and the whole set of X, respectively.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on a non empty set X is a family τ of IFSs in X satisfying the following axioms:

(a) $0_\sim, 1_\sim \in \tau$,

(b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i \in J\} \subseteq \tau$.

In this case, the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

$int(A) = \cup \{ G / G$ is an IFOS in X and $G \subseteq A \}$, $cl(A) = \cap \{ K / K$ is an IFCS in X and $A \subseteq K \}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 2.5: An IFS $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$ in an IFTS (X, τ) is said to be

(a) intuitionistic fuzzy semi closed set [6] (IFSCS in short) if $int(cl(A)) \subseteq A$,

(b) intuitionistic fuzzy α-closed set [6] (IFαCS in short) if $cl(int(cl(A))) \subseteq A$,

(c) intuitionistic fuzzy pre-closed set [6] (IFPCS in short) if $cl(int(A)) \subseteq A$,

(d) intuitionistic fuzzy regular closed set [6] (IFRCS in short) if $cl(int(A)) = A$,

(e) intuitionistic fuzzy generalized closed set [16] (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,

(f) intuitionistic fuzzy generalized semi closed set [15] (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,

(g) intuitionistic fuzzy α generalized closed set [13] (IFαGCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,

(h) intuitionistic fuzzy γ closed set [5] (IFγ CS in short) if $int(cl(A)) \cap cl(int(A)) \subseteq A$.

An IFS A is called intuitionistic fuzzy semi open set, intuitionistic fuzzy α-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set, intuitionistic fuzzy α generalized open set and intuitionistic fuzzy γ open set (IFSOS, IFαOS, IFPOS, IFROS, IFGOS, IFGSOS, IFαGOS and IFγ OS) if the complement A^c is an IFSCS, IFαCS, IFPCS, IFRCS, IFGCS, IFGSCS, IFαGCS and IFγ CS respectively.

Definition 2.6: [8] An IFS $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$ in an IFTS (X, τ) is said to be an intuitionistic fuzzy regular weakly generalized closed set (IFRWGCS in short) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X .

The family of all IFRWGCSs of an IFTS (X, τ) is denoted by IFRWGC(X).

Definition 2.7: [8] An IFS $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$ is said to be an intuitionistic fuzzy regular weakly generalized open set (IFRWGOS in short) in (X, τ) if the complement A^c is an IFRWGCS in X.

The family of all IFRWGOSs of an IFTS (X, τ) is denoted by IFRWGO(X).

Result 2.8: [8] Every IFCS, IFαCS, IFGCS, IFRCS, IFPCS, IFαGCS is an IFRWGCS but the converses need not be true in general.

Definition 2.9: [9] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy regular weakly generalized interior and an intuitionistic fuzzy regular weakly generalized closure are defined by

$wgint(A) = \cup \{G \mid G$ is an IFRWGOS in X and $G \subseteq A\}$, $wgcl(A) = \cap \{K \mid K$ is an IFRWGCS in X and $A \subseteq K\}$.

Definition 2.10: [3] Let f be a mapping from an IFTS (X, τ) into an IFTS(Y, σ). If $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle \mid y \in Y\}$ is an IFS in Y, then the pre-image of B under f denoted by $f^{-1}(B)$, is the IFS in X defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x))\rangle \mid x \in X\}$, where $f^{-1}(\mu_B(x)) = \mu_B(f(x))$.

If $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ is an IFS in X, then the image of A under f denoted by f(A) is the IFS in Y defined by $f(A) = \{\langle y, f(\mu_A(y)), f\_(\nu_A(y))\rangle \mid y \in Y\}$ where $f\_(\nu_A) = 1 - f(1 - \nu_A)$.

Definition 2.11: Let f be a mapping from an IFTS (X, τ) into an IFTS (Y,σ). Then f is said to be

(a) intuitionistic fuzzy continuous [4] (IF continuous in short) if $f^{-1}(B) \in IFO(X)$ for every $B\in \sigma$,
(b) intuitionistic fuzzy α continuous [6] (IFα continuous in short) if $f^{-1}(B) \in IF\alpha O(X)$ for every $B\in \sigma$,
(c) intuitionistic fuzzy pre continuous [6] (IFP continuous in short) if $f^{-1}(B) \in IFPO(X)$ for every $B\in \sigma$,
(d) intuitionistic fuzzy generalized continuous [16] (IFG continuous in short) if $f^{-1}(B) \in IFGO(X)$ for every $B\in \sigma$,
(e) intuitionistic fuzzy α generalized continuous [14] (IFαG continuous in short) if $f^{-1}(B) \in IF\alpha GO(X)$ for every $B\in \sigma$,
(f) intuitionistic fuzzy regular weakly generalized continuous [10] (IFRWG continuous in short) if $f^{-1}(B) \in IFRWGO(X)$ for every $B\in \sigma$,
(g) intuitionistic fuzzy almost continuous [17] (IFA continuous in short) if $f^{-1}(B) \in IFO(X)$ for every IFROS $B\in \sigma$,
(h) intuitionistic fuzzy almost regular weakly generalized continuous [11] (IFARWG continuous in short) if $f^{-1}(B) \in IFRWGO(X)$ for every IFROS $B\in \sigma$,
(i) intuitionistic fuzzy regular weakly generalized irresolute [9] (IFRWG irresolute in short) if $f^{-1}(B) \in IFRWGO(X)$ for every IFRWGOS $B\in \sigma$,
(j) intuitionistic fuzzy totally continuous mapping [7] if the inverse image of every IFCS in Y is an intuitionistic fuzzy clopen subset in X,

Definition 2.12: [8] An IFTS (X, τ) is said to be an intuitionistic fuzzy rwT1/2 ((IF rwT1/2 in short) space if every IFRWGCS in X is an IFCS in X.

Definition 2.13: **[8]** An IFTS (X, τ) is said to be an intuitionistic fuzzy rwgT1/2 ((IF rwgT1/2 in short) space if every IFRWGCS in X is an IFPCS in X.

III. Intuitionistic Fuzzy Perfectly Regular Weakly Generalized Continuous Mappings

In this section, we introduce intuitionistic fuzzy perfectly regular weakly generalized continuous mappings and study some of their properties.

Definition 3.1: A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be an intuitionistic fuzzy perfectly regular weakly generalized continuous (IF perfectly RWG continuous in short) mapping if the inverse image of every IFRWGCS of Y is intuitionistic fuzzy clopen in X.

Theorem 3.2: Every intuitionistic fuzzy perfectly regular weakly generalized continuous mapping is an intuitionistic fuzzy continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFRWGCS, A is an IFRWGCS in Y. By hypothesis, $f^{-1}(A)$ is intuitionistic fuzzy clopen in X. Thus $f^{-1}(A)$ is an IFCS in X. Therefore f is an intuitionistic fuzzy continuous mapping.

Example 3.3: Let X = {a, b}, Y = {u, v} and T_1 = ⟨ x, (0.6, 0.3), (0.6, 0.4)⟩ , T_2 = ⟨ y, (0.6, 0.3), (0.6, 0.4)⟩ . Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Consider a mapping f : (X, τ) → (Y, σ) defined as f (a) = u and f (b) = v. This f is an intuitionistic fuzzy continuous mapping but not an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping, since the IFS B = ⟨ y, (0.4, 0.6), (0.3, 0.2)⟩ is an IFRWGCS in Y but f^{-1} (B) = ⟨ x, (0.4, 0.6), (0.3, 0.2)⟩ is not intuitionistic fuzzy clopen in X.

Theorem 3.4: Every intuitionistic fuzzy perfectly regular weakly generalized continuous mapping is an intuitionistic fuzzy α continuous mapping but not conversely.

Proof: Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFRWGCS, A is an IFRWGCS in Y. By hypothesis, f^{-1}(A) is intuitionistic fuzzy clopen in X. Thus f^{-1}(A) is an IFCS in X. Since every IFCS is an IFαCS, f^{-1}(A) is an IFαCS in X. Hence f is an intuitionistic fuzzy α continuous mapping.

Example 3.5: Let X = {a, b}, Y = {u, v} and T_1 = ⟨ x, (0.3, 0.5), (0.4, 0.5)⟩ , T_2 = ⟨ y, (0.3, 0.5), (0.4, 0.5)⟩ . Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Consider a mapping f : (X, τ) → (Y, σ) defined as f (a) = u and f (b) = v. This f is an intuitionistic fuzzy α continuous mapping but not an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping, since the IFS B = ⟨ y, (0.4, 0.6), (0.3, 0.1)⟩ is an IFRWGCS in Y but f^{-1}(B) = ⟨ x, (0.4, 0.6), (0.3, 0.1) ⟩ is not intuitionistic fuzzy clopen in X.

Theorem 3.6: Every intuitionistic fuzzy perfectly regular weakly generalized continuous mapping is an intuitionistic fuzzy pre continuous mapping but not conversely.

Proof: Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFRWGCS, A is an IFRWGCS in Y. By hypothesis, f^{-1}(A) is intuitionistic fuzzy clopen in X. Thus f^{-1}(A) is an IFCS in X. Since every IFCS is an IFPCS, f^{-1}(A) is an IFPCS in X. Hence f is an intuitionistic fuzzy pre continuous mapping.

Example 3.7: Let X = {a, b}, Y = {u, v} and T_1 = ⟨ x, (0.3, 0.4), (0.4, 0.5)⟩ , T_2 = ⟨ y, (0.3, 0.4), (0.4, 0.5)⟩ . Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y, respectively. Consider a mapping f : (X, τ) → (Y, σ) defined as f (a) = u and f (b) = v. This f is an intuitionistic fuzzy pre continuous mapping but not an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping, since the IFS B = ⟨ y, (0.7, 0.3), (0.3, 0.2)⟩ is an IFRWGCS in Y but f^{-1}(B) = ⟨ x, (0.7, 0.3), (0.3, 0.2)⟩ is not intuitionistic fuzzy clopen in X.

Theorem 3.8: Every intuitionistic fuzzy perfectly regular weakly generalized continuous mapping is an intuitionistic fuzzy generalized continuous mapping but not conversely.

Proof: Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFRWGCS, A is an IFRWGCS in Y. By hypothesis, f^{-1}(A) is intuitionistic fuzzy clopen in X. Thus f^{-1}(A) is an IFCS in X. Since every IFCS is an IFGCS, f^{-1}(A) is an IFGCS in X. Hence f is an intuitionistic fuzzy generalized continuous mapping.

Example 3.9: Let X = {a, b}, Y = {u, v} and T_1 = ⟨ x, (0.3, 0.3), (0.4, 0.5)⟩ , T_2 = ⟨ y, (0.3, 0.3), (0.4, 0.5)⟩ . Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Consider a mapping f : (X, τ) → (Y, σ) defined as f (a) = u and f (b) = v. This f is an intuitionistic fuzzy generalized continuous mapping but not an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping, since the IFS B = ⟨ y, (0.5, 0.6), (0.4, 0.2)⟩ is an IFRWGCS in Y but f^{-1}(B) = ⟨ x, (0.5, 0.6), (0.4, 0.2)⟩ is not intuitionistic fuzzy clopen in X.

Theorem 3.10: Every intuitionistic fuzzy perfectly regular weakly generalized continuous mapping is an intuitionistic fuzzy α generalized continuous mapping but not conversely.

Proof: Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFRWGCS, A is an IFRWGCS in Y. By hypothesis, f^{-1}(A) is intuitionistic fuzzy clopen in X. Thus f^{-1}(A) is an IFCS in X. Since every IFCS is an IFαGCS, f^{-1}(A) is an IFαGCS in X. Hence f is an intuitionistic fuzzy α generalized continuous mapping.

Example 3.11: Let X = {a, b}, Y = {u, v} and T_1 = ⟨ x, (0.3, 0.3), (0.5, 0.5)⟩ , T_2 = ⟨ y, (0.3, 0.3), (0.5, 0.5)⟩ . Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Consider a mapping f : (X, τ) → (Y, σ) defined as f (a) = u and f (b) = v. This f is an intuitionistic fuzzy α generalized continuous mapping but not an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping, since the IFS B = ⟨ y, (0.4, 0.4), (0.3, 0.3)⟩ is an IFRWGCS in Y but $f^{-1}(B)$ = ⟨ x, (0.4, 0.4), (0.3, 0.3)⟩ is not intuitionistic fuzzy clopen in X.

Theorem 3.12: Every intuitionistic fuzzy perfectly regular weakly generalized continuous mapping is an intuitionistic fuzzy almost regular weakly generalized continuous mapping but not conversely.

Proof: Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping. Let A be an IFRCS in Y. Since every IFRCS is an IFRWGCS, A is an IFRWGCS in Y. By hypothesis, $f^{-1}(A)$ is intuitionistic fuzzy clopen in X. Thus $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFRWGCS, $f^{-1}(A)$ is an IFRWGCS in X. Hence f is an intuitionistic fuzzy almost regular weakly generalized continuous mapping.

Example 3.13: Let X = {a, b}, Y = {u, v} and T_1 = ⟨ x, (0.4, 0.4), (0.6, 0.6)⟩ , T_2 = ⟨ y, (0.4, 0.4), (0.6, 0.6)⟩ . Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Consider a mapping f : (X, τ) → (Y, σ) defined as f (a) = u and f (b) = v. This f is an intuitionistic fuzzy almost regular weakly generalized continuous mapping but not an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping, since the IFS B = ⟨ y, (0.6, 0.7), (0.4, 0.3)⟩ is an IFRWGCS in Y but $f^{-1}(B)$ = ⟨ x, (0.6, 0.7), (0.3, 0.2)⟩ is not intuitionistic fuzzy clopen in X.

Theorem 3.14: Every intuitionistic fuzzy perfectly regular weakly generalized continuous mapping is an intuitionistic fuzzy almost continuous mapping but not conversely.

Proof: Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping. Let A be an IFRCS in Y. Since every IFRCS is an IFRWGCS, A is an IFRWGCS in Y. By hypothesis, $f^{-1}(A)$ is intuitionistic fuzzy clopen in X. Thus $f^{-1}(A)$ is an IFCS in X. Hence f is an intuitionistic fuzzy almost continuous mapping.

Example 3.15: Let X = {a, b}, Y = {u, v} and T_1 = ⟨ x, (0.4, 0.4), (0.4, 0.5)⟩ , T_2 = ⟨ y, (0.4, 0.4), (0.4, 0.5)⟩ . Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Consider a mapping f : (X, τ) → (Y, σ) defined as f (a) = u and f (b) = v. This f is an intuitionistic fuzzy almost continuous mapping but not an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping, since the IFS B = ⟨ y, (0.6, 0.6), (0.4, 0.3)⟩ is an IFRWGCS in Y but $f^{-1}(B)$ = ⟨ x, (0.6, 0.6), (0.4, 0.3)⟩ is not intuitionistic fuzzy clopen in X.

Theorem 3.16: Every intuitionistic fuzzy perfectly regular weakly generalized continuous mapping is an intuitionistic fuzzy regular weakly generalized continuous mapping but not conversely.

Proof: Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFRWGCS, A is an IFRWGCS in Y. By hypothesis, $f^{-1}(A)$ is intuitionistic fuzzy clopen in X. Thus $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFRWGCS, $f^{-1}(A)$ is an IFRWGCS in X. Hence f is an intuitionistic fuzzy regular weakly generalized continuous mapping.

Example 3.17: Let X = {a, b}, Y = {u, v} and T_1 = ⟨ x, (0.3, 0.2), (0.5, 0.5)⟩ , T_2 = ⟨ y, (0.3, 0.2), (0.5, 0.5)⟩ . Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y, respectively. Consider a mapping f : (X, τ) → (Y, σ) defined as f (a) = u and f (b) = v. This f is an intuitionistic fuzzy regular weakly generalized continuous mapping but not an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping, since the IFS B = ⟨ y, (0.7, 0.6), (0.3, 0.3)⟩ is an IFRWGCS in Y but $f^{-1}(B)$ = ⟨ x, (0.7, 0.6), (0.3, 0.3)⟩ is not intuitionistic fuzzy clopen in X.

Theorem 3.18: Let f : (X, τ) → (Y, σ) be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then the following statements are equivalent.

(a) f is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping,
(b) $f^{-1}(B)$ is intuitionistic fuzzy clopen in X for every IFRWGOS B in Y.

Proof: (a) ⇒ (b): Let B be an IFRWGOS in Y. Then B^c is an IFRWGCS in Y. Since f is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping, $f^{-1}(B^c)$ = $(f^{-1}(B))^c$ is intuitionistic fuzzy clopen in X. This implies $f^{-1}(B)$ is intuitionistic fuzzy clopen in X.

(b) ⇒ (a): Let B be an IFRWGCS in Y. Then B^c is an IFRWGOS in Y. By hypothesis, $f^{-1}(B^c) = (f^{-1}(B))^c$ is intuitionistic fuzzy clopen in X, which implies $f^{-1}(B)$ is intuitionistic fuzzy clopen in X. Therefore f is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping.

Theorem 3.19: If f : (X, τ)→ (Y, σ) is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping from an IFTS (X, τ) into an IFTS (Y, σ), then f(cl(A)) ⊆ rwgcl(f(A)) for every IFS A in X.

Proof: Let A be an IFS in X. Then rwgcl(f(A)) is an IFRWGCS in Y. Since f is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping, f^{-1}(rwgcl(f(A))) is intuitionistic fuzzy clopen in X. Thus f^{-1}(rwgcl(f(A))) is an IFCS in X. Clearly A ⊆ f^{-1}(rwgcl(f(A))). Therefore, cl(A) ⊆ cl(f^{-1}(rwgcl(f(A)))) = f^{-1}(rwgcl(f(A))). Hence f(cl(A)) ⊆ rwgcl(f(A)) for every IFS A in X.

Theorem 3.20: If f : (X, τ) → (Y, σ) is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping from an IFTS (X, τ) into an IFTS (Y, σ), then cl(f^{-1}(B)) ⊆ f^{-1}(rwgcl(B)) for every IFS B in Y.

Proof: Let B be an IFS in Y. Then rwgcl(B) is an IFRWGCS in Y. By hypothesis, f^{-1}(rwgcl (B)) is intuitionistic fuzzy clopen in X. Thus f^{-1}(rwgcl(B)) is an IFCS in X. Clearly B ⊆ rwgcl(B) implies f^{-1}(B) ⊆ f^{-1}(rwgcl(B)). Therefore cl(f^{-1}(B)) ⊆ cl(f^{-1}(rwgcl (B))) = f^{-1}(rwgcl(B)). Hence cl(f^{-1}(B)) ⊆ f^{-1}(rwgcl(B)) for every IFS B in Y.

Theorem 3.21: If f : (X, τ) → (Y, σ) is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping from an IFTS (X, τ) into an IFTS (Y,σ), then f^{-1}(rwgint(B)) ⊆ int(f^{-1}(B)) for every IFS B in Y.

Proof: Let B be an IFS in Y. Then rwgint(B) is an IFRWGOS in Y. By hypothesis, f^{-1}(rwgint(B)) is intuitionistic fuzzy clopen in X. Thus f^{-1}(rwgint(B)) is an IFOS in X. Clearly wgint(B) ⊆ B implies f^{-1}(rwgint(B)) ⊆ f^{-1}(B). Therefore int(f^{-1}(rwgint(B))) ⊆ int(f^{-1}(B)). Hence, f^{-1}(rwgint(B)) ⊆ int(f^{-1}(B)) for every IFS B in Y.

Theorem 3.22: The composition of two intuitionistic fuzzy perfectly regular weakly generalized continuous mapping is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping in general.

Proof: Let A be an IFRWGCS in Z. By hypothesis, g^{-1}(A) is intuitionistic fuzzy clopen in Y and hence an IFCS in Y. Since every IFCS is an IFRWGCS, g^{-1}(A) is an IFRWGCS in Y. Further, since f is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping, f^{-1} (g^{-1} (A)) = $(gof)^{-1}$(A) is intuitionistic fuzzy clopen in X. Hence gof is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping.

Theorem 3.23: Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, δ) be any two mappings. Then the following statements hold.

(i) Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy continuous mapping and g : (Y, σ) → (Z, δ) an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping. Then their composition gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping.

(ii) Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping and g : (Y, σ) → (Z, δ) an intuitionistic fuzzy continuous mapping [respectively intuitionistic fuzzy α continuous mapping, intuitionistic fuzzy pre continuous mapping, intuitionistic fuzzy α generalized continuous mapping and intuitionistic fuzzy generalized continuous mapping]. Then their composition gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy continuous mapping.

(iii) Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping and g : (Y, σ) → (Z, δ) an intuitionistic fuzzy regular weakly generalized continuous mapping. Then their composition gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy continuous mapping.

Proof: (i) Let A be an IFRWGCS in Z. By hypothesis, g^{-1}(A) is intuitionistic fuzzy clopen in Y and hence an IFCS in Y. Since f is an intuitionistic fuzzy continuous mapping, f^{-1} (g^{-1} (A)) = $(gof)^{-1}$(A) is an IFCS in X. Hence gof is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping.

(ii) Let A be an IFCS in Z. By hypothesis, g^{-1}(A) is an IFCS [respectively IFαCS, IFPCS, IFαGCS and IFGCS] in Y. Since every IFCS [respectively IFαCS, IFPCS, IFαGCS and IFGCS] is an IFRWGCS, g^{-1}(A) is an IFRWGCS in Y. Then f^{-1} (g^{-1} (A)) = $(gof)^{-1}$(A) is intuitionistic fuzzy clopen in X, by hypothesis. Thus $(gof)^{-1}$(A) is an IFCS in X. Hence gof is an intuitionistic fuzzy continuous mapping.

(iii) Let A be an IFCS in Z. By hypothesis, g^{-1}(A) is an IFRWGCS in Y. Since f is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is intuitionistic fuzzy clopen in X. Thus $(gof)^{-1}(A)$ is an IFCS in X. Hence gof is an intuitionistic fuzzy continuous mapping.

Theorem 3.24: If f : (X, τ) → (Y, σ) is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping and g : (Y, σ) → (Z, δ) is an intuitionistic fuzzy regular weakly generalized irresolute mapping, then gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping.

Proof: Let A be an IFRWGCS in Z. By hypothesis, $g^{-1}(A)$ is an IFRWGCS in Y. Since f is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is intuitionistic fuzzy clopen in X. Hence gof is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping.

Theorem 3.25: Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping and g : (Y, σ) → (Z, δ) be any mapping. Then gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping if and only if g is an intuitionistic fuzzy regular weakly generalized irresolute mapping.

Proof: Let g : (Y, σ) → (Z, δ) be an intuitionistic fuzzy regular weakly generalized irresolute mapping. Then the proof follows from the theorem 3.24.

Conversely, let gof : (X, τ) → (Z, δ) be an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping. Let A be an IFRWGCS in Z. Since gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping, $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy clopen in X. Since f is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping, $g^{-1}(A)$ is an IFRWGCS in Y. Thus the inverse image of each IFRWGCS in Z is an IFRWGCS in Y. Hence g is an intuitionistic fuzzy regular weakly generalized irresolute mapping.

IV. Intuitionistic Fuzzy Perfectly Regular Weakly Generalized Open Mappings

In this section, we introduce intuitionistic fuzzy perfectly regular weakly generalized open mappings and study some of their properties.

Definition 4.1: A mapping f : (X, τ) → (Y, σ) is said to be an intuitionistic fuzzy perfectly regular weakly generalized open mapping if the image of every IFRWGOS in X is intuitionistic fuzzy clopen in Y.

Theorem 4.2: Let f : (X, τ) → (Y, σ) be a mapping from an IFTS (X, τ) into an IFTS (Y, σ), then the following statements are equivalent.
(a) f is an intuitionistic fuzzy perfectly regular weakly generalized open mapping,
(b) f(B) is intuitionistic fuzzy clopen in Y for every IFRWGCS B in X.

Proof: (a) ⇒ (b): Let B be an IFRWGCS in X. Then B^c is an IFRWGOS in X. Since f is an intuitionistic fuzzy perfectly regular weakly generalized open mapping, $f(B^c) = (f(B))^c$ is intuitionistic fuzzy clopen in Y. This implies f(B) is intuitionistic fuzzy clopen in Y.

(b) ⇒ (a): Let B be an IFRWGOS in X. Then B^c is an IFRWGCS in X. By hypothesis, $f(B^c) = (f(B))^c$ is intuitionistic fuzzy clopen in Y, which implies that f(B) is intuitionistic fuzzy clopen in Y. Therefore f is an intuitionistic fuzzy perfectly regular weakly generalized open mapping.

Theorem 4.3: Let f : (X, τ) → (Y,σ) be a bijective mapping from an IFTS (X, τ) into an IFTS (Y,σ), then the following statements are equivalent.

(a) Inverse of f is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping.
(b) f is an intuitionistic fuzzy perfectly regular weakly generalized open mapping.

Proof: (a) ⇒ (b): Let A be an IFRWGOS of X. By assumption, $(f^{-1})^{-1}(A) = f(A)$ is intuitionistic fuzzy clopen in Y. Hence f is an intuitionistic fuzzy perfectly regular weakly generalized open mapping.

(b) ⇒ (a): Let B be an IFRWGOS in X. Then f(B) is intuitionistic fuzzy clopen in Y. That is

$(f^{-1})^{-1}(B) = f(B)$ is intuitionistic fuzzy clopen in Y. Therefore f^{-1} is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping.

Theorem 4.4: The composition of two intuitionistic fuzzy perfectly regular weakly generalized open mapping is again an intuitionistic fuzzy perfectly regular weakly generalized open mapping.

Proof: Suppose f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, δ) are any two intuitionistic fuzzy perfectly regular weakly generalized open mapping. Let A be an IFRWGOS in X. Since f is an intuitionistic fuzzy perfectly regular weakly generalized open mapping, f(A) is intuitionistic fuzzy clopen in Y. Hence it is an IFOS in Y. But every IFOS is an IFRWGOS, which implies f(A) is an IFRWGOS in Y. Since g is an intuitionistic fuzzy perfectly regular weakly generalized open mapping, g(f(A)) = (gof)(A) is intuitionistic fuzzy clopen in Z. Thus the image of each IFRWGOS in X is intuitionistic fuzzy clopen in Z. Therefore gof: (X, τ) → (Z, δ) is intuitionistic fuzzy perfectly regular weakly generalized open mapping.

Theorem 4.5: If f : (X, τ) → (Y, σ) is an intuitionistic fuzzy regular weakly generalized * open mapping and g : (Y, σ) → (Z, δ) is an intuitionistic fuzzy perfectly regular weakly generalized open mapping, then their composition gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy perfectly regular weakly generalized open mapping.

Proof: Let A be an IFRWGOS in X. Since f is an intuitionistic fuzzy regular weakly generalized * open mapping, f(A) is an IFRWGOS in Y. Further, since g is an intuitionistic fuzzy perfectly regular weakly generalized open mapping, g(f(A)) = (gof)(A) is intuitionistic fuzzy clopen in Z. Hence gof : (X, τ) → (Z,δ) is an intuitionistic fuzzy perfectly regular weakly generalized open mapping.

Theorem 4.6: Let f : (X, τ) → (Y, σ) and g : (Y,σ) → (Z, δ) be two mappings such that gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy perfectly regular weakly generalized open mapping. Then the following statements hold.

(a) If f is an intuitionistic fuzzy regular weakly generalized irresolute mapping and surjective, then g is an intuitionistic fuzzy perfectly regular weakly generalized open mapping.
(b) If g is an intuitionistic fuzzy totally continuous mapping and injective, then f is an intuitionistic fuzzy perfectly regular weakly generalized open mapping.

Proof: (a) Let A be an IFRWGOS in Y. Then $f^{-1}(A)$ is an IFRWGOS in X, because f is an intuitionistic fuzzy regular weakly generalized irresolute mapping. Since (gof) is an intuitionistic fuzzy perfectly regular weakly generalized open mapping, $(gof)(f^{-1}(A)) = g(A)$ is intuitionistic fuzzy clopen in Z. This shows that g is an intuitionistic fuzzy perfectly regular weakly generalized open mapping.

(b) Since g is injective, we have, $f(A) = g^{-1}(gof)(A)$ is true for every subset A of X. Let B be an IFRWGOS in X. Therefore (gof)(B) is intuitionistic fuzzy clopen in Z and hence an IFOS in Z. Since g is intuitionistic fuzzy totally continuous, $g^{-1}(gof)(A) = f(A)$ is intuitionistic fuzzy clopen in Y. This shows that f is an intuitionistic fuzzy perfectly regular weakly generalized open mapping.

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