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(M, N)-Jordan Left Derivation on Matrix Ring

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Abstract: *In this paper ,we introduced a new definition which is the definition of (m,n)-Jordan left derivation and we prove that any (m,n)-Jordan left derivation on the full matrix ring is identically zero also we describe the structure of (m,n)-Jordan left derivation on the upper triangular matrix ring .*
Keywords: *Left derivation , Jordan Left Derivation.*

I. Introduction

Throughout this paper ,R will represent an associative ring with center Z(R).A ring R is n-Torsion free where $n > 1$ is an integer ,in case nx=0 , x∈ R implies x=0.Let R be a ring and let M be a left R-module .An additive mapping D:R→ M is said to be left derivation (resp.Jordan left derivation)if D(xy)=xD(y)+yD(x) holds $\forall x, y \in R$. (if $D(x^2) = 2xD(x) \forall x \in R$).obviously any left derivation is a Jordan left derivation but in general the converse is not true (see [11] ,example 1.1).the concepts of left derivation and Jordan left derivation were introduced by Bresar and Vukman [1],one can easly prove that the existence of a non-zero left derivation D:R→ R ,where R is prime ring of ChR≠ 2 ,forces the ring R to be commutative .Moreover ,any Jordan derivation which maps a non-commutative prime ring R of ChR≠ 2 into it self is zero. this result was first proved by Bresar and Vukman in [1] under the additional assumption that R is also of ChR≠ 3.Later Deng[2] removed the assumption that R is of ChR≠ 3 .(see also [6]). In [10]Vukman introduced the definition of (m,n)-Jordan derivation and study it on prime ring with char(R)≠ $2mn(m + n)$.Recently ,Vukman [9] has proved that ,in case D:R→ R is a Jordan left derivation ,where R is 2-torsion free semi-prime ring ,then D is a derivation which maps R into Z(R) .In [3] authors prove that if R is a 2-torsion free ring with identity ,then any Jordan left derivation (hence , any left derivation)on the full matrix ring $M_n(R), n \geq 2$ is identically zero . In this paper ,we give a new definitions which is definition of (m,n)-Jordan left derivation and we prove that any (m,n)-Jordan left derivation on the full matrix ring is identically zero also we describe the structure of (m,n)-Jordan left derivation on the upper triangular matrix ring .

II. Main Result And Proofs

In this section, after we proof the main results we introduced a new definition which is definition of (m,n)-Jordan left derivation

Definition 2.1:- Let $m \geq 0, n \geq 0$ be some fixed integers with $m + n \neq 0$.An additive map $D: R \to R$ is called (m,n)-Jordan left derivation if the following condition
$(m+n)D(x^2) = 2mxD(x) + 2nxD(x) \quad \forall x \in R$ holds …………(1)
It is easy to see that (1,0)- Jordan left derivation and (0,1)- Jordan left derivation be an ordinary Jordan left derivation.

Now we shall prove the main results in this paper .
Theorem2.2:- Let R be a 2(m+n)-torsion free ring with identity and let $ń \geq 2$.Then any (m,n)-Jordan left derivation D on the ring $M_{ń}(R)$ is identically zero .

Proof:- Since $(m+n)D(x^2) = 2mxD(x) + 2nxD(x) \quad \forall x \in R$
Replace x by x +y
$(m+n)D((x + y)^2) = 2(m + n)(x + y)D(x + y)$
$=2(m+n)(xD(x)+yD(y)+xD(y)+yD(x))$
On the other hand
$(m+n)D((x + y)^2) = (m+n) D(x^2) + (m + n)D(xy + yx) + (m + n)D(y^2)$
Then by comparing these two expression we get
$(m+n)D(xy+yx)=2(m+n)xD(y)+2(m+n)yD(x) \quad \forall x, y \in R$…………(2)

If $(a_{rs}) \in M_{ń}(R)$ the following conclusion holds
If$(m+n)(a_{rs}) = 2(m + n)E_{ii}(a_{rs})$ then $(a_{rs}) = 0$……………………..(3)

Fix $i \in N$,Since $E_{ii}^2 = E_{ii}$
(m+n)$D(E_{ii}{}^2) = 2(m+n)E_{ii}D(E_{ii})$
(m+n)$D(E_{ii}) = 2(m+n)E_{ii}D(E_{ii})$
Then by (3),we have
$D(E_{ii}) = 0 \quad \forall\, 1 \le i \le ń$ ………………..(4)

Now ,fix $i \neq j$ in N. from
(m+n)E_{ij}=(m+n)($E_{ij}E_{jj} + E_{jj}E_{ij}$)
(m+n)$D(E_{ij})$=(m+n)D ($E_{ij}E_{jj} + E_{jj}E_{ij}$)
=2(m+n)(E_{ij} D (E_{jj}) + E_{jj} D(E_{ij}))
=2(m+n)E_{jj} D(E_{ij})
Then by (3),we have
$D(E_{ij}) = 0 \quad \forall\, 1 \le i, j \le ń$ ....………….(5)

Next ,we show that $\forall\, r \in R, i \neq j$ in N, $D(r E_{ij}) = 0$
(m+n) rE_{ij}=(m+n)($rE_{ij}E_{jj} + E_{jj}\, rE_{ij}$)
(m+n) $D(rE_{ij})$=2(m+n) $rE_{ij}D(E_{jj}) + 2(m+n)E_{jj}\, D(rE_{ij})$
(m+n) $D(rE_{ij})$=2(m + n)E_{jj} D(r E_{ij})
Then by (3),we have
$D(rE_{ij}) = 0 \;\forall\, r \in R, i \neq j$ in N … … …. (6)

In the next step ,we show that for any $r \in R$ and $i \in$N, $D(r E_{ii}) = 0$
Fix $i \neq j$ in N. and set
E=$E_{ii} + E_{jj}$
(m+n)D(rE)= (m+n)D($rE_{ii} + rE_{jj}$)
=(m+n)(D((r E_{ij})E_{ji} + E_{ji}(rE_{ij})))
=2(m+n)(r E_{ij})$D(E_{ji}) + 2(m+n)E_{ji}D(rE_{ij})$)
Then (m+n)D(rE)=0
2(m+n)$rE_{ii} = 2(m+n)rEE_{ii} = (m+n)(rEE_{ii} + E_{ii}(rE))$
2(m+n)$D(rE_{ii}) = 2(m+n)(rE\, D(E_{ii}) + E_{ii}\, D(rE))$
$D(rE_{ii}) = 0$………………(7)

Then D=0 on $M_{ń}(R)$■

Let R and S be a 2(m+n)-torsion free ring with identity ,M be a 2(m+n)-torsion free (R,S)-bimodule ,and T be the upper triangular matrix ring $\begin{bmatrix} R & M \\ 0 & S \end{bmatrix}$ with the usual addition and multiplication of matrices .the following theorem describes the structure of (m,n)-Jordan left derivation of T .

Theorem 2.3:- Let the ring T be as above ,and let $D: T \to T$ be a (m,n)-Jordan left derivation .then there exist (m,n)-Jordan left derivations $\delta: R \to R, \lambda: R \longrightarrow M. \gamma: S \to S$ such that $M\,\gamma(S) = 0$ *and*
$$D\begin{bmatrix} r & ḿ \\ 0 & s \end{bmatrix} = \begin{bmatrix} \delta(r) & \lambda(r) \\ 0 & \gamma(s) \end{bmatrix}$$

Proof:-Linearizing (m+n)$D(x^2) = 2mxD(x) + 2nxD(x)$
(m+n)D(xy+yx)=2(m+n)xD(y)+2(m+n)yD(x)
Applying D on $I^2 = I$ *and* $E_{ii}^2 = E_{ii} \quad (i = 1,2)$
$D(E_{11}) = D(E_{22}) = D(I) = 0$
Let $ḿ \in M.from$
$ḿE_{12}$=$E_{11}(ḿE_{12}) + (ḿE_{12})E_{11}$
(m+n)D($ḿE_{12}$)=$(m+n)D(E_{11}(ḿE_{12}) + (ḿE_{12})E_{11})$
D($ḿE_{12}$)=2$(m+n)E_{11}\, D(ḿE_{12})$+2$(m+n)ḿE_{12}$D(E_{11})
By (3), we have
D($ḿE_{12}$) $= 0 \;\forall m \in M$ ...…………………(8)

Now ,let s$\in S$ and suppose that
D(sE_{22}) $= (a_{ij}) \in T$

2(m+n) s$E_{22} = (m+n)((sE_{22})E_{22} + E_{22}(sE_{22}))$
2(m+n)D(s$E_{22}) = 2(m+n)\ (sE_{22})D(E_{22}) + 2(m+n)E_{22}D(sE_{22})$
2(m+n)D(s$E_{22}) = 2(m+n)E_{22}D(sE_{22})$
Since 2(m+n)$a_{11} = 0 = 2(m+n)a_{12}$ *then* $a_{11} = a_{12} = 0$.
And so D induced a map $\gamma: S \to S$
D(s$E_{22}) = \gamma(s)E_{22}\ \forall s \in S$ ……………………..(9)

Since D is additive ,so is γ.
Since $s^2E_{22} = (sE_{22})^2$
(m+n)$s^2E_{22} = (m+n)(sE_{22})^2$
(m+n)$D(s^2E_{22})$=2(m+n) $sE_{22}\ D(sE_{22})$
(m+n)$D(s^2E_{22})$= 2(m+n)$sE_{22}\ \gamma(s)E_{22}$
(m+n) $\gamma(s^2)E_{22} = 2(\mathrm{m+n})\ s\,\gamma(s)E_{22}$
(m+n) $\gamma(s^2) = 2(\mathrm{m+n})\ s\,\gamma(s)$
Proving that γ *is* $(m,n) -$ *Jordan left derivation on* S .

Next, let r$\in R$ and assume that
D(r$E_{11}) = (b_{ij}) \in T$.
2(m+n)r$E_{11} = (m+n)(rE_{11}.E_{11} + E_{11}.rE_{11})$
2(m+n)D(r$E_{11}) = 2(m+n)rE_{11}.D(E_{11}) + 2(m+n)\ E_{11}.D(rE_{11})$
2(m+n)D(r$E_{11}) = 2(m+n)\ E_{11}.D(rE_{11})$
$b_{22} = 0$ *then* D induced $\delta: R \to R, \lambda: R \longrightarrow M$.
D(r$E_{11}) = \delta(r)E_{11} + \lambda(r)E_{12}$ ………………(10)

By similar argument as above ,one can show that δ and λ are also (m,n)-Jordan left derivation .Now ,in view of (8),(9) and (10) for every $\begin{bmatrix} r & \acute{m} \\ 0 & s \end{bmatrix}$ in T, we have
D$\begin{bmatrix} r & \acute{m} \\ 0 & s \end{bmatrix}$ = D(rE_{11}) + D($\acute{m}E_{12}$) +D(sE_{22})
= $\delta(r)E_{11} + \lambda(r)E_{12} + \gamma(s)E_{22}$
=$\begin{bmatrix} \delta(r) & \lambda(r) \\ 0 & \gamma(s) \end{bmatrix}$
Since $\acute{m}sE_{12} = (\acute{m}E_{12})(sE_{22}) + (sE_{22})\acute{m}E_{12}$
(m+n)D($\acute{m}sE_{12}) = 2(\mathrm{m+n})(\acute{m}E_{12})D(sE_{22}) + 2(\mathrm{m+n})(sE_{22})D(\acute{m}E_{12})$
0=$2(\mathrm{m+n})(\acute{m}E_{12})(\gamma(s)E_{22})$
and since M is 2(m+n)-torsion free module,then
0=$\acute{m}\,\gamma(s)E_{12}E_{22}$ and so $\acute{m}\,\gamma(s)E_{12} = 0$ then
$\acute{m}\,\gamma(s)$=0■

Corollary2.4 :- Let T and D as above and assume that M is a faithful right S-module then $\gamma = 0$.

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