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Properties of NANO GB-Closed Maps

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Abstract: *The aim of this paper is to introduce a new class of maps called nano gb-closed maps in nano topological spaces. Also some characterizations and several properties concerning nano gb-closed maps and strongly nano gb-closed maps are derived. 2010 AMS Subject Classification: 54A05, 54C08.*
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I. Introduction

Levine [8] introduced the concept of generalized closed sets in topological space. The notion of generalized b-closed sets and its various characterizations were given by Ahmad Al.Omari and Mohd.Salmi Md. Noorani in 2009 [2]. The concept of nano topology was introduced by Lellis Thivagar [9] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established the weak forms of nano open sets namely nano α -open sets, nano semi open sets and nano pre open sets in a nano topological space. Since the advent of these notions several research papers with interesting results in different respects came to existence.

The purpose of the present paper is to introduce and investigate some of the fundamental properties of nano gb-closed maps and strongly nano gb-closed maps.

II. Preliminaries

Definition 2.1[12] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$. That is
$L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by $x \in U$.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is
$B_R(X) = U_R(X) – L_R(X)$.

Definition 2.2[10] If (U, R) is an approximation space and X, Y $\subseteq$ U, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$
(ii) $L_R(\phi) = U_R(\phi) =$ and $L_R(U) = U_R(U) = U$
(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
(viii) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
(ix) $U_RU_R(X) = L_RU_R(X) = U_R(X)$
(x) $L_RL_R(X) = U_RL_R(X) = L_R(X)$

Definition 2.3[9] Let U be an universe, R be an equivalence relation on U and $\tau_R(X)$ = {U, ϕ, $L_R(X)$, $U_R(X)$, $B_R(X)$} where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms:

(i) U and $\phi \in \tau_R(X)$

(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$. That is, $\tau_R(X)$ forms a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called nano open sets.

Definition 2.4[9] Let $(U, \tau_R(X))$ be a nano topological space and A$\subseteq$U. Then, A is said to be
(i) nano semi-open if A $\subseteq$ Ncl(Nint(A))
(ii) nano pre-open if A$\subseteq$ Nint(Ncl(A))
(iii) nano α -open if A$\subseteq$ Nint(Ncl(Nint(A)))
(iv) nano semi pre-open if A $\subseteq$ Ncl(Nint(Ncl(A)))
(v) Nano b-open if A $\subseteq$ Ncl(Nint(A)) $\cup$ Nint(Ncl(A))

Definition 2.5 A subset A of a topological space $(U, \tau_R(X))$ is called

(i) nano generalized closed (briefly, nano g-closed)[4] if Ncl(A) $\subseteq$ G whenever A$\subseteq$G and G is nano open in U.
(ii) nano semi-generalized closed (briefly nano sg-closed)[5] if Nscl(A) $\subseteq$ G whenever A $\subseteq$G and G is nano semi-open in U.
(iii) nano generalized b-closed (briefly nano gb-closed), if Nbcl(A) $\subseteq$G whenever A$\subseteq$G and G is nano open in U.

Definition 2.6[10] Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be nano topological spaces, then a map $f:(U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be

(i) nano continuous if $f^{-1}(V)$ is nano closed in $(U, \tau_R(X))$ for each nano closed set V in $(V, \tau_{R'}(Y))$.
(ii) nano b-continuous if $f^{-1}(V)$ is nano b-closed in $(U, \tau_R(X))$ for each nano closed set V in $(V, \tau_{R'}(Y))$.

III. Nano gb-Closed And Strongly Nano gb-Closed Maps

This section defines two types of nano gb-closed maps via nano gb-closed sets, derives their properties and establishes their relationship.

Definition 3.1 A map $f:(U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be nano gb-closed if the image of every nano closed set in $(U, \tau_R(X))$ is nano gb-closed in $(V, \tau_{R'}(Y))$.

Definition 3.2 A map $f:(U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be nano gb-open if f(A) is nano gb-open for each nano open set A in $(U, \tau_R(X))$.

Definition 3.3 A map $f:(U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be strongly nano gb-closed if the image f(A) is nano gb-closed set in $(V, \tau_{R'}(Y))$ for each nano gb-closed set A in $(U, \tau_R(X))$.

Theorem 3.4 Every strongly nano gb-closed map is nano gb-closed but not conversely.

Proof Since every nano closed set is nano gb-closed the result follows.

Example 3.5 Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X = {a, b}, then $\tau_R(X)$ = {{U, ϕ ,{a}, {a, b, d}, {b, d}}. Let V = {x, y, z, w} with V/R' = {{x}, {y, w}, {z}} and Y = {y, w}, then $\tau_{R'}(Y)$ = {U, ϕ, {y, w}}. Define $f: U \to V$ as f(a) = x, f(b) = y, f(c) = z, f(d) = w. Then, f is nano gb-closed but not strongly nano gb-closed since for the nano gb-closed set {b, d}, f({b, d}) = {y, w} which is not nano gb-closed in V.

Definition 3.6 A map $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is said to be strongly nano gb-open if f(A) is nano gb-open in $(V,\tau_{R'}(Y))$ for each nano gb-open set A in $(U,\tau_R(X))$.

Theorem 3.7 A nano-continuous, nano b-closed function maps nano gb-closed sets into nano gb-closed sets.

Proof Let $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ be nano continuous, and nano b-closed and let A $\subseteq$ U be nano gb-closed. Let G be nano open in V such that f(A) $\subseteq$ G. Then A $\subseteq$ $f^{-1}(G)$, which is nano open in U, since f is nano continuous. Then Nbcl(A) $\subseteq$ Nbcl($f^{-1}(G)$) = $f^{-1}(G)$, since A is nano b-closed in U. Then, f(Nbcl(A)) $\subseteq$G. Since, f is nano b-closed and Nbcl(A) is nano b-closed in U, f(Nbcl(A)) is nano b-closed in V. Therefore f(Nbcl(A)) = Nbcl(f(Nbcl(A))) $\subseteq$ G and hence Nbcl(f(Nbcl(A))) $\subseteq$ G. But Nbcl(f(A)) $\subseteq$ Nbcl(f(Nbcl(A))) $\subseteq$ G. Therefore, Nbcl(f(A)) $\subseteq$ G, whenever G is nano open and f(A) $\subseteq$ G. Thus, f(A) is nano gb-closed in V. Hence f maps nano gb-closed sets into nano gb-closed sets.

Remark 3.8 A nano-continuous, nano b-closed map does not map nano gb-open sets into nano gb-open sets.

Example 3.9 Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X = {a, b}, then $\tau_{R'}(Y)$ = {{U, ϕ ,{a}, {a, b, d}, {b, d}}.Let V = {x, y, z, w} with V/R' = {{x}, {y, w}, {z}} and Y = {y, w}, then $\tau_{R'}(Y)$ = {U, ϕ, {y, w}}. Define $f: U \to V$ as f(a) = y, f(b) = x, f(c) = x, f(d) = z. Then, f is nano continuous and nano b-closed map. {b, d} is nano gb-open in U, but f({b, d}) = {x, z} is not nano gb-open in V.

Theorem 3.10 The inverse image of a nano gb-closed set is nano gb-closed under a nano b-continuous, nano closed map.

Proof Let $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ be nano b-continuous, and nano closed map. Let A be nano gb-closed in V. Let G be nano open in U such that $f^{-1}(A)$ $\subseteq$ G. Then, $G^c \subseteq$ $[f^{-1}(A)]^c$ = $f^{-1}(A^c)$. Also Nbcl($f^{-1}(A)$)$\subseteq$ f^{-1}(Ncl(A)), since f is nano b-continuous. Therefore, Nbcl($f^{-1}(A)$)$\cap$ G^c $\subseteq$ f^{-1} (Ncl(A)) $\cap$ G^c $\subseteq$ f^{-1} (Ncl(A)) $\cap$ ($f^{-1}(A^c)$), since G^c $\subseteq$ $f^{-1}(A^c)$ = f^{-1} [Ncl(A) $\cap$ A^c]. Thus, f(Nbcl($f^{-1}(A)$)$\cap$ G^c) $\subseteq$ Ncl(A) $\cap$ A^c = Ncl(A)-A. Since A is nano gb-closed by Theorem 3.8, f(Nbcl($f^{-1}(A)$ $\cap$ G^c)= ϕ. Thus, Nbcl($f^{-1}(A)$ $\cap$ G^c)= ϕ and hence Nbcl($f^{-1}(A)$)$\subseteq$ G. Therefore $f^{-1}(A)$ is nano gb-closed in U.

Theorem 3.11 The inverse image of a nano gb-open set is nano gb-open under a nano b-continuous nano closed map.

Proof follows from the previous theorem by taking complements.

Remark 3.12 The image of nano gb-closed set is not nano gb-closed under nano b-continuous nano open map.

Example 3.13 Let U = {a, b, c, d} with U/R = {{a}, {b, d}, {c}} and X = {b, d}, then $\tau_R(X)$ = {{U, ϕ, {b, d}}. Let V = {x, y, z, w} with V/R' = {{x, w}, {y}, {z}} and Y = {x, z}, then $\tau_{R'}(Y)$ = {U, ϕ, {z}, {x, z, w}, {x, w}}. Define $f: U \to V$ as f(a) = z, f(b) = z, f(c) = z, f(d) = z. Then, f is nano b-continuous and nano open map. Let A = {b, d} is nano gb-closed in U, but f({b, d}) = {z} is not nano gb-closed in V.

Remark 3.14 The image of nano gb-open set is not nano gb-open under nano b-continuous nano open map.

Example 3.15 Let U = {a, b, c, d} with U/R = {{a},{b, d}, {c}} and X = {b, d}, then $\tau_R(X)$ = {{U, ϕ, {b, d}}. Let V = {x, y, z, w} with V/R' = {{x}, {y, w}, {z}} and Y = {x, y}, then $\tau_{R'}(Y)$ = {U, ϕ, {x}, {x, y, w}, {y, w}}. Define $f: U \to V$ as f(a) = x, f(b) = x, f(c) = z, f(d) = x. Then, f is nano b-continuous and nano open map. Let A = {c} is nano gb-open in U, but f({c}) = {z} is not nano gb-open in V.

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