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Newsboy Problem with Lost Sales Recapture As Function of $\left(\frac{r}{p}\right)^m$ and Beta Distributed Demand Error

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Abstract: *We consider an extension to the lost sale recapture model in a newsvendor framework developed earlier by the authors. As in real practice, we have considered that there may be an opportunity to backlog the lost sales, by offering some incentive for waiting. The back log fill rate is modeled as a function of proportion of rebate relative to the price. The retailer's decision includes selling price, order quantity and the rebate that will maximize its expected profit. Sensitivities of the demand errors in the form of beta distribution rather than the uniform and the normal distribution serve as an extension to the previous work by the authors.*
Keywords: *newsvendor problem, lost sales, rebates, price dependent demand, beta distribution*

I. Introduction And Literature Review

This paper considers the buying and ordering policies of a newsvendor-type retailer, faced with the possibility of backordering at least some of the shortages incurred from demand underestimation. The backordering occurs through an emergency purchase of the items in question at some premium over the regular purchasing cost. In turn, the retailer offers to the end-customers left out of the initial sale a rebate incentive upon purchase of each item backordered. The problem of backordering shortage items has been considered by Weng (2004) and Zhou and Wang (2009). A different model of lost sales recapture was discussed by Arcelus, Gor and Srinivasan (2012). This paper is similar in lines of and Patel and Gor (2013, 2014(a),(b),(c)). Here, we use an entirely different fill rate function than Arcelus, Gor and Srinivasan (2012) and Patel and Gor (2013) and include sensitivities to the beta distribution over and above the one for uniform and normal distribution discussed in Patel and Gor (2014(b),(c)). We describe the characteristics of the model, develop the objective function and derive the profit-maximizing optimality conditions that are shown to be unique. We present a numerical example. In addition to illustrating the main features of the model and discussing some comparative statics of interest, this section attempts to conjecture the behavioural relationship between various parameters and variables. A conclusions section completes the paper. Table 1 lists the notations used throughout the paper.

Table 1: Notation

p	The selling price per unit (decision variable)
v	The salvage value per unsold unit
q	The order quantity (decision variable)
r	The rebate per backordered item (decision variable)
c	The acquisition cost per unit
s	The shortage penalty per unsold unit
D	The total demand rate per unit of time
g, ε	The deterministic and stochastic components, respectively, of D
a,b	The upper and lower values, respectively, of ε
μ, σ	The mean and standard deviation, respectively, of ε
f, F	The density function and the cumulative distribution function, respectively, of ε
δ_0, δ_1	The intercept and slope, respectively, of the deterministic linear demand function
	The intercept and the demand elasticity, respectively, of the iso-elastic deterministic demand function
Ω	The fill rate of backlogged demand
d	The premium on the purchase price of each backlogged unit acquired
z	The stocking factor
Λ, Φ	The expected number of leftovers and shortages, respectively
e	The price elasticity of demand
Iε	The generalized failure rate function

π(p,q,r)	The retailer's profit function
E(p,q,r)	The retailer's expected profit function
U. D.	Uniform Distribution
N. D.	Normal Distribution
B 13 D.	Beta 13 Distribution
B 33 D.	Beta 33 Distribution
B 31 D.	Beta 31 Distribution
AELD	Additive Error and Linear Demand
MEID	Multiplicative Error and Iso-elastic Demand

II. Model Formulation

In this section, we describe the key characteristics of the model, formulate the retailer's profit-maximizing objective function and derive the optimality conditions. Observe that, in the development of the models, the arguments of the functions are omitted whenever possible, to simplify notation.

III. Characteristics of the model

Characteristic 1: **Key properties of the demand function**.

The random single-period total demand, D(p,ε), is of the form:

$$\begin{aligned} D(p,\varepsilon) &= g(p)+\varepsilon, \quad \text{if additive error} \\ & g(p)\varepsilon, \qquad \text{if multiplicative error} \end{aligned} \tag{1}$$

g(p) has an IPE or increasing price elasticity, e, which satisfies the following condition:

$$e_p' = \frac{\partial e}{\partial p} \geq 0, \quad \text{where} \quad e = \frac{\partial g}{\partial p}\frac{p}{g}$$

ε has a GSIFR or generalized strictly increasing failure rate, I_ε, since

$$I_\varepsilon' = \partial I_\varepsilon / \partial \varepsilon \geq 0, \quad \text{where} \quad I_\varepsilon = \varepsilon f/(1-F)$$

Characteristic 2: **A fill rate, Ω, given by the following expression**:

$$\Omega = (\text{r/p})\text{^m}, \quad \text{where} \quad 0 < r < p, \quad 0 < \Omega < 1, \; 0 < m < \infty \tag{2}$$

Characteristic 3: **The stocking factor, z**

$$\begin{aligned} z &= q-g, \quad \text{if additive} \\ &= q/g, \quad \text{if multiplicative} \\ \Phi &= \int_z^B (\varepsilon - z) f(\varepsilon) d\varepsilon \\ \Lambda &= \int_A^z (z-\varepsilon) f(\varepsilon) d\varepsilon = \Phi + z - \mu \end{aligned} \tag{3}$$

Detailed discussion on the above three characteristics can be found in Patel and Gor (2013), Patel and Gor (2014 (a),(b),(c)).

The retailer's profit-maximizing objective

The retailer profit function is decomposable into two parts, depending upon whether the retailer order quantity exceeds or understates the demand for the product. If the first, then q exceeds D and the retailer sells D units at p per unit, disposes of the rest at a salvage value of v per unit and incurs an acquisition cost of c for each of the q units ordered. If the second, q is below D, in which case the retailer buys and sells the q units at a profit margin of (p-c) per unit, acquires a fraction Ω of the shortage demand at a premium d per unit, sells it at (p-r), the regular selling price, p, net of the per unit rebate offered, r, and pays a shortage penalty of s per unit on the rest of the merchandise. Formally, the functional form of the retailer's profit function, π(p,q,r), is as follows:

$$\begin{aligned} \pi(p,q,r) &= pD - cq + v(q-D), \quad \text{if } q \geq D \\ &= (p-c)q + \left[(p-r)-(c+d)\right]\Omega(D-q) - s(1-\Omega)(D-q), \quad \text{if } q \leq D \end{aligned} \tag{4}$$

The objective is to find the levels of p, q and r that maximizes E(p,q,r), the retailer's expected profit. Using (3) and (4), it can be readily seen that E may be written as follows:

$$E(p,q,r) = (p-c)(g+\mu) - (c-v)\Lambda - [(p-c+s)(1-\Omega) + \Omega(r+d)]\Phi, \text{ if additive}$$
$$= (p-c)g\mu - g(c-v)\Lambda - g[(p-c+s)(1-\Omega) + \Omega(r+d)]\Phi, \text{ if multiplicative} \quad (5)$$

First-order optimality conditions: To simplify the explanation, only the additive-error/linear-demand case will be discussed. The multiplicative case can be developed along the same lines. Let $E_i^{'} = \partial E / \partial i, i = p, r, Q$ be the first derivative of the expected profit with respect to each of the decision variables. Setting these derivatives to zero, we obtain the following first-order optimality conditions.

$$E_p^{'} = 0 = (g+\mu) + g_p^{'}(p-c) - (1-\Omega)\Phi + (p-c+s-r-d)\Phi\Omega_p^{'}$$
$$E_r^{'} = 0 = \Phi\Omega_r^{'}(p-c+s-r-d) - \Phi\Omega \quad (6)$$
$$E_z^{'} = 0 = -(c-v) - \Phi_z^{'}[(p-v+s) - \Omega(p-c+s-r-d)]$$

where $\Omega_p^{'}$ and $\Omega_r^{'}$ are defined in (3). The detailed economic interpretations of the optimality conditions above can be found in Patel and Gor (2014(b)).

Numerical Analysis: Given the central objective of the paper, our numerical analysis centers on the impact of fluctuations in base*m* of the fill rate function, upon the fill rate, *Ω*, and through it, upon the retailer's profit-maximizing pricing, ordering, rebate policies. All computations were carried out with MAPLE's Optimization toolbox.

Base-case numerical structure: We consider three components of the beta distribution, which may be left-skewed (Table 2.1), symmetrical (Table 2.2) or right-skewed (Table 2.3). This aspect is the most significant contribution in this paper. Each component consists of two sets of examples that serve as the base-case for the analysis of this section. The first (second) set, denoted by *AL (MI)*, assumes the deterministic demand, *g*, to be linear (iso-elastic) and its error, additive (multiplicative), i.e.

$$D(p) = \delta_0 - \delta_1 p + \varepsilon, \quad \delta_0 > 0, \quad \delta_1 > 0, \quad \text{for AL total demand}$$
$$\gamma_0 p^{-\gamma_1}\varepsilon, \quad \gamma_0 > 0, \quad 0 > \gamma_1 > 1, \quad \text{for MI total demand} \quad (7)$$

For comparability purposes, this section operates with the parameter values of Patel and Gor (2014(b),(c)) to which suitable values for the remaining parameters have been added.

The beta distribution has been selected because of its flexibility to accommodate observed phenomenon, through appropriate changes in the distribution parameters. The probability density function of the general, four-parameter beta distribution, denoted by f(y/a,b,α,β), and its corresponding standard beta density function, are listed in the following characteristic.

Characteristic 4: The beta distribution function

$$\text{Let } B(\alpha,\beta) = \int_0^1 t^{(\alpha-1)}(1-t)^{(\beta-1)}\,dt \text{ be the beta function.}$$

The generalized beta density function is

$$f(y/a,b,\alpha,\beta) = \frac{(y-a)^{(\alpha-1)}(b-y)^{(\beta-1)}}{\mathrm{B}(\alpha,\beta)(b-a)^{(\alpha+\beta-1)}}, \quad a \le y \le b; \quad \alpha,\beta > 0, \quad (8)$$

and the standard beta density function, is:

$$f(x/0,1,\alpha,\beta) = \frac{x^{(\alpha-1)}(1-x)^{(\beta-1)}}{\mathrm{B}(\alpha,\beta)}, \quad 0 \le x \le 1; \quad \alpha,\beta > 0,$$

through the variable transformation: $x = \dfrac{y-a}{b-a}$

Where b(a)is the largest (smallest) value of y; α and β, the shape parameters of the beta distribution; and B(α,β), the beta function, designed to ensure that the total area under the density curves identified by the first two equations in (8) equals 1.

Further for maximum comparability among probability distributions, all cases are related to random variable uniformly distributed over the interval (-3,500, 1,500), for the AL demand model and (0.7, 1.1), for its MI counterpart. Either support interval describes the Beta Distribution completely. To simplify the explanation, only the AL case is described in detail. The MI case follows a similar procedure. Further, U(-3,500, 1,500) yields a mean of μ_u=(b+a)/2= -1,000 and a standard deviation of $\sigma_u = (b-a)/\sqrt{12} \sim 1{,}440$. For the normal, we use the same mean of μ_n=μ_u=-1000 and assume that the (b-a) interval covers 6 standard deviations of the normal distribution, thereby $\sigma_n = 5000/6 \sim 800$. Hence, N(-1000, 800) also describes the distribution completely and ensures that the support of the uniform distribution, (-3500, 1500), covers in excess of 99% of the normal distribution. Finally, the generalized beta distribution has as values of (a,b) = (-3500, 1500), the upper and lower limits of the uniform support interval. Similarly, through the transformation in (8), the standard beta case yields (a,b)=(0, 1). The values of α and β will vary to simulate whether the distribution is left skewed (α<β), right skewed (α>β) or symmetrical (α=β). With respect to the means of the beta distributions, the mean of the standard beta, $\mu_x =$ α/ (α +β), which amounts to ¼, 1 or ¾, for the left skewed (α=1; β=3), symmetrical (α=β=3) and right-skewed (α=3; β=1)cases, respectively. For the generalized beta, with (a,b)=(-3500, 1500), we use the variable transformation of (9), to obtain the mean of y, $(\mu_y)$from(μ_x), as follows :

$$y = (b-a)x + a = 5000x - 3500$$

$$\mu_y = (b-a)\frac{\alpha}{\alpha+\beta} + a = 5000\frac{\alpha}{\alpha+\beta} - 3500 \qquad (9)$$

As shown in Tables 2.1 – 2.3 (9) leads to a value for μ_y of -2250, -1000 or 250, for the left-skewed, the symmetrical and the right-skewed beta distributions, respectively.

Table 2.1 Base Case Optimal Policies(BETA 1,3 left skewed)

DISTRIBUTION	Support[A,B] & Mean			
BETA 1,3 DISTRIBUTION With α=1,β=3 Additive Error and Linear Demand A>-a Multiplicative Error and Iso-elastic demand A>0	[-3500, 1500] , Mean = -2250 [0.7, 1.1], Mean = 0.8			
Additive Error Linear Demand Parameter values: γ0 =100000 ; γ 1 =1500 ; c = 35; d = 3; v = 10; s = 3				
Profit	p	Q	Λ	Φ
325767	49.88	22502	181	608
Multiplicative Error Iso-Elastic Demand Parameter values: γ0 = 500000000; γ 1 = 2.5; c = 35; d = 3; v = 10; s = 3				
Profit	p	Q	Λ	Φ
328813	59.95	14165	454	660

Table 2.2Base Case Optimal Policies (BETA 3,3symmetrical)

DISTRIBUTION	Support[A,B] & Mean			
BETA 3,3 DISTRIBUTION With α=3,β=3 Additive Error and Linear Demand A>-a Multiplicative Error and Iso-elastic demand A>0	[-3500, 1500] , Mean = -1000 [0.7, 1.1], Mean = 0.9			
Additive Error Linear Demand Parameter values: γ0 =100000 ; γ 1 =1500 ; c = 35; d = 3; v = 10; s = 3				
Profit	p	Q	Λ	Φ
343748	50.33	23294	295	501

Multiplicative Error Iso-Elastic Demand Parameter values: γ0 = 500000000; γ 1 = 2.5; c = 35; d = 3; v = 10; s = 3				
Profit	p	Q	Λ	Φ
373823	60.14	16153	614	502

Table 2.3Base Case Optimal Policies (BETA 3,1right skewed)

DISTRIBUTION	Support[A,B] & Mean			
BETA 3,1 DISTRIBUTION With α=3,β=1 Additive Error and Linear Demand A>-a Multiplicative Error and Iso-elastic demand A>0	[-3500, 1500] , Mean = 250 [0.7, 1.1], Mean = 1.0			
Additive Error Linear Demand Parameter values: γ0 =100000 ; γ 1 =1500 ; c = 35; d = 3; v = 10; s = 3				
Profit	p	Q	Λ	Φ
362670	50.78	24087	404	383
Multiplicative Error Iso-Elastic Demand Parameter values: γ0 = 500000000; γ 1 = 2.5; c = 35; d = 3; v = 10; s = 3				
Profit	p	Q	Λ	Φ
419418	60.25	18163	762	339

Numerical example and Sensitivity Analysis

The optimal results using MAPLE for the fill rate model with varied base on $\left(\frac{r}{p}\right)^m$ are shown in Table 3.1-3.3 (Beta D.). The reader can refer to Patel & Gor(2014(b),(c)) for comparability purposes with Uniform and Normal distribution cases. Both the cases Additive Error Linear Demand and Multiplicative Error Iso-elastic Demand are showcased to highlight the variations in the optimal solutions too.

Table 3.1 Optimal Policies for fill rate $\Omega = \left(\frac{r}{p}\right)^m$ (Beta 1,3 D.)

Additive Error Linear Demand							
m	Π*	p*	q*	r*	Ω*	Λ*	Φ*
0.5	327772	49.92	22333	4.97	0.31	140	672
1	326455	49.89	22436	7.44	0.14	167	630
2	325887	49.88	22488	9.92	0.03	179	612
3	325792	49.88	22498	11.16	0.01	181	609
Multiplicative Error Iso-Elastic Demand							
m	Π*	p*	q*	r*	Ω*	Λ*	Φ*
0.5	333261	59.79	14018	8.26	0.37	336	787
1	330587	59.94	14076	12.47	0.20	405	706
2	329240	59.98	14127	16.65	0.07	442	669
3	328939	59.97	14149	18.72	0.03	450	662

Table 3.2 Optimal Policies for fill rate $\Omega = \left(\frac{r}{p}\right)^m$ (Beta 3,3 D.)

Additive Error Linear Demand							
m	Π*	p*	q*	r*	Ω*	Λ*	Φ*
0.5	345503	50.36	23122	5.12	0.31	244	577

1	344349	50.34	23230	7.67	0.15	277	526
2	343855	50.33	23281	10.22	0.04	292	505
3	343771	50.33	23291	11.50	0.01	294	502
Multiplicative Error Iso-Elastic Demand							
m	Π*	p*	q*	r*	Ω*	Λ*	Φ*
0.5	377324	59.97	16024	8.32	0.37	497	629
1	375201	60.11	16080	12.55	0.20	566	548
2	374153	60.15	16123	16.77	0.07	603	512
3	373921	60.15	16141	18.86	0.03	611	505

Table 3.3 Optimal Policies for fill rate $\boldsymbol{\Omega = \left(\frac{r}{p}\right)^m}$ (Beta 3,1 D.)

Additive Error Linear Demand							
m	Π*	p*	q*	r*	Ω*	Λ*	Φ*
0.5	364108	50.80	23918	5.26	0.32	345	469
1	363157	50.79	24026	7.89	0.15	384	411
2	362757	50.79	24075	10.52	0.04	400	388
3	362689	50.78	24084	11.84	0.01	403	384
Multiplicative Error Iso-Elastic Demand							
m	Π*	p*	q*	r*	Ω*	Λ*	Φ*
0.5	421905	6.07	18053	8.35	0.37	646	464
1	420372	60.20	18107	12.60	0.20	716	384
2	419644	60.25	18142	16.83	0.07	750	349
3	419484	60.2.5	18154	18.94	0.03	758	342

The following observations and interpretations are made:

(a) The optimal policy for the fill rate model with m=2, as shown in row 4 of Table 3.2 (Beta. 3,3 D.) in Additive Error Linear Demand case, consists of the retailer acquiring q*=23281 units at a unit cost of c=$35 and selling them at a unit price of p*=$50.33. With respect to the fill rate, approximately Ω*= 4% of the shortages are recaptured at an extra purchasing cost of d=$3.00 to the retailer, who allows a rebate of r*=$10.22 per unit backlogged. Afterwards, all unsold units, i.e. $[(\mathbf{1}-\boldsymbol{\Omega}^*)(\mathbf{D}-\mathbf{q}^*)]$, will be assigned a unit shortage penalty of s=$3.
On the other hand, when demand falls below the q*=23281 units ordered and all purchased at the cost of c=$35 per unit, D units are sold at the regular unit price of p*=$50.33 and the remaining, at the salvage value of v=$10.00 per unit.
The resulting optimal policy is π*[p*, q*, r*] = $343855.
As show earlier in Table 2.2, these results contrast with the optimal solution for the AL certainty case of π*[p*; q*] =$343748 [$50.33; 23294]
Similar comparisons can be made for the other corresponding tables too.

(b) The differences in the profitability, optimal order quantity, rebates etc. can be very well observed across the three forms of Beta Distributions.

(c) Extending the earlier work done by the authors [Refer (2014 (b),(c))], the percentage change in the base case policies when the Beta Distributions is used in place of Uniform or Normal, is shown in Tables 4.1-4.3.

Table 4.1.% change in the Base Case Optimal Policies when Beta distribution (B 1,3 D.) is used

Additive Error Linear Demand					
Distribution	Profit	P	Q	Λ	Φ
Beta 1,3 D.	325767	49.88	22502	181	608
Uniform Dist.	339096	50.22	23276	444	836
% CHANGE	4.09↑	0.68↑	3.43↑	145.3↑	37.5↑
Multiplicative Error Iso-Elastic Demand					
Distribution	Profit	P	Q	Λ	Φ
Beta 1,3 D.	328813	59.95	14165	454	660
Uniform Dist.	356419	61.41	15496	988	713
% CHANGE	8.39↑	2.43↑	9.39↑	117.62↑	8.03↑
Additive Error Linear Demand					
Distribution	Profit	P	Q	Λ	Φ
Beta 1,3 D.	325767	49.88	22502	181	608
Normal Dist.	346866	50.36	23295	245	399
% CHANGE	6.47↑	0.96↑	3.52↑	35.35↑	34.37↓
Multiplicative Error Iso-Elastic Demand					
Distribution	Profit	P	Q	Λ	Φ
Beta 1,3 D.	328813	59.95	14165	454	660
Normal Dist.	377413	59.90	16290	538	452
% CHANGE	14.78↑	0.08↓	15.0↑	18.5↑	31.51↓

Table 4.2.% change in the Base Case Optimal Policies when Beta distribution (B 3,3 D.) is used

Additive Error Linear Demand					
Distribution	Profit	P	Q	Λ	Φ
Beta 3,3 D.	343748	50.33	23294	295	501
Uniform Dist.	339096	50.22	23276	444	836
% CHANGE	1.35↓	0.21↓	0.07↓	50.5↑	66.86↑
Multiplicative Error Iso-Elastic Demand					
Distribution	Profit	P	Q	Λ	Φ
Beta 3,3 D.	373823	60.14	16153	614	502
Uniform Dist.	356419	61.41	15496	988	713
% CHANGE	4.65↓	2.11↑	4.06↓	60.91↑	42.03↑
Additive Error Linear Demand					
Distribution	Profit	P	Q	Λ	Φ
Beta 3,3 D.	343748	50.33	23294	295	501
Normal Dist.	346866	50.36	23295	245	399
% CHANGE	0.90↑	0.05↑	0.004↑	16.94↓	20.35↓
Multiplicative Error Iso-Elastic Demand					
Distribution	Profit	P	Q	Λ	Φ
Beta 3,3 D.	373823	60.14	16153	614	502
Normal Dist.	377413	59.90	16290	538	452
% CHANGE	0.96↑	0.39↓	0.84↑	12.37↓	9.96↓

Table 4.3.% change in the Base Case Optimal Policies when Beta distribution (B 3,1 D.) is used

Additive Error Linear Demand					
Distribution	Profit	P	Q	Λ	Φ
Beta 3,1 D.	362670	50.78	24087	404	383
Uniform Dist.	339096	50.22	23276	444	836
% CHANGE	6.50↓	1.10↓	3.36↓	9.90↑	118.2↑
Multiplicative Error Iso-Elastic Demand					
Distribution	Profit	P	Q	Λ	Φ
Beta 3,1 D.	419418	60.25	18163	762	339
Uniform Dist.	356419	61.41	15496	988	713
% CHANGE	15.02↓	1.92↑	14.68↓	29.65↑	110.3↑
Additive Error Linear Demand					
Distribution	Profit	P	Q	Λ	Φ
Beta 3,1 D.	362670	50.78	24087	404	383
Normal Dist.	346866	50.36	23295	245	399
% CHANGE	4.35↓	0.82↓	3.28↓	39.35↓	4.17↑
Multiplicative Error Iso-Elastic Demand					
Distribution	Profit	P	Q	Λ	Φ
Beta 3,1 D.	419418	60.25	18163	762	339
Normal Dist.	377413	59.90	16290	538	452
% CHANGE	10.01↓	0.58↓	10.31↓	29.39↓	33.33↑

Contrasting retailer's policies across distributions

With respect to the second issue, contrasting retailer's policies across distributions, we perceive strong directional differences in profitability, due to the degree of asymmetry embedded in each distribution. In fact, even in the case of uniform, normal and symmetrical beta, with equal mean and support, one cannot expect exact comparisons, since the normal case needs, of necessity, to be truncated in such a way that the uniform support covers the overwhelming normal range. Nevertheless, in the simple numerical case reported here, we modified (α,β), from (1,3) to (3,1). The outcome observed in all of these AL numerical experiments, regardless of the size or the direction of the parameters changed to generate various types of asymmetric distributions is quite clear: the left-skewed (right-skewed)beta distribution yields the least (most) profitable retailer policies, with the symmetrical cases, in between.Further, changes in the direction of the selling price (p*↓, for the right-skewed beta; p*↑, for the remaining cases) differ across distributions.

Some Concluding Comments

The primary contribution of this paper has been to consider the impact upon the ordering and pricing policies of a newsvendor-type, profit-maximizing retailer, faced with the possibility of backordering at least some of the shortages incurred from demand underestimation, by offering some rebate incentives for waiting. The backlog fill rate, representing the probability of the end-customers returning to satisfy their unfilled demand, is modelled as a function of the size of the rebate offered relative to the selling price. The decision variables are the selling price, the order size and the rebate offered as an incentive to satisfy at least a portion of the unfulfilled demand. Sensitivities of the demand errors in the form of beta distribution rather than the uniform and normal distribution serve as an extension to the previous work by the authors.

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