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# The Geometric Inverse Burr Distribution: Model, Properties and Simulation

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**Abstract**: *In this paper, we introduced a new model called the geometric inverse burr distribution. Statistical measures and their properties are derived and discussed. In particular, explicit expression for the density, rth moment and entropy are obtained. The method of maximum likelihood estimator is used to obtain the estimate values of the parameters and provide the information matrix. Simulation studies are performed for different parameter values and sample sizes to assess the finite sample behavior of the MLEs.*
***Key Words:*** *probability density function(pdf), probability mass function(pmf), geometric inverse burr(GIB) hazard function, cumulative distribution function(cdf), moments, maximum likelihood estimation(MLEs) and quartile.*

## I. Introduction

Burr (1942) introduced the system of distributions that comprises the Burr XII as the most useful of these distributions. If the random variable X has the Burr XII distribution, then the inverse of X has the Burr III distribution. The cdf of the inverse Burr is given by

$$F_{\alpha,\beta} = \left(\frac{w^{\alpha}}{1+w^{\alpha}}\right)^{\beta} \quad where$$

$\alpha > 0$, $\beta > 0$ are shape parameters.

While the *pdf* of the inverse burr is given by

$$f_{\alpha,\beta}(w) = \frac{\alpha\beta}{w^{\alpha+1}}\left(\frac{w^{\alpha}}{1+w^{\alpha}}\right)$$

The inverse Burr distribution is been used in various fields of sciences. In the actuarial literature it is known as the Burr III distribution (see, e.g., Klugman et al., 1998) and as the kappa distribution in the meteorological literature (Mielke, 1973; Mielke and Johnson, 1973). It has also been used in finance, environmental studies, and survival analysis and reliability theory (see
,Sherrick et al., 1996; Lindsay et al., 1996; Al-Dayian, 1999; Shao, 2000; Hose, 2005; Mokhlis
Gove et al., 2008). Further, Shao et al. (2008) proposed an extended inverse Burr distribution in ;2005
Low-flow frequency analysis where its lower tail is of main interest. A bivariate extension of the
.(Inverse Burr distribution had been given by Rodriguez 1980)

Several authors proposed a new distribution in the literature to model lifetime data by combining geometric and other well known distributions. Adamidis and Loukas (1998) introduced the two-parameter exponential-geometric (EG) distribution with decreasing failure rate. Kus (2007) introduced the exponential-Poisson distribution (following the same idea of the EG distribution) with decreasing failure rate and discussed several of its properties. Marshall and Olkin (1997) presented a method for adding a parameter to a family of distributions with application to the exponential and Weibull families. Adamidis et al. (2005) proposed the extended exponential-geometric (EEG) distribution which generalizes the EG distribution and discussed several of its statistical properties along with its reliability features. The hazard function of the EEG distribution can be monotone decreasing, increasing or constant. Wagner et al. (2008) proposed the weibull geometric distribution. In this paper, we introduced a new distribution by combining geometric and inverse burr distribution .to form geometric inverse burr distribution (GIB)

The rest of the paper is organized as follows. Section 2, provides the new distribution. Statistical properties of this class of distribution are given in section 3. Statistical inferences and entropy are given in Sections 4 and 5 respectively. Section 6 gives the simulation studies for different parameter values and sample sizes. Conclusion of the paper is given in Section 7

## II. The geometric inverse burr distribution

In this section, we introduce the new class of distribution call the geometric inverse burr distribution (GIB). The pdf of the class is a decreasing and unimodal, while the hazard rate function is decreasing, increasing and a bathtube shape depending on the parameter values. Figures 1 and 2 below have clearly shown the shape of the .pdf and the hazard rate function

Let $z$ be a geometric random variable with pmf $P(z,p) = (1-p)p^{z-1}$ for $z \in \mathbb{N}$ and $p \in (0,1)$. Define $X = min(w_1, ..., w_z)$. The marginal pdf of $X$ is

$$f(x;p,\alpha,\beta) = \frac{\alpha\beta(1-p)x^{-(\alpha+1)}\left(1+x^{-\alpha}\right)^{-(\beta+1)}}{\left[1-p(1-(1+x^{-\alpha})^{-\beta})\right]^2} \quad x>0 \tag{1}$$

which defines the pdf of the GIB. The cdf for the GIB becomes

$$F_{\alpha,\beta}(x;p,\alpha,\beta) = \left(\frac{x^{\alpha}}{1+x^{\alpha}}\right)^{\beta}\left[1-p(1-(1+x^{-\alpha})^{-\beta})\right]^{-1} \quad x>0 \tag{2}$$

The hazard rate function of the GIB is

$$h(x,p,\alpha,\beta) = \frac{\alpha\beta(1-p)x^{-(\alpha+1)}(1+x^{-\alpha})^{-(\beta+1)}\left[1-p(1-(1+x^{-\alpha})^{-\beta})\right]^{-1}}{\left[1-p(1-(1+x^{-\alpha})^{-\beta})\right]-(1+x^{-\alpha})^{-\beta}} \quad x>0 \tag{3}$$

The survival function of the GIB is

$$s(x,p,\alpha,\beta) = 1-\left(\frac{x^{\alpha}}{1+x^{\alpha}}\right)^{\beta}\left[1-p(1-(1+x^{-\alpha})^{-\beta})\right]^{-1} \quad x>0 \tag{4}$$

**Proposition 2.1**

The Pdf of the GIB is decreasing for o<α<1 and unimodal for α>1.

**Proposition 2.2**

The limiting behaviour of the hazard rate function of the GIB is as follows

**a.** if $0 < \beta < 1$, then

$$\lim_{x \to 0} h(x) = \begin{cases} \infty & \text{if } 0 < \alpha \leq 1 \\ 0 & \text{if } \alpha > 0 \end{cases} \tag{5}$$

and $\lim_{x \to \infty} h(x) = 0$

**b.** for $\beta = 1$,

$$\lim_{x \to 0} h(x) = \begin{cases} \infty & \text{if } 0 < \alpha < 1 \\ \frac{1}{1-p} & \alpha = 1 \\ 0 & \text{if } \alpha > 1 \end{cases} \tag{6}$$

and $\lim_{x \to \infty} h(x) = 0$

for $\beta > 1$, $\lim_{x \to 0} h(x) = 0$, for each value $\alpha > 0$ $\lim_{x \to \infty} h(x) = \infty$

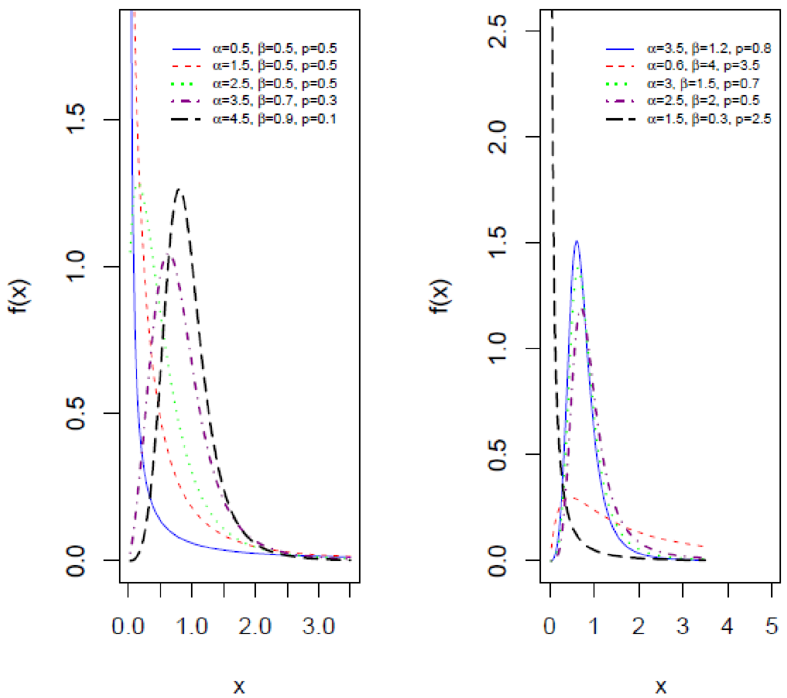


Figure 1: pdf of the GIB for different parameter values

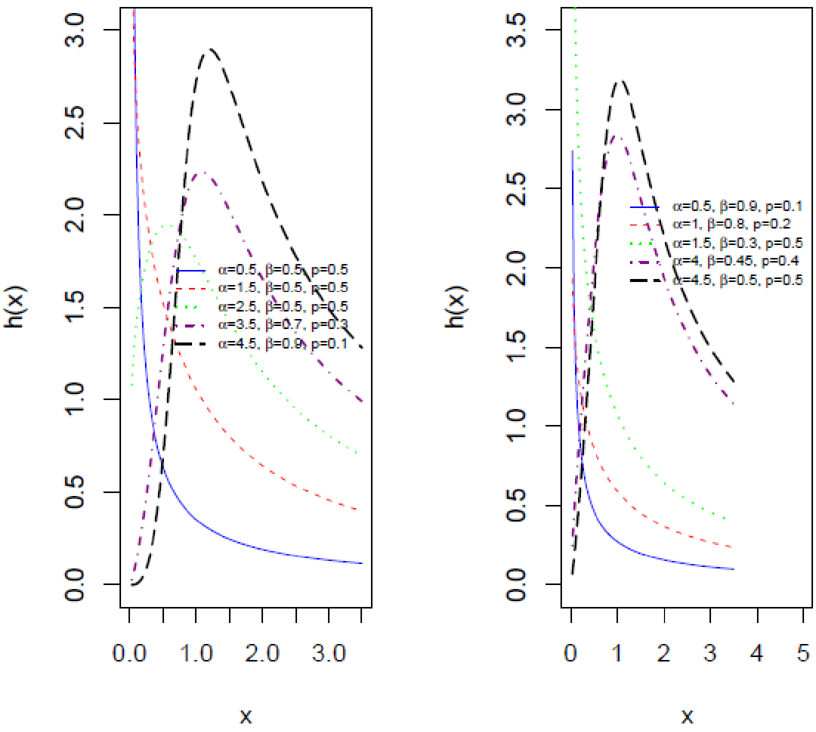


Figure 2: hazard rate function of the class for different parameter values

**Proposition 2.3**

The hazard rate function is decreasing for $0 < \alpha \leq 1$ and for $\alpha > 1$ it can take different forms. Now, for $|z| < 1$ and $p > 0$, the power series expansion is given by

$$(1-z)^{-p} = \sum_{j=0}^{\infty} \frac{\Gamma(p+j) z^j}{\Gamma(p) j!} \qquad (7)$$

We use (7) for the derivation of the properties of the geometric inverse burr (GIB) distribution. Using (7) in (1) and thereafter applying binomial expansion we obtain the following

$$f(x;p,\alpha,\beta) = (1-p)\sum_{i=0}^{\infty}(-1)^i(i+1)^{-1}\sum_{j=i}^{\infty}(j+1)p^j\binom{j}{i}f_{\alpha,B(i+1)}(x) \qquad (8)$$

by letting $m_i = (1-p)(-1)^i(i+1)^{-1}\sum_{j=i}^{\infty}(j+1)p^j\binom{j}{i}$, equation (8) becomes

$$f(x;p,\alpha,B) = \sum_{i=0}^{\infty} m_i f_{\alpha,B(i+1)}(x) \qquad (9)$$

where $f_{\alpha,B(i+1)}(x)$ is the pdf of the inverse burr.

## III. Statistical properties

In this section, we discuss some of the statistical properties of the GIB among which include the following

### 3.1 moments

**Theorem 3.1** *If $Z \sim IB(\alpha, \beta)$, the $(r,n)^{th}$ probability weighted moment (pwm) of $Z$ becomes*
$m(r,n) = aB(a(n+1) + \frac{r}{\alpha}, 1 - \frac{r}{\alpha})$.

where $a = \beta$

Proof: see appendix Ai

**Theorem 3.2** *If $X_1 \sim GIB(p, \alpha, \beta)$, the $r^{th}$ moment (pwm) of $X_1$ is given by*
$E(X^r) = a\sum_{i=0}^{\infty} m_i(i+1)B(a(i+1) + \frac{r}{\alpha}, 1 - \frac{r}{\alpha})$.

where $a = \beta$

Proof: see appendix Aii

## 3.2 mean and variance

$$E(X) = a\sum_{i=0}^{\infty} m_i(i+1)B(a(i+1) + \frac{1}{\alpha}, 1 - \frac{1}{\alpha})$$

$$E(X^2) = a\sum_{i=0}^{\infty} m_i(i+1)B(a(i+1) + \frac{2}{\alpha}, 1 - \frac{2}{\alpha})$$

$$Var(X) = a\sum_{i=0}^{\infty} m_i(i+1)B(a(i+1) + \frac{2}{\alpha}, 1 - \frac{2}{\alpha}) - \left\{a\sum_{i=0}^{\infty} m_i(i+1)B(a(i+1) + \frac{1}{\alpha}, 1 - \frac{1}{\alpha})\right\}^2$$

## 3.3 Quatile and median

by inverting the cdf of the class we obtained the quantile function $(for\ \ 0 < q < 1)$ as

$$x_q = \left\{\left(\frac{1-pq}{q(1-p)}\right)^{\frac{1}{\beta}} - 1\right\}^{-(\frac{1}{\alpha})} \tag{10}$$

(10) is used for the simulation of the class. Therefore, we can have the median as

$$x_{0.5} = \left\{\left(\frac{2-p}{(1-p)}\right)^{\frac{1}{\beta}} - 1\right\}^{-(\frac{1}{\alpha})} \tag{11}$$

# IV. Statistical inference

In this section, we discuss the estimation problem about the unknown parameters of the proposed model. For the estimation problem, we discuss the most popular method of estimation used in statistical science namely, the method of maximum likelihood estimators (MLEs). This is because the MLEs possesses under fairly regular condition of some optimal properties

## 4.1 estimation

Let $X_1,...,X_n$ be a random sample with observed values $x_1,...,x_n$ from the class with parameters $p$, $\alpha$, $\beta$. Let $\Theta = (p,\alpha,\beta)^T$ be the parameter vector.The log likelihood function is given by

$$l(\theta) = \log \alpha\beta(1-p)-(\alpha+1)\sum_{i=1}^{n}\log x_i-(\beta+1)\sum_{i=1}^{n}\log(1+x_i^{-\alpha})-2\sum_{i=1}^{n}\log\left(1-p[1-(1+x^{-\alpha})]\right) \tag{14}$$

The log-likelihood can be maximized either directly or by solving the nonlinear likelihood equation obtained by differentiating $l(x;p,\alpha,\beta)$ above. The components of the score vector $U = (\frac{\partial l}{\partial p}, \frac{\partial l}{\partial \alpha}, \frac{\partial l}{\partial \beta})^T$ are given by

$$\frac{\partial l}{\partial p} = -\frac{n}{1-p} - 2\sum_{i=1}^{n}\frac{-1+\alpha\beta x_i^{-(\alpha+1)}t_i^{-(\beta+1)}}{\left(1-p[1-t_i^{-\beta}]\right)} \tag{15}$$

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n}\log x_i + \alpha(\beta+1)\sum_{i=1}^{n}\frac{x_i^{-(\alpha+1)}}{t_i^{-(\beta+1)}} - 2\alpha\beta p\sum_{i=1}^{n}\frac{x_i^{-(\alpha+1)} - t_i^{-(\beta+1)}}{\left(1-p[1-t_i^{-\beta}]\right)} \tag{16}$$

$$\frac{\partial l}{\partial \beta} = -\frac{n}{\beta} - \sum_{i=1}^{n}\log t_i + 2\alpha\beta p\sum_{i=1}^{n}\frac{x_i^{-(\alpha+1)}t_i^{-(\beta+1)}}{\left(1-p[1-t_i^{-\beta}]\right)} \tag{17}$$

where $t_i = (1+x_i^{-\alpha})$

**Theorem 4.1** *The maximum likelihood estimators* $(\hat{p},\hat{\alpha},\hat{\beta})$ *are consistent estimators, and* $\sqrt{n}\left(\hat{p}-p,\hat{\alpha}-\alpha,\hat{\beta}-\beta\right)^T$ *is asymptotically normal with mean vector* $\boldsymbol{o}$ *and the variance covariance matrix* $\boldsymbol{I}^{-1}$, *where* $\boldsymbol{I} = -\frac{1}{n}E\left(\frac{\partial l_1^2(\Theta)}{\partial\Theta\partial\Theta^T}\right)$.

The $3\times3$ observed information matrix is given by

$$\mathbf{J} = \begin{pmatrix} J_{PP} & J_{p\alpha} & J_{p\beta} \\ J_{\alpha\beta} & J_{\alpha\alpha} & J_{\alpha P} \\ J_{\beta p} & J_{\beta\alpha} & J_{\beta\beta} \end{pmatrix}$$

where the expressions for the elements of J are given in Appendix B

## V. Entropy

Statistical entropy is a probabilistic measure of uncertainty about the outcome of a random experiment and is a measure of a reduction in that uncertainty. Numerous entropy and information indices, among them the Renyi entropy have been developed and used in various disciplines and contexts. Information theoretic principles and methods have become integral parts of probability and statistics and have been applied in various branches of ,statistics and related fields.

Entropy has been used in various situations in science and Engineering. The entropy of a random variable X is a .measure of variation of the uncertainty. Renyi entropy is defined by

$$I_R(r) = \frac{1}{1-r} \log \left[ \int_R f^r(x)dx \right] \tag{18}$$

where $r > 0$ and $r \neq 1$

Now,

$$\int_R f^r(x)dx = (\alpha\beta(1-p))^r \sum_{i=0}^{\infty} \sum_{j=i}^{\infty} (i+1)^{-1}(-1)^i \frac{\Gamma(2r+j)}{\Gamma(2r)j!} p^j \binom{j}{i} \int_0^{\infty} f_{\alpha,B(i+1)}(x)$$

where

$$\int_0^{\infty} f_{\alpha,B(i+1)}(x) = \frac{1}{\alpha} B\left( a(i+r) + r - 1 - \frac{(\alpha+1)(r-1)}{\alpha}, 1 + \frac{(\alpha+1)(r+1)}{\alpha} \right)$$

Consequentlty,

$$I_R(r) = \frac{1}{1-r} \log \left[ (\alpha\beta(1-p))^r \sum_{i=0}^{\infty} \sum_{j=i}^{\infty} (i+1)^{-1}(-1)^i \frac{\Gamma(2r+j)}{\Gamma(2r)j!} p^j \binom{j}{i} \frac{1}{\alpha} \right.$$

$$\left. \times B\left( a(i+r) + r - 1 - \frac{(\alpha+1)(r-1)}{\alpha}, 1 + \frac{(\alpha+1)(r+1)}{\alpha} \right) \right]$$

The shannon entropy is defined as $E[-\log f(x)]$. Therefore, by taking the negative log of the pdf of the GIB, we obatain

$$E(-\log f(x)) = -\log \alpha\beta(1-p) + (\alpha+1)E(\log x) + (\beta+1)E(\log(1+x^{-\alpha})) + 2E\left\{ \log\left(1 - p[1-(1+x^{-\alpha})]\right) \right\}.$$

## VI. Simulation

In this section, we assess the finite sample performance of the MLEs of θ = (p; α; β). The results are obtained from generating N samples from GIB. For each replication, a random sample of size n=50,100, 200 and 300 is drawn from GIB and the parameters are then estimated by using the method of maximum likelihood. The GIB random generation number was performed by using equation 10. We used four different true parameter values. The number of replication is set to be N = 10000; four different true parameter values are used in the data simulation process. Table 1 reports the average MLEs for the three parameters of the proposed model along with mean squared error. The result reported in Table 1, from the Table, we can see that there are convergences and consistency and this emphasize the numerical stability of the MLE method. Also, as the sample size increases, .the mean squared error decreases

Table 1: The average of 10000 MLEs and standard error simulated from GIB

| | | AE | | | SD | | |
|---|---|---|---|---|---|---|---|
| n | $(p, \alpha, \beta)$ | $\hat{p}$ | $\hat{\alpha}$ | $\hat{\beta}$ | $sd(\hat{p})$ | $sd(\hat{\alpha})$ | $sd(\hat{\beta})$ |
| 50 | (0.5, 0.5, 2) | 0.582 | 0.761 | 4.485 | 0.291 | 0.857 | 5.528 |
| | (1.0, 2.0, 1.0) | 1.581 | 2.085 | 1.779 | 0.543 | 1.451 | 3.795 |
| | (3.0, 2.0, 1.0) | 4.599 | 4.403 | 1.257 | 8.694 | 2.671 | 1.475 |
| | (3.0, 3.0, 3.0) | 3.152 | 4.503 | 3.429 | 1.625 | 2.045 | 3.205 |
| 100 | (0.5, 0.5, 2) | 0.504 | 0.702 | 3.722 | 0.171 | 0.834 | 4.473 |
| | (1.0, 2.0, 1.0) | 1.072 | 2.033 | 1.685 | 0.358 | 1.329 | 2.722 |
| | (3.0, 2.0, 1.0) | 4.380 | 4.372 | 1.112 | 6.426 | 2.572 | 1.452 |
| | (3.0, 3.0, 3.0) | 3.068 | 3.784 | 3.311 | 1.037 | 1.932 | 3.007 |
| 200 | (0.5, 0.5, 2) | 0.498 | 0.711 | 3.579 | 0.101 | 0.204 | 3.875 |
| | (1.0, 2.0, 1.0) | 1.032 | 2.022 | 1.369 | 0.248 | 0.588 | 1.407 |
| | (3.0, 2.0, 1.0) | 3.358 | 4.112 | 1.063 | 2.996 | 2.275 | 1.393 |
| | (3.0, 3.0, 3.0) | 2.899 | 3.779 | 3.612 | 0.835 | 1.931 | 3.001 |
| 300 | (0.5, 0.5, 2) | 0.488 | 0.701 | 2.459 | 0.084 | 0.105 | 1.902 |
| | (1.0, 2.0, 1.0) | 1.004 | 2.006 | 1.264 | 0.196 | 0.577 | 1.203 |
| | (3.0, 2.0, 1.0) | 3.070 | 4.003 | 0.999 | 1.311 | 2.177 | 0.976 |
| | (3.0, 3.0, 3.0) | 2.778 | 3.767 | 3.054 | 0.644 | 1.922 | 1.940 |

## VII. Conclusion

We introduce a new class of lifetime distributions called the geometric inverse burr distribution (GIB), which is obtained by compounding the geometric distribution (GD) and inverse burr (IB) distribution. The ability of the new proposed model is in covering five possible hazard rate function i.e., increasing, decreasing, upside-down bathtub (unimodal), bathtub and increasing-decreasing-increasing shaped. Several properties of the GIB distributions such as moments, maximum likelihood estimation procedure and inference for a large sample, are discussed in this paper. In order to show the flexibility and potentiality of the new distributions, Simulation studies are performed for different parameter values and sample sizes to assess the finite sample behaviour of .the MLEs

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# Appendix A

## i Proof.

$$\begin{aligned} m(r,n) &= E[X^r F^n(x)] \\ &= \int_0^\infty x^r f_{\alpha,\beta}(x) F^n_{\alpha,\beta} dx \\ &= \alpha\beta \int_0^\infty x^r x^{-(\alpha+1)} (1+x^{-(\alpha)})^{-\beta(n+1)-1} dx \end{aligned}$$

let $k = (1+x^{-(\alpha)})^{-\beta(n+1)-1}$ after some algebra, we abtain

$$m(r,n) = \frac{\beta}{\beta(n+1)+1} \int_0^1 \left(k^{\frac{1}{\beta(n+1)+1}}\right)^{\frac{r}{\alpha}} k^{\frac{1}{-\beta(n+1)-1}} \left(1 - \left(k^{\frac{1}{\beta(n+1)+1}}\right)^{-\frac{r}{\alpha}}\right) dk$$

Transforming let $u = k^{\frac{1}{\beta(n+1)+1}}$ consequently

$$m(r,n) = aB(a(n+1) + \frac{r}{\alpha}, 1 - \frac{r}{\alpha})$$

∎

## ii Proof.

$$\begin{aligned} \mu &= E[X^r] \\ &= \int_0^\infty x^r f(x) dx \\ &= \sum_{i=1}^{\infty} m_i \alpha\beta \int_0^\infty x^r x^{-(\alpha+1)} (1+x^{-(\alpha)})^{-\beta(n+1)-1} dx \end{aligned}$$

after some algebra, we obtain

$$E(X^r) = a \sum_{i=0}^{\infty} m_i (i+1) B(a(i+1) + \frac{r}{\alpha}, 1 - \frac{r}{\alpha})$$

∎where $a = \beta$

# Appendix B

let $t_i = (1 + x_i^{-\alpha})$

$$J_{pp} = -\frac{n}{(1-p)^2} - 2\sum_{i=1}^{n} \frac{(1-t_i)^{-(\beta)})}{\left(1 - p[1 - t_i^{-\beta}]\right)}$$

$$J_{\alpha\alpha} = -\frac{n}{\alpha^2} + \alpha(\beta+1)\sum_{i=1}^{n}\frac{\alpha(\alpha+1)t_i\left[x_i^{-\alpha} + x_i^{-(\alpha+1)}\right] - \alpha^2 x_i^{-2(\alpha+1)}}{(t_i)^2}$$
$$-2\beta p\sum_{i=1}^{n}\frac{\left(1-p[1-t_i^{-\beta}]\right)t_i^{-(\beta+1)}[\alpha(\alpha+1)x_i^{-\alpha} + x_i^{-(\alpha+1)}] - \alpha^2\beta p x_i^{-2(\alpha+1)}t_i^{-2(\beta+1)}}{\left(1-p[1-t_i^{-\beta}]\right)^2}$$

$$J_{\beta\beta} = -\frac{n}{\beta^2} - 2\alpha p\sum_{i=1}^{n}\frac{x_i^{-(\alpha+1)}t_i^{-(\beta)}}{\left(1-p[1-t_i^{-\beta}]\right)}$$

$$J_{p\alpha} = -2\sum_{i=1}^{n}\frac{\left[\alpha^2\beta(\beta+1)x_i^{-2(\alpha+1)}t_i^{-(\beta+2)} - \alpha(\alpha+1)t_i^{-(\beta+1)}x_i^{-(\alpha+2)} + x_i^{-(\alpha+1)}\right]}{\left(1-p[1-t_i^{-\beta}]\right)}$$
$$-2\alpha^2\beta^2 p\sum_{i=1}^{n}\frac{x_i^{-2(\alpha+1)}t_i^{-2(\beta+1)}}{\left(1-p[1-t_i^{-\beta}]\right)^2}$$

$$J_{p\beta} = -2\sum_{i=1}^{n}\frac{x_i^{-(\alpha+1)}t_i^{-(\beta+1)}}{\left(1-p[1-t_i^{-\beta}]\right)}$$

$$J_{\alpha p} = -2\alpha\beta\sum_{i=1}^{n}\frac{x_i^{-\alpha}t_i^{-(\beta+1)}}{\left(1-p[1-t_i^{-\beta}]\right)} - 2\alpha\beta p\sum_{i=1}^{n}\frac{x_i^{-(\alpha+1)}t_i^{-(\beta+1)}(1-t_i)^{-\beta}}{\left(1-p[1-t_i^{-\beta}]\right)^2}$$

$$J_{\alpha\beta} = \alpha\sum_{i=1}^{n}\frac{x_i^{-(\alpha+1)}}{t_i} - 2\alpha p\sum_{i=1}^{n}\frac{x_i^{-(\alpha+1)}t_i^{-(\beta+1)}}{\left(1-p[1-t_i^{-\beta}]\right)}$$

$$J_{\beta p} = -2\alpha\beta\sum_{i=1}^{n}\frac{p x_i^{-(\alpha+1)}t_i^{-(\beta+1)}(1-t_i)^{-\beta}}{\left(1-p[1-t_i^{-\beta}]\right)^2} - 2\alpha\beta\sum_{i=1}^{n}\frac{x_i^{-(\alpha+1)}t_i^{-(\beta+1)}}{\left(1-p[1-t_i^{-\beta}]\right)}$$

$$J_{\beta\alpha} = \alpha\sum_{i=1}^{n}\frac{x_i^{-(\alpha+1)}}{t_i} + 2\beta p\sum_{i=1}^{n}\frac{t_i^{-(\beta+1)}\left(x_i^{-(\alpha+1)} - \alpha(\alpha+1)x_i^{-(\alpha+2)}\right) + \alpha^2(\beta+1)x_i^{-2(\alpha+1)}}{\left(1-p[1-t_i^{-\beta}]\right)^2}$$
$$+\alpha^2\beta p\sum_{i=1}^{n}\frac{x_i^{-2(\alpha+1)}t_i^{-2(\beta+1)}}{\left(1-p[1-t_i^{-\beta}]\right)^2}$$

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