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Unsteady MHD Oscillatory Free Convective and Chemical Reactive Fluid Flow through Porous Medium between Parallel Plates with Radiation and Temperatue Gradient Dependent Heat Source

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Abstract: *Aim of the paper is to investigate unsteady oscillatory free convective flow of a viscous incompressible electrically conducting fluid through a non-homogenous porous medium between two vertical parallel plates in slip flow regime in the presence of radiation, chemical reaction and temperature gradient dependent heat source. A uniform magnetic field is applied in the direction normal to the plate. The effect of free convection and chemical reaction on unsteady MHD oscillatory flow of fluid in vertical media is encountered in a wide range of engineering and industrial applications. The oscillatory time-dependent coupled non-linear equations are solved for the fluid velocity, temperature and concentration by using regular perturbation technique. Numerical results for the velocity, temperature, concentration profiles, local skin friction coefficient, local Nusselt number and Sherwood number for various physical parameters are discussed numerically and presented graphically.*

Keywords: *Unsteady, MHD, porous medium, chemical reaction, free-convection, temperature gradient dependant heat source, suction, radiation, skin-friction coefficient, Nusselt number and Sherwood number.*

I. Introduction

Analysis of fluid flow between parallel plates attracts the interest of many researchers in last few decades because of its application in science and technology. It has many applications such as geothermal energy recovery, thermal energy storage, flow through filtering devices, glass production, furnace design, thermonuclear fusion, drying processes, nuclear waste disposal and in the control of pollutant spread in ground water. Further, MHD free convection has become important in the control of mountain iron flow in the steady industrial liquid metal cooling in nuclear reactors and magnetic separation of molecular semi conducting materials etc.

Verma and Bansal (1966) studied flow of a viscous incompressible fluid between two parallel plates, one in uniform motion and the other at rest with uniform suction at the stationary plate. Hassanien and Mansour (1990) analyzed unsteady magnetohydrodynamic flow through a porous medium between two infinite parallel plates. Attia and Kotb (1996) presented MHD flow between two parallel plates with heat transfer. Sharma and Kumar (1998) investigated unsteady flow and heat transfer between two horizontal plates in the presence of transverse magnetic field. Acharya, Dash and Singh (2000) discussed magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Singh (2000) explained unsteady flow of liquid through a channel with pressure gradient changing exponentially under the influence of inclined magnetic field. Chung (2002) presented Computational Fluid Dynamics. Sharma and Chaturvedi (2003) analyzed unsteady flow and heat transfer of an electrically conducting viscous incompressible fluid between two non-conducting parallel porous plates under uniform transverse magnetic field. Sharma, Chawla and Singh (2005) explained unsteady plane poiseuille flow and heat transfer in the presence of oscillatory temperature of the lower plate. Makinde and Mhone (2005) considered heat transfer to MHD oscillatory flow in a channel filled with porous medium. Ganesh and Krishnambal (2006) studied magnetohydrodynamic flow of viscous fluid between two parallel porous plates. Sharma and Sharma (2007) analyzed effect of oscillatory suction and heat source on heat and mass transfer in MHD flow along a vertical moving porous plate bounded by porous medium.

Sharma and Singh (2008) investigated unsteady MHD free convective flow and heat transfer along a vertical porous plate with variable suction and internal heat generation. Ajadi, Adegoke and Aziz (2009) studied slip boundary layer flow of non-Newtonian fluid over a flat plate with convective thermal boundary condition. Sharma, Kumar and Sharma (2010) discussed unsteady MHD convective flow and heat transfer between heated inclined plates with magnetic field in the presence of radiation effects. Pal and Talukdar (2010) explained perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Jha and Apere (2011) presented magnetohydrodynamic free convective Couette flow with suction and injection. Sharma and

Dadheech (2012) presented effect of volumetric heat generation / absorption on convective heat and mass transfer in porous medium in between two vertical plates. Devika, Satyanarayana and Venkataramana (2013) described MHD oscillatory flow of a visco elastic fluid in a porous channel with chemical reaction. Kesavaiah, Satyanarayana and Sudhakaraiah (2013) discussed effects of radiation and free convection currents on unsteady Couette flow between two vertical parallel plates with constant heat flux and heat source through porous medium. Kirubhashankar and Ganesh (2014) unsteady MHD flow of a Casson fluid in a parallel plate channel with heat and mass transfer of chemical reaction.

Aim of the present paper is to investigate oscillatory MHD flow of fluid through porous medium in slip flow regime between two vertical parallel plates in the presence of radiation, chemical reaction, free convection and temperature gradient dependent heat source. Equations of momentum, energy and diffusion, which govern the fluid flow, heat and mass transfer are solved by using perturbation method. The effects of various physical parameters on fluid velocity, temperature, concentration, skin friction coefficient, Nusselt number and Sherwood number at the plates are derived, discussed numerically and shown through graphs.

II. Mathematical Analysis

Consider MHD free convective and chemical reactive oscillatory flow of an incompressible viscous fluid between two infinite vertical parallel plates at a distance d apart filled with porous medium. The flow is considered in presence of radiation, chemical reaction, free convection and temperature gradient dependent heat source. The x^* - axis is taken along the plate (when $y^* = 0$) and the y^* - axis is taken normal to the plate. A transverse magnetic field of uniform strength B_0 is applied normal to the plate. The magnetic Reynolds number is taken to be small so that the induced magnetic field is neglected. Similarly, for small velocity the viscous dissipation and Darcy's dissipation are neglected. As the plates are infinite in length, the velocity, temperature and concentration fields are functions of y^* and t^* only. Under the above assumptions and usual Boussinesq approximation, the governing equations of motion, energy and mass conservation are as follows

$$\frac{\partial v^*}{\partial y^*} = 0, \qquad \text{...(1)}$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = g\beta_T\left(T^* - T_0\right) + g\beta_C\left(C^* - C_0\right) + \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\upsilon}{K^*}u^* - \frac{\sigma B_0{}^2}{\rho}u^*, \qquad \text{...(2)}$$

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y^*} + \frac{Q_0}{\rho C_p}\frac{\partial T^*}{\partial y^*}, \qquad \text{...(3)}$$

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial y^*} = D_M \frac{\partial^2 C^*}{\partial y^{*2}} + D_T \frac{\partial^2 T^*}{\partial y^{*2}} - K_C\left(C^* - C_0\right), \qquad \text{...(4)}$$

where u^* and v^* are the fluid velocity components in x^* and y^* - directions, respectively; t^* the time, T^* the fluid temperature, C^* the fluid concentration, β_T the coefficient of the thermal expansion, β_C the coefficient of the mass expansion, g acceleration due to gravity, υ the kinematic viscosity, B_0 the strength of the magnetic field, σ the electric conductivity, ρ the fluid density, K^* the permeability of porous medium, κ the thermal conductivity, C_p the specific heat at constant pressure, q_r the radiative heat flux in the y^* - direction, Q_0 the heat generation/ absorption constant, D_M the mass diffusion coefficient, D_T the thermal diffusion coefficient and K_C the chemical reaction coefficient.

The boundary conditions are given by

$$y^* = 0 : u^* = L^*\left(\frac{\partial u^*}{\partial y^*}\right), T^* = T_w + \varepsilon\left(T_w - T_0\right)e^{-n^* t^*}, C^* = C_w + \varepsilon\left(C_w - C_0\right)e^{-n^* t^*};$$

$$y^* = d : u^* = 0, T^* = T_0, C^* = C_0, \qquad \text{...(5)}$$

where T_0 and C_0 are the temperature and species concentration at the plate (when $y^* = 0$), respectively; T_w and C_w are the temperature and species concentration at the plate (when $y^* = d$), respectively and L^* is the slip parameter.
From the equation (1), it is noted that the suction velocity at the plate is either a constant or a function of time only. Here, the suction velocity normal to the plate is assumed to be in the form

$$v^* = V_0\left(1+\varepsilon e^{-n^* t^*}\right). \qquad ...(6)$$

It is assumed that the fluid is optically thin with a relative low density and radiative heat flux is according to Cogley et al. (1968) and given by

$$\frac{\partial q_r}{\partial y^*} = 4\alpha^2\left(T^* - T_0\right), \qquad ...(7)$$

where α is the mean radiation absorption coefficient.
and permeability of the porous medium is considered in the following form

$$K^* = \frac{K_0}{\left(1+\varepsilon e^{-n^* t^*}\right)}. \qquad ...(8)$$

Introducing the following dimensionless quantities

$$y = \frac{y^*}{d},\ u = \frac{u^*}{V_0},\ t = \frac{V_0}{4d}t^*,\ n = \frac{4d}{V_0}n^*,\ \theta = \frac{T^* - T_0}{T_w - T_0},\ C = \frac{C^* - C_0}{C_w - C_0},\ Gr = \frac{g\beta_T d^2\left(T_w - T_0\right)}{\upsilon V_0},$$

$$Gc = \frac{g\beta_C d^2\left(C_w - C_0\right)}{\upsilon V_0},\ Ha^2 = \frac{\sigma_e B_0^2 d^2}{\upsilon\rho},\ K^2 = \frac{d^2}{K^*},\ \mathrm{Re} = \frac{V_0 d}{\upsilon},\ \mathrm{Pr} = \frac{\upsilon\rho C_p}{\kappa},\ R^2 = \frac{4\alpha^2 d^2}{\upsilon\rho C_p},$$

$$S = \frac{Q_0 d^2}{\upsilon\rho C_p},\ Sc = \frac{\upsilon}{D_M},\ So = \frac{D_M}{\upsilon}\left(\frac{T_w - T_0}{C_w - C_0}\right),\ Kr = \frac{d^2}{\upsilon}K_C,\ L = \frac{V_0}{d}L^*. \qquad ...(9)$$

into the equations (2) to (4), we get

$$\frac{1}{4}\frac{\partial u}{\partial t} + \left(1+\varepsilon e^{-nt}\right)\frac{\partial u}{\partial y} = \frac{Gr}{\mathrm{Re}}\theta + \frac{Gc}{\mathrm{Re}}C + \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial y^2} - \frac{K^2}{\mathrm{Re}}\left(1+\varepsilon e^{-nt}\right)u - \frac{Ha^2}{\mathrm{Re}}u, \qquad ...(10)$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} + \left(1+\varepsilon e^{-nt}\right)\frac{\partial \theta}{\partial y} = \frac{1}{\mathrm{Pr}\,\mathrm{Re}}\frac{\partial^2 \theta}{\partial y^2} - \frac{R^2}{\mathrm{Re}}\theta + \frac{S}{\mathrm{Re}}\frac{\partial \theta}{\partial y}, \qquad ...(11)$$

$$\frac{1}{4}\frac{\partial C}{\partial t} + \left(1+\varepsilon e^{-nt}\right)\frac{\partial C}{\partial y} = \frac{1}{Sc\,\mathrm{Re}}\frac{\partial^2 C}{\partial y^2} + \frac{So}{\mathrm{Re}}\frac{\partial^2 \theta}{\partial y^2} - \frac{Kr}{\mathrm{Re}}C. \qquad ...(12)$$

where u is dimensionless velocity along x-axis, t the dimensionless time, y dimensionless coordinate axis normal to the plates, θ dimensionless temperature, C dimensionless concentration, Gr Grashof number, Gc modified Grashof number, Re cross flow Reynolds number, K permeability parameter, Ha Hartmann number, Pr Prandtl number, R Radiation parameter, S heat source parameter Sc Schmidt number, So Soret number and Kr chemical reaction parameter.
The dimensionless boundary conditions are given by

$$y = 0: u = h\frac{\partial u}{\partial y},\ \theta = 1+\varepsilon e^{-nt},\ C = 1+\varepsilon e^{-nt};$$

$$y = 1: u = 0,\ \theta = 0,\ C = 0. \qquad ...(13)$$

where h is the slip parameter.

III. Method Of Solution

Equations (10), (11) and (12) are second order coupled partial differential equations with boundary conditions (13). Since ε<<1, therefore fluid velocity, temperature and concentration can be expanded as given below

$$u(y,t)=u_0(y)+\varepsilon e^{-nt}u_1(y)+O(\varepsilon^2),$$
$$\theta(y,t)=\theta_0(y)+\varepsilon e^{-nt}\theta_1(y)+O(\varepsilon^2),$$
$$C(y,t)=C_0(y)+\varepsilon e^{-nt}C_1(y)+O(\varepsilon^2). \qquad ...(14)$$

Using (14) into the equations (10), (11) and (12), equating the coefficients of the harmonic and non-harmonic terms and neglecting the coefficients of ε^2, we obtain

$$\frac{d^2u_0}{dy^2}-\mathrm{Re}\frac{du_0}{dy}-a_7u_0=-Gr\theta_0-GcC_0, \qquad ...(15)$$

$$\frac{d^2u_1}{dy^2}-\mathrm{Re}\frac{du_1}{dy}-a_8u_1=-Gr\theta_1-GcC_1+\mathrm{Re}\frac{du_0}{dy}-K^2u_0, \qquad ...(16)$$

$$\frac{d^2\theta_0}{dy^2}+a_1\frac{d\theta_0}{dy}-a_2\theta_0=0, \qquad ...(17)$$

$$\frac{d^2\theta_1}{dy^2}+a_1\frac{d\theta_1}{dy}-a_3\theta_1=\frac{d\theta_0}{dy}, \qquad ...(18)$$

$$\frac{d^2C_0}{dy^2}-a_4\frac{dC_0}{dy}-a_5C_0=-SoSc\frac{d^2\theta_0}{dy^2}, \qquad ...(19)$$

$$\frac{d^2C_1}{dy^2}-a_4\frac{dC_1}{dy}-a_6C_1=Sc\,\mathrm{Re}\frac{dC_0}{dy}-SoSc\frac{d^2\theta_1}{dy^2}. \qquad ...(20)$$

where $a_1=\Pr(S-\mathrm{Re})$, $a_2=R^2\Pr$, $a_3=\left(R^2-\frac{n}{4}\mathrm{Re}\right)\Pr$, $a_4=Sc\,\mathrm{Re}$, $a_5=KrSc$,

$a_6=\left(Kr-\frac{n}{4}\mathrm{Re}\right)Sc$, $a_7=(K^2+Ha^2)$ and $a_8=\left(K^2+Ha^2-\frac{n}{4}\mathrm{Re}\right)$.

Now, the corresponding boundary conditions are reduced to

$$y=0: u_0=h\frac{du_0}{dy},\ u_1=h\frac{du_1}{dy},\ \theta_0=1,\ \theta_1=1,\ C_0=1,\ C_1=1;$$
$$y=1: u_0=0,\ u_1=0,\ \theta_0=0,\ \theta_1=0,\ C_0=0,\ C_1=0. \qquad ...(21)$$

Equations (15) to (20) are ordinary second order coupled differential equations and solved under the boundary conditions (21). Through straight forward calculations $u_0(y)$, $u_1(y)$, $\theta_0(y)$, $\theta_1(y)$, $C_0(y)$ and $C_1(y)$ are known and given by

$$u_0(y)=A_{25}e^{m_9y}+A_{26}e^{m_{10}y}+A_{27}e^{m_1y}+A_{28}e^{m_2y}+A_{29}e^{m_5y}+A_{30}e^{m_6y}, \qquad ...(22)$$

$$u_1(y)=A_{33}e^{m_{11}y}+A_{34}e^{m_{12}y}+A_{35}e^{m_1y}+A_{36}e^{m_2y}+A_{37}e^{m_3y}+A_{38}e^{m_4y}+A_{39}e^{m_5y}+A_{40}e^{m_6y}$$
$$+A_{41}e^{m_7y}+A_{42}e^{m_8y}+A_{43}e^{m_9y}+A_{44}e^{m_{10}y}, \qquad ...(23)$$

$$\theta_0(y)=A_1e^{m_1y}+A_2e^{m_2y}, \qquad ...(24)$$

$$\theta_1(y)=A_3e^{m_3y}+A_4e^{m_4y}+A_5e^{m_1y}+A_6e^{m_2y}, \qquad ...(25)$$

$$C_0(y)=A_9e^{m_5y}+A_{10}e^{m_6y}+A_{11}e^{m_1y}+A_{12}e^{m_2y}, \qquad ...(26)$$

$$C_1(y)=A_{15}e^{m_7y}+A_{16}e^{m_8y}+A_{17}e^{m_1y}+A_{18}e^{m_2y}+A_{19}e^{m_3y}+A_{20}e^{m_4y}+A_{21}e^{m_5y}+A_{22}e^{m_6y}, \qquad ...(27)$$

Finally, the expression of $u(y,t)$, $\theta(y,t)$ and $C(y,t)$ are known and given by

$$u(y,t)=A_{25}e^{m_9y}+A_{26}e^{m_{10}y}+A_{27}e^{m_1y}+A_{28}e^{m_2y}+A_{29}e^{m_5y}+A_{30}e^{m_6y}+\varepsilon e^{-nt}\left(A_{33}e^{m_{11}y}\right.$$

$$+A_{34}e^{m_{12}y}+A_{35}e^{m_1y}+A_{36}e^{m_2y}+A_{37}e^{m_3y}+A_{38}e^{m_4y}+A_{39}e^{m_5y}+A_{40}e^{m_6y}+A_{41}e^{m_7y}$$
$$+A_{42}e^{m_8y}+A_{43}e^{m_9y}+A_{44}e^{m_{10}y}\Big), \qquad ...(28)$$

$$\theta(y,t)=A_1e^{m_1y}+A_2e^{m_2y}+\varepsilon e^{-nt}\left(A_3e^{m_3y}+A_4e^{m_4y}+A_5e^{m_1y}+A_6e^{m_2y}\right), \qquad ...(29)$$

$$C(y,t)=A_9e^{m_5y}+A_{10}e^{m_6y}+A_{11}e^{m_1y}+A_{12}e^{m_2y}+\varepsilon e^{-nt}\Big(A_{15}e^{m_7y}+A_{16}e^{m_8y}+A_{17}e^{m_1y}$$
$$+A_{18}e^{m_2y}+A_{19}e^{m_3y}+A_{20}e^{m_4y}+A_{21}e^{m_5y}+A_{22}e^{m_6y}\Big). \qquad ...(30)$$

where A_1 to A_{46} are constants, whose expressions are not given here for the sake of brevity.

IV. Skin-Friction Coefficient

The dimensionless stress tensor in terms of skin-friction coefficient at both the plates are given by

$$C_f=\frac{du}{dy}=\frac{du_0}{dy}+\varepsilon e^{-nt}\frac{du_1}{dy} \text{ at } y=0 \text{ and } y=1, \qquad …(31)$$

Hence, skin-friction coefficient at the plate when $y=0$ is given by

$$\left(C_f\right)_0=A_{25}m_9+A_{26}m_{10}+A_{27}m_1+A_{28}m_2+A_{29}m_5+A_{30}m_6+\varepsilon e^{-nt}\big(A_{33}m_{11}+A_{34}m_{12}$$
$$+A_{35}m_1+A_{36}m_2+A_{37}m_3+A_{38}m_4+A_{39}m_5+A_{40}m_6+A_{41}m_7+A_{42}m_8+A_{43}m_9$$
$$+A_{44}m_{10}\big), \qquad ...(32)$$

The skin-friction coefficient at the plate when $y=1$ is given by

$$\left(C_f\right)_1=A_{25}m_9e^{m_9}+A_{26}m_{10}e^{m_{10}}+A_{27}m_1e^{m_1}+A_{28}m_2e^{m_2}+A_{29}m_5e^{m_5}+A_{30}m_6e^{m_6}$$
$$+\varepsilon e^{-nt}\Big(A_{33}m_{11}e^{m_{11}}+A_{34}m_{12}e^{m_{12}}+A_{35}m_1e^{m_1}+A_{36}m_2e^{m_2}+A_{37}m_3e^{m_3}+A_{38}m_4e^{m_4}$$
$$+A_{39}m_5e^{m_5}+A_{40}m_6e^{m_6}+A_{41}m_7e^{m_7}+A_{42}m_8e^{m_8}+A_{43}m_9e^{m_9}+A_{44}m_{10}e^{m_{10}}\Big). \qquad ...(33)$$

V. Nusselt Number

The dimensionless rate of heat transfer in terms of the Nusselt number at both the plates is given by

$$Nu=-\left(\frac{\partial\theta}{\partial y}\right)=-\left(\frac{d\theta_0}{dy}+\varepsilon e^{-nt}\frac{d\theta_1}{dy}\right) \text{ at } y=0 \text{ and } y=1, \qquad ...(34)$$

Hence, the Nusselt number at the plate when $y=0$ is given by

$$(Nu)_0=A_1m_1+A_2m_2+\varepsilon e^{-nt}\left(A_3m_3+A_4m_4+A_5m_1+A_6m_2\right), \qquad ...(35)$$

The Nusselt number at the plate when $y=1$ is given by

$$(Nu)_1=A_1m_1e^{m_1}+A_2m_2e^{m_2}+\varepsilon e^{-nt}\left(A_3m_3e^{m_3}+A_4m_4e^{m_4}+A_5m_1e^{m_1}+A_6m_2e^{m_2}\right). \qquad …(36)$$

VI. Sherwood Number

The dimensionless rate of mass transfer in terms of the Sherwood number at both the plates is given by

$$Sh=-\left(\frac{\partial C}{\partial y}\right)=-\left(\frac{dC_0}{dy}+\varepsilon e^{-nt}\frac{dC_1}{dy}\right) \text{ at } y=0 \text{ and } y=1, \qquad …(37)$$

Hence, the Sherwood number at the plate when $y=0$ is given by

$$(Sh)_0=A_9m_5+A_{10}m_6+A_{11}m_1+A_{12}m_2+\varepsilon e^{-nt}\big(A_{15}m_7+A_{16}m_8+A_{17}m_1+A_{18}m_2+A_{19}m_3$$
$$+A_{20}m_4+A_{21}m_5+A_{22}m_6\big), \qquad …(38)$$

The Sherwood number at the plate when $y=1$ is given by

$$(Sh)_1=A_9m_5e^{m_5}+A_{10}m_6e^{m_6}+A_{11}m_1e^{m_1}+A_{12}m_2e^{m_2}+\varepsilon e^{-nt}\Big(A_{15}m_7e^{m_7}+A_{16}m_8e^{m_8}$$
$$+A_{17}m_1e^{m_1}+A_{18}m_2e^{m_2}+A_{19}m_3e^{m_3}+A_{20}m_4e^{m_4}+A_{21}m_5e^{m_5}+A_{22}m_6e^{m_6}\Big). \qquad …(39)$$

VII. Results And Discussion

The effects of radiation, free convection, chemical reaction and temperature gradient dependent heat source on oscillatory flow of an incompressible viscous electrically conducting fluid through a non-homogenous porous medium between two vertical parallel plates in slip flow regime are investigated. Equations of momentum, energy and diffusion, which govern the fluid flow and heat and mass transfer are solved by using perturbation method. The effects of various physical parameters on fluid velocity, temperature, concentration, skin friction coefficient, Nusselt number and Sherwood number at the plates are observed, discussed numerically and shown through graphs.

It is observed from figure 1 to 8 that the velocity of fluid increases with the increase of Grashof number, modified Grashof number, Schmidt number, Soret number, Prandtl number, radiation parameter, heat source parameter or slip parameter. It is noted from figure 9 to 11 that the velocity of fluid decreases for Hartmann number, permeability parameter or chemical reaction parameter. Figure 12 shows that as the cross flow Reynolds number increases the velocity of fluid decreases near the plate at $y=0$ and it increases near the plate at $y=1$.

It is noted from figure 13 to 15 that the temperature of fluid decreases due to increase in Prandtl number, radiation parameter or heat source parameter. It is revealed from figure 16 that the temperature of fluid increases as cross flow Reynolds number increases.

It is observed from figures 17 to 21 that the concentration of fluid increases with the increase of Schmidt number, Soret number, Prandtl number, radiation parameter or heat source parameter. It is seen from figure 22 that the concentration of the fluid is inversely proportional to the value of chemical reaction parameter, thus increasing Kr reduces the concentration of the system. It is noted from figure 23 that with the increase of cross flow Reynolds number the concentration of fluid decreases near the plate at $y=0$ and increases near the plate at $y=1$.

It is observed from Table 1 that the skin-friction coefficient at the plate when $y=0$ increases as the values of Grashof number, modified Grashof number, Schmidt number, Soret number, Prandtl number, radiation parameter or heat source parameter, while it decreases due to increase in the values of Hartmann number, permeability parameter, chemical reaction parameter, cross flow Reynolds number or slip parameter. The skin-friction coefficient at the plate when $y=1$ increases due to increase of Hartmann number, permeability parameter or chemical reaction parameter, while it decreases due to increase of Grashof number, modified Grashof number, Schmidt number, Soret number, Prandtl number, radiation parameter, heat source parameter, cross flow Reynolds number or slip parameter.

It is noted from Table 2 that the Nusselt number at the plate when $y=0$ increases due to increase in Prandtl number, radiation parameter or heat source parameter, while it decreases due to increase in the value of cross flow Reynolds number. The Nusselt number at the plate when $y=1$ increases as cross flow Reynolds number, while it decreases due to increase of Prandtl number, radiation parameter or heat source parameter.

From table 3, it is observed that the Sherwood number at the plate when $y=0$ increases as chemical reaction parameter or cross flow Reynolds number, while it decreases as the Schmidt number, Soret number, Prandtl number, radiation parameter or heat source parameter increases. The Sherwood number at the plate when $y=1$ increases due to increase in Schmidt number, Soret number, Prandtl number, radiation parameter, heat source parameter or cross flow Reynolds number increases, while it decreases due to increase in the values of chemical reaction parameter.

VIII. Conclusions

Effect of heat and mass transfer on oscillatory free convective and chemical reactive flow of an incompressible viscous electrically conducting fluid through porous medium in slip flow regime between parallel plates with radiation and temperatue gradient dependent heat source is investigated. The velocity, temperature and concentration profiles are obtained and shown through graphs. Dimensionless rate of shear stress, rate of heat and mass transfer at the plates are derived and their numerical values are presented through tables. In view of the above, the following conlusions are made

(i) The velocity of the fluid increases due to increase in Grashof number, modified Grashof number or slip parameter.

(ii) The velocity and concentration of the fluid increase due to increase in Schmidt number or Soret number.

(iii) As Prandtl number, radiation parameter or heat source parameter increases, the velocity and concentration of the fluid increase, while temperature of the fluid decreases.

(iv) The velocity and concentration of the fluid increases near the plate at $y=1$ as cross flow Reynolds number increases, while opposite behaviour is observed on the other plate.

(v) As Grashof number, modified Grashof number, Schmidt number or Soret number increases, the skin-friction coefficient increases at the plate when $y=0$ and they decreases at the plate when $y=1$.

(vi) Due to an increase in Prandtl number, radiation parameter or heat source parameter the skin-friction coefficient and Nusselt number increase at the plate when $y=0$ and they decrease at the plate when $y=1$

(vii) The skin-friction coefficient at both the plates decreases as cross flow Reynolds number increases.

(viii) As cross flow Reynolds number increases, the Nusselt number decreases at the plate when $y=0$, while increases at the plate when $y=1$.

(ix) The Sherwood number at the plate when $y=0$ decreases due to increase in Schmidt number, Soret number, Prandtl number, radiation parameter or heat source parameter and it decreases as chemical reaction parameter increases, while the opposite behaviour is observed at the plate when $y=1$.

(x) The Sherwood number at both the plates decreases as cross flow Reynolds number increases.

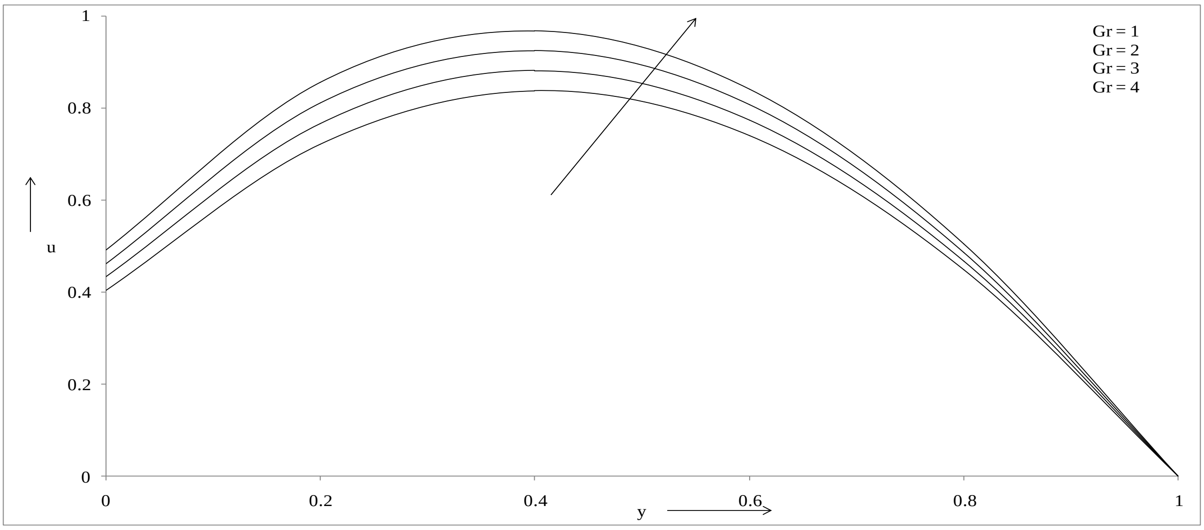

Figure 1. Velocity profiles versus y for different values of Gr when Gc=5, Sc=0.6, So=3, Pr=0.71, R=2, S=5, Ha=1, K=0.2, Kr=2, Re=1, h=0.2, ε=0.001, t=0.1, n=1.

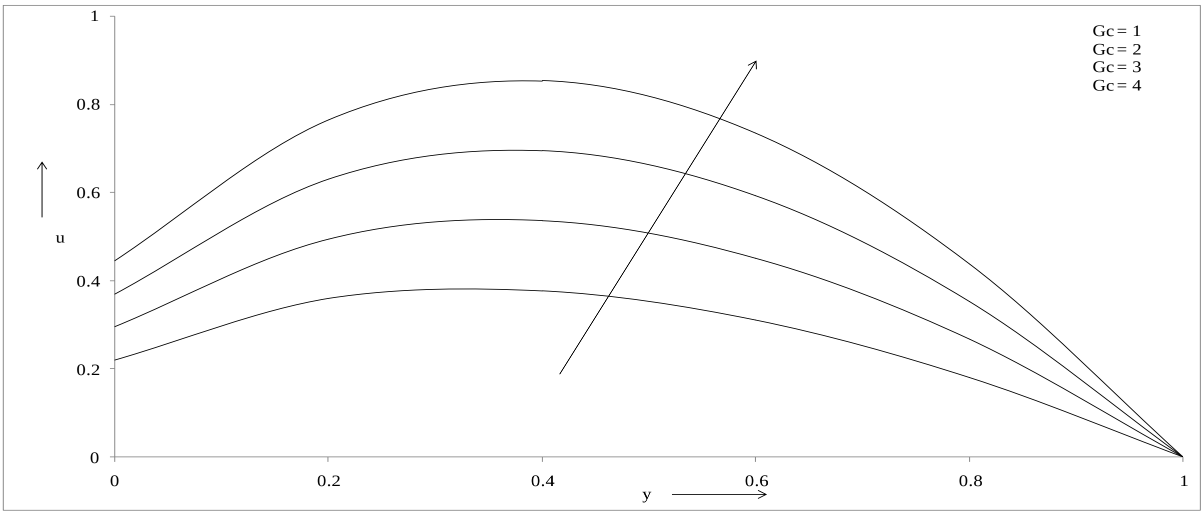

Figure 2. Velocity profiles versus y for different values of Gc when Gr=5, Sc=0.6, So=3, Pr=0.71, R=2, S=5, Ha=1, K=0.2, Kr=2, Re=1, h=0.2, ε=0.001, t=0.1, n=1.

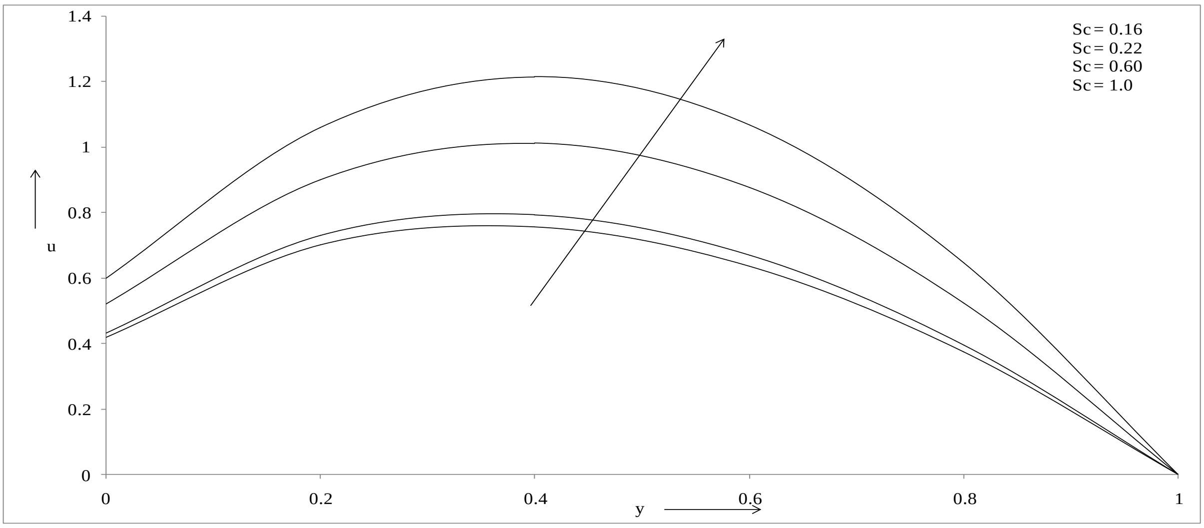

Figure 3. Velocity profiles versus y for different values of Sc when Gr=5, Gc=5, So=3, Pr=0.71, R=2, S=5, Ha=1, K=0.2, Kr=2, Re=1, h=0.2, ε=0.001, t=0.1, n=1.

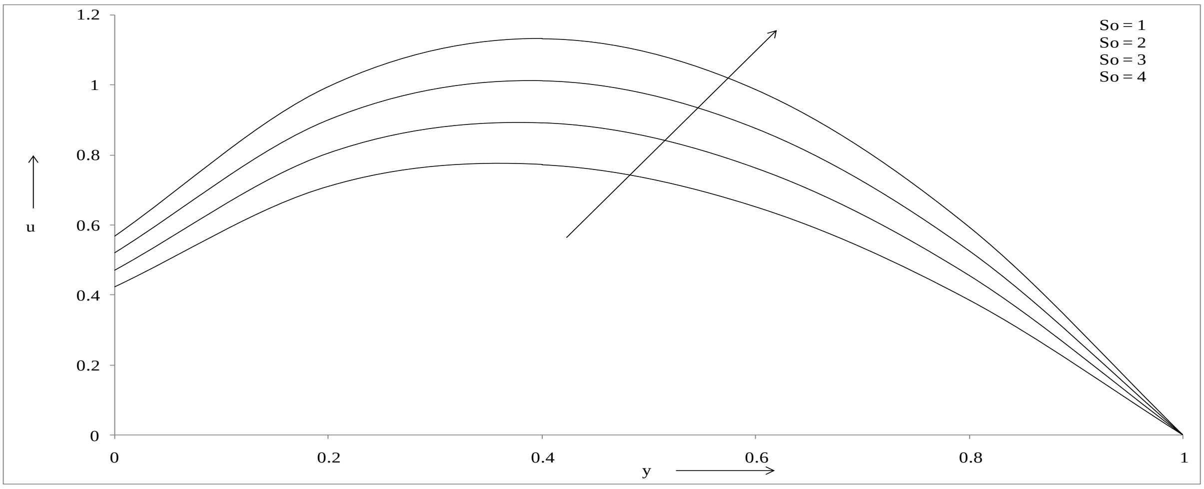

Figure 4. Velocity profiles versus y for different values of So when Gr=5, Gc=5, Sc=0.6, Pr=0.71, R=2, S=5, Ha=1, K=0.2, Kr=2, Re=1, h=0.2, ε=0.001, t=0.1, n=1.

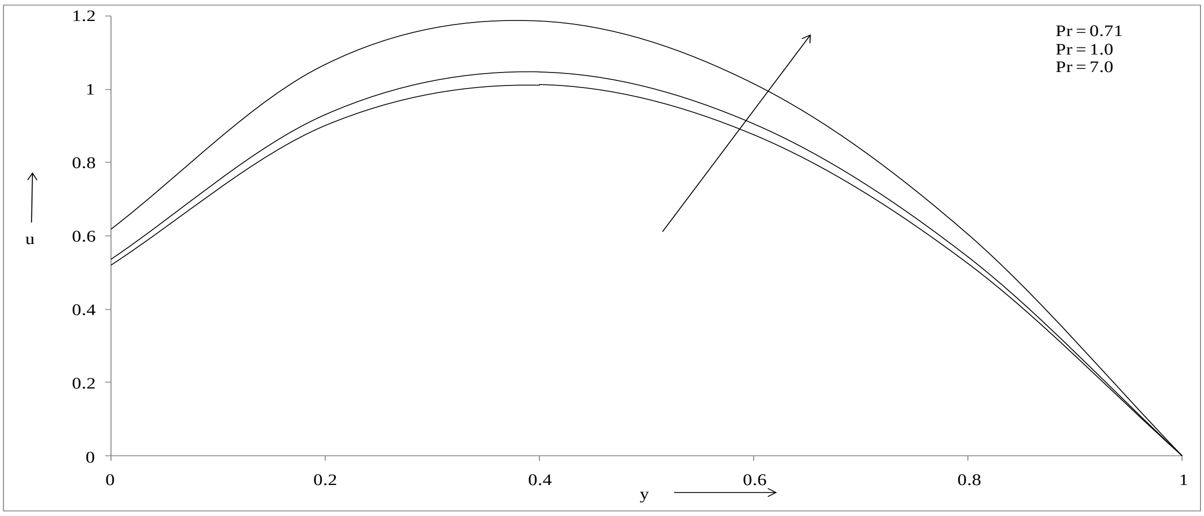

Figure 5. Velocity profiles versus y for different values of $\Pr$ when Gr=5, Gc=5, Sc=0.6, So=3, R=2, S=5, Ha=1, K=0.2, Kr=2, Re=1, h=0.2, ε=0.001, t=0.1, n=1.

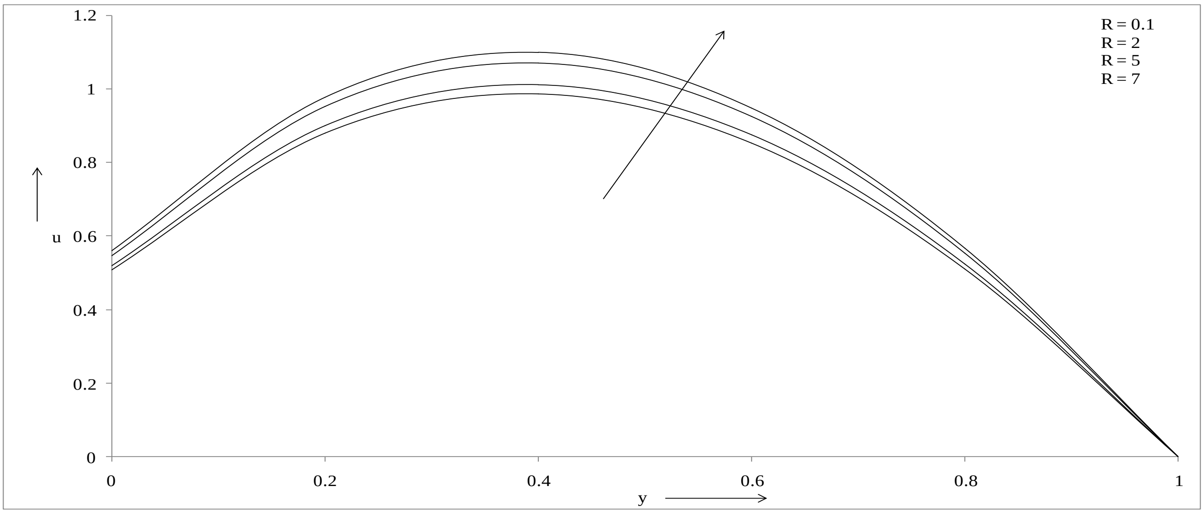

Figure 6. Velocity profiles versus y for different values of R when Gr=5, Gc=5, Sc=0.6, So=3, Pr=0.71, S=5, Ha=1, K=0.2, Kr=2, Re=1, h=0.2, ε=0.001, t=0.1, n=1.

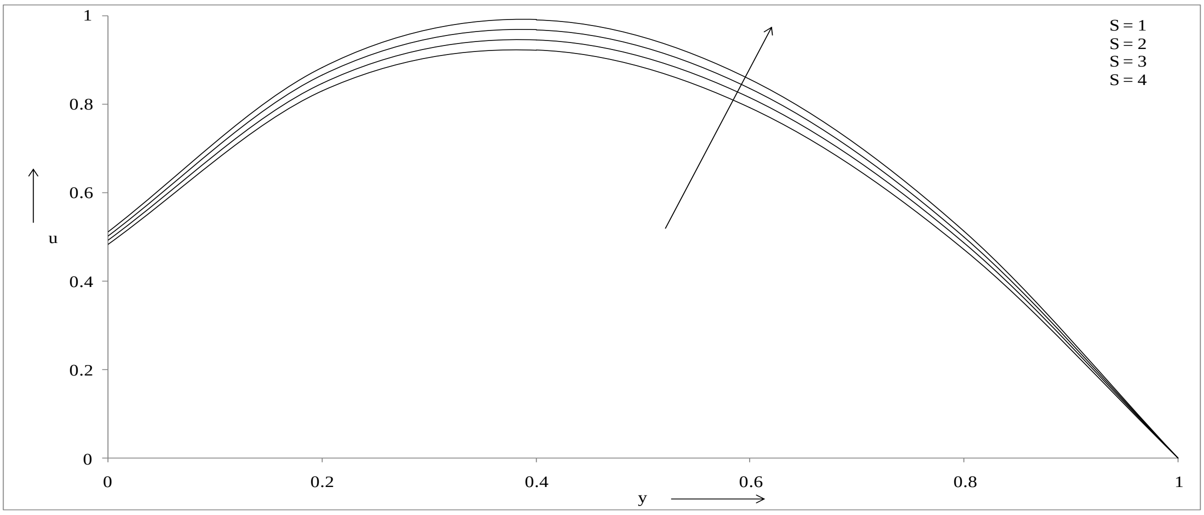

Figure 7. Velocity profiles versus y for different values of S when Gr=5, Gc=5, Sc=0.6, So=3, Pr=0.71, R=2, Ha=1, K=0.2, Kr=2, Re=1, h=0.2, ε=0.001, t=0.1, n=1.

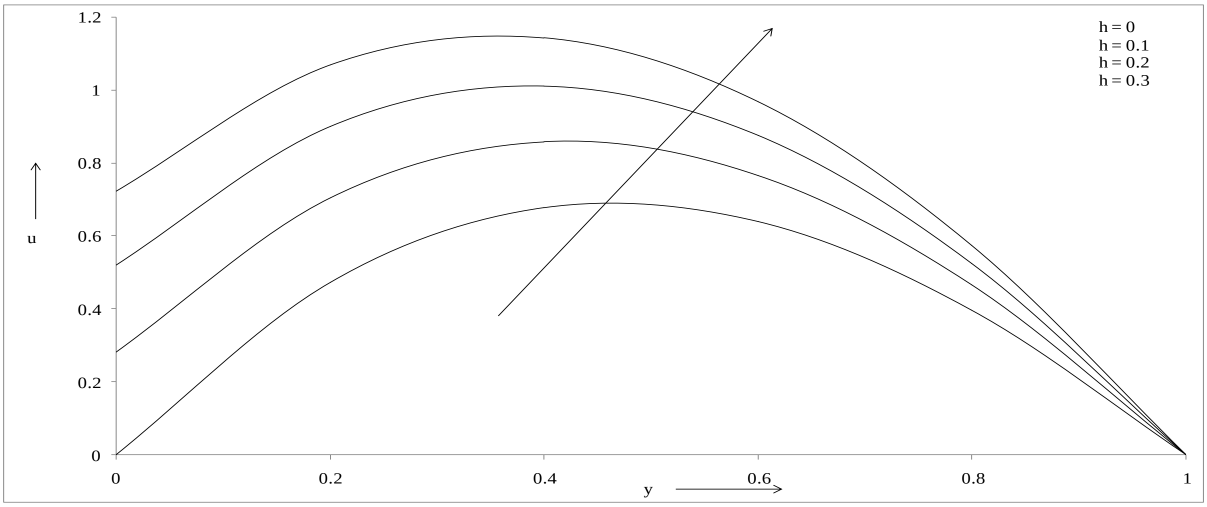

Figure 8. Velocity profiles versus y for different values of h when Gr=5, Gc=5, Sc=0.6, So=3, Pr=0.71, R=2, S=5, Ha=1, K=0.2, Kr=2, Re=1, ε=0.001, t=0.1, n=1.

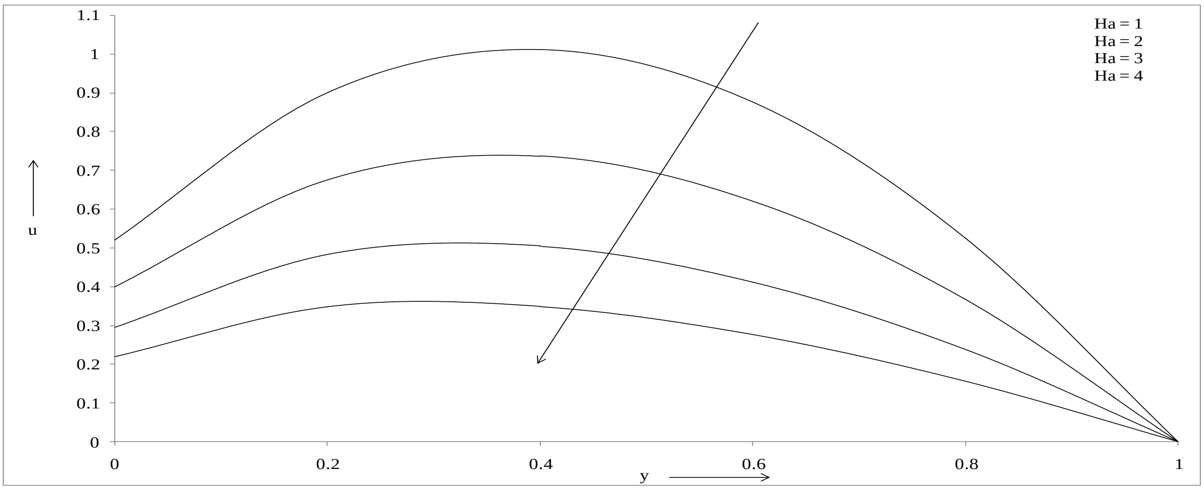

Figure 9. Velocity profiles versus y for different values of Ha when Gr=5, Gc=5, Sc=0.6, So=3, Pr=0.71, R=2, S=5, K=0.2, Kr=2, Re=1, h=0.2, ε=0.001, t=0.1, n=1.

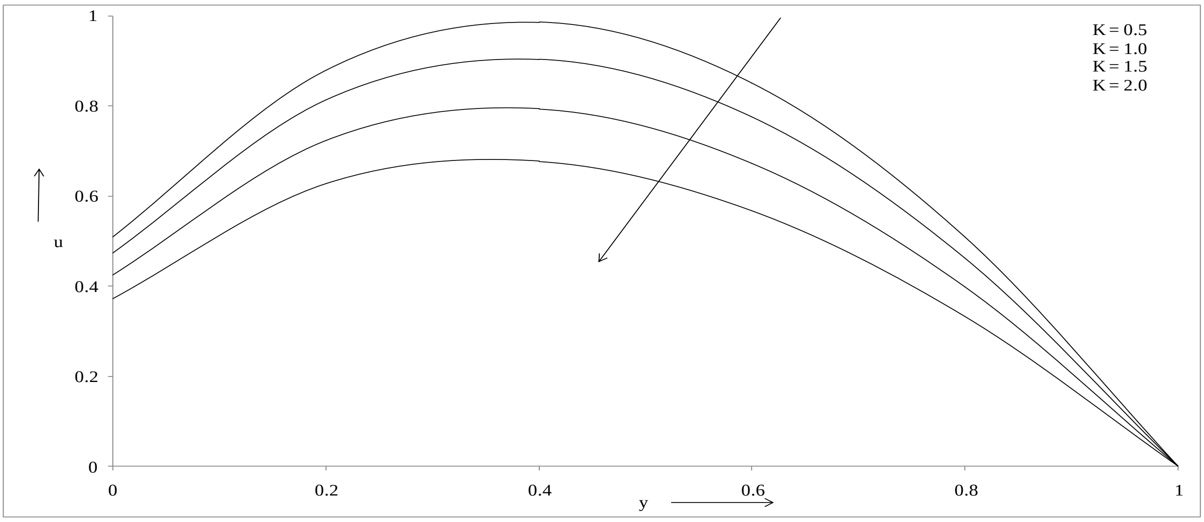

Figure 10. Velocity profiles versus y for different values of K when Gr=5, Gc=5, Sc=0.6, So=3, Pr=0.71, R=2, S=5, Ha=1, Kr=2, Re=1, h=0.2, ε=0.001, t=0.1, n=1.

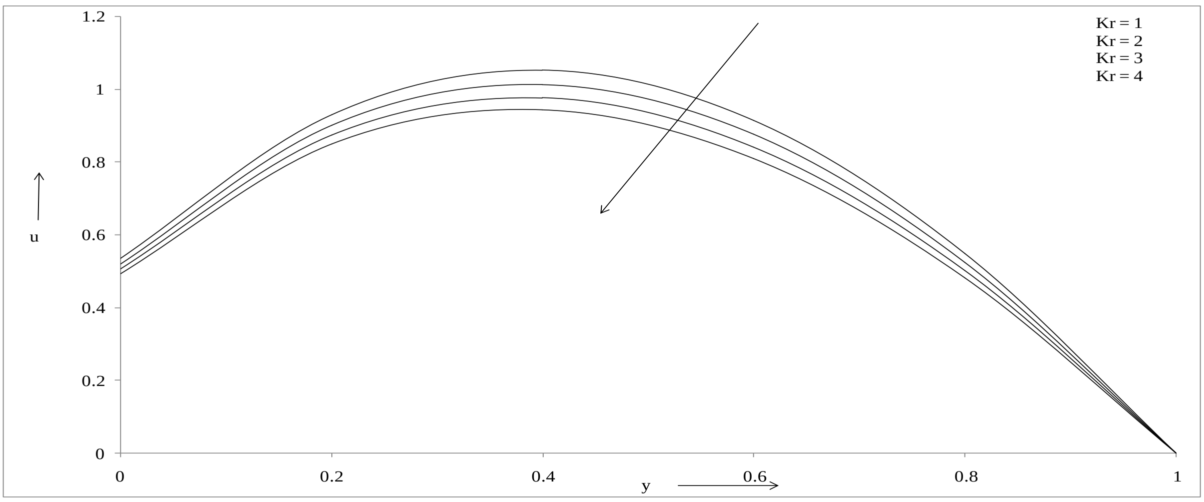

Figure 11. Velocity profiles versus y for different values of Kr when Gr=5, Gc=5, Sc=0.6, So=3, Pr=0.71, R=2, S=5, Ha=1, K=0.2, Re=1, h=0.2, ε=0.001, t=0.1, n=1.

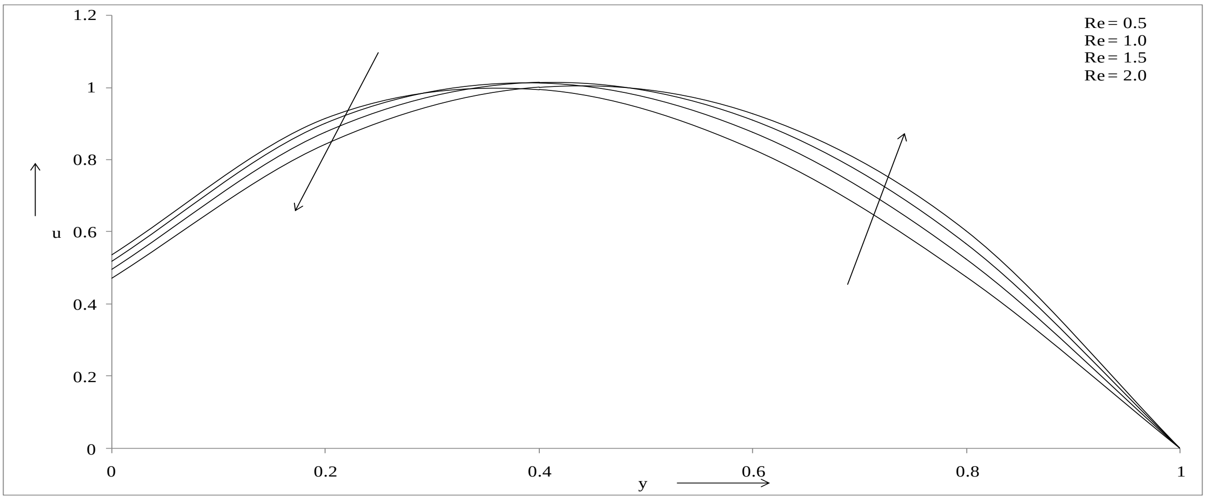

Figure 12. Velocity profiles versus y for different values of Re when Gr=5, Gc=5, Sc=0.6, So=3, Pr=0.71, R=2, S=5, Ha=1, K=0.2, Kr=2, h=0.2, ε=0.001, t=0.1, n=1.

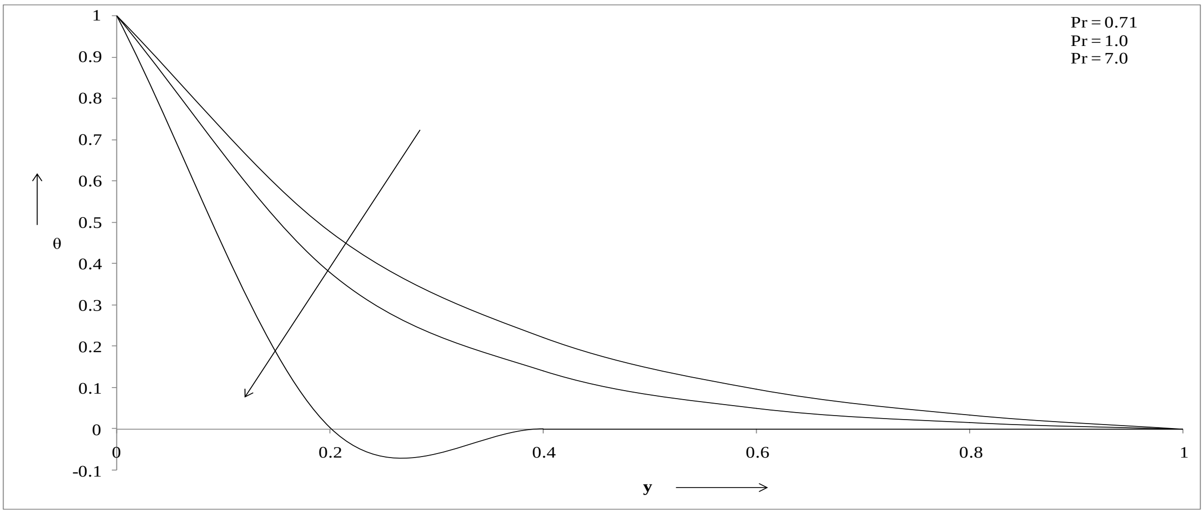

Figure 13. Temperature profiles versus y for different values of Pr when Gr=5, Gc=5, Sc=0.6, So=3, R=2, S=5, Ha=1, K=0.2, Kr=2, Re=1, h=0.2, ε=0.001, t=0.1, n=1.

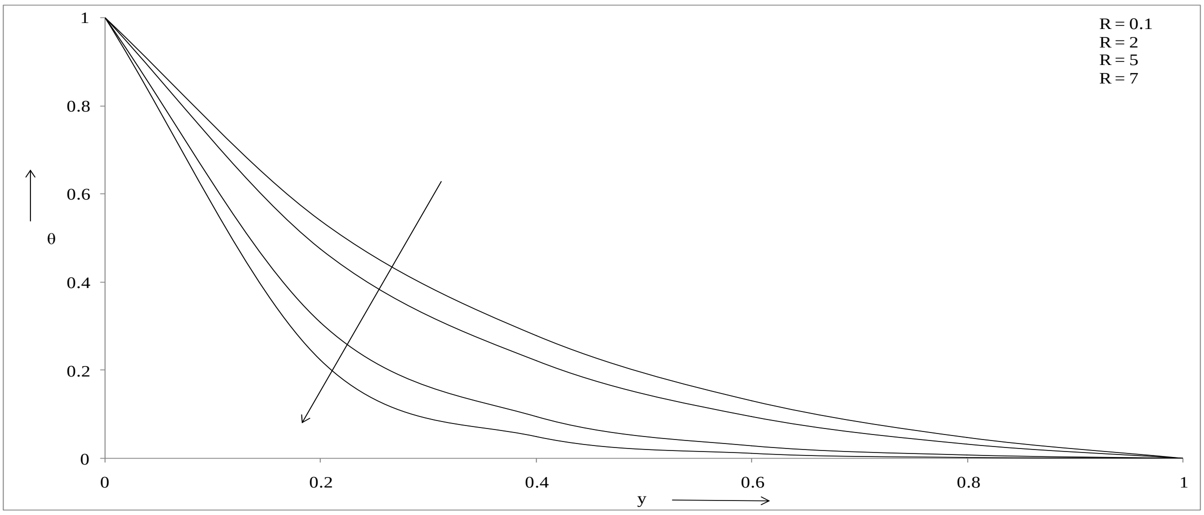

Figure 14. Temperature profiles versus y for different values of R when Gr=5, Gc=5, Sc=0.6, So=3, Pr=0.71, S=5, Ha=1, K=0.2, Kr=2, Re =1, h=0.2, ε=0.001, t=0.1, n=1.

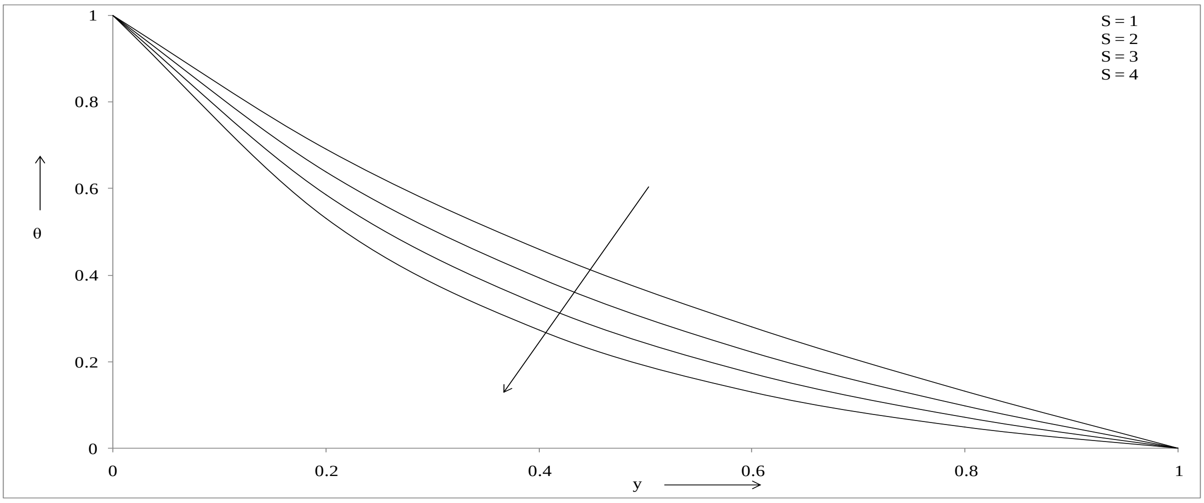

Figure 15. Temperature profiles versus y for different values of S when Gr=5, Gc=5, Sc=0.6, So=3, Pr=0.71, R=2, Ha=1, K=0.2, Kr=2, Re =1, h=0.2, ε=0.001, t=0.1, n=1.

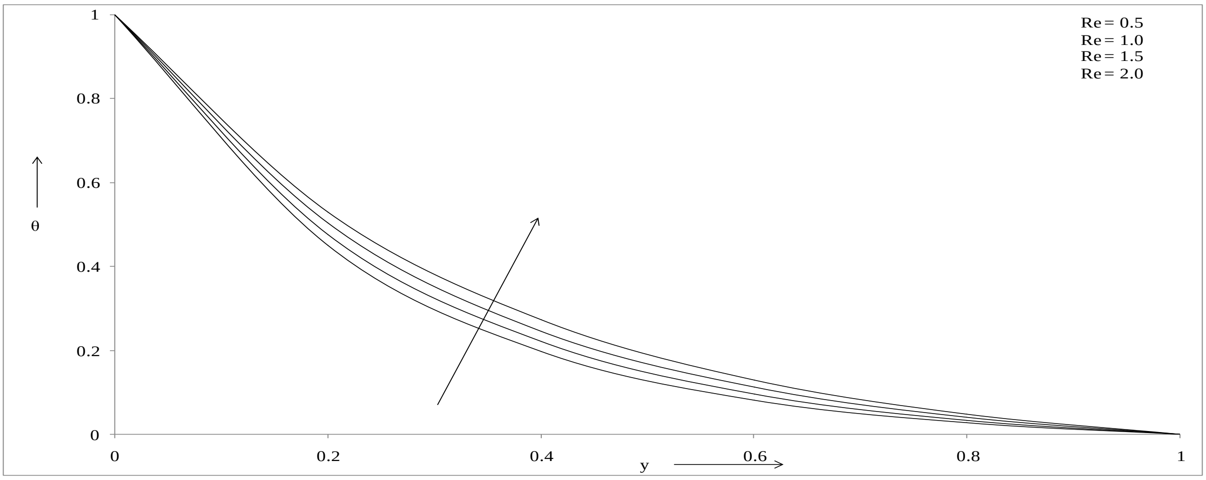

Figure 16. Temperature profiles versus y for different values of Re when Gr=5, Gc=5, Sc=0.6, So=3, Pr=0.71, R=2, S=5, Ha=1, K=0.2, Kr=2, h=0.2, ε=0.001, t=0.1, n=1.

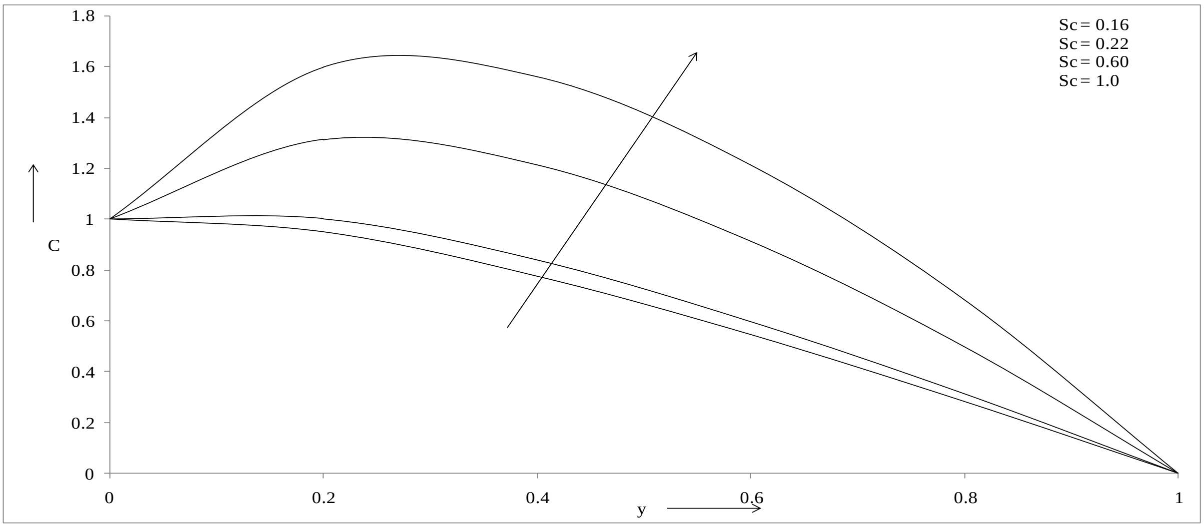

Figure 17. Concentration profiles versus y for different values of Sc when Gr=5, Gc=5, So=3, Pr=0.71, R=2, S=5, Ha=1, K=0.2, Kr=2, Re =1, h=0.2, ε=0.001, t=0.1, n=1.

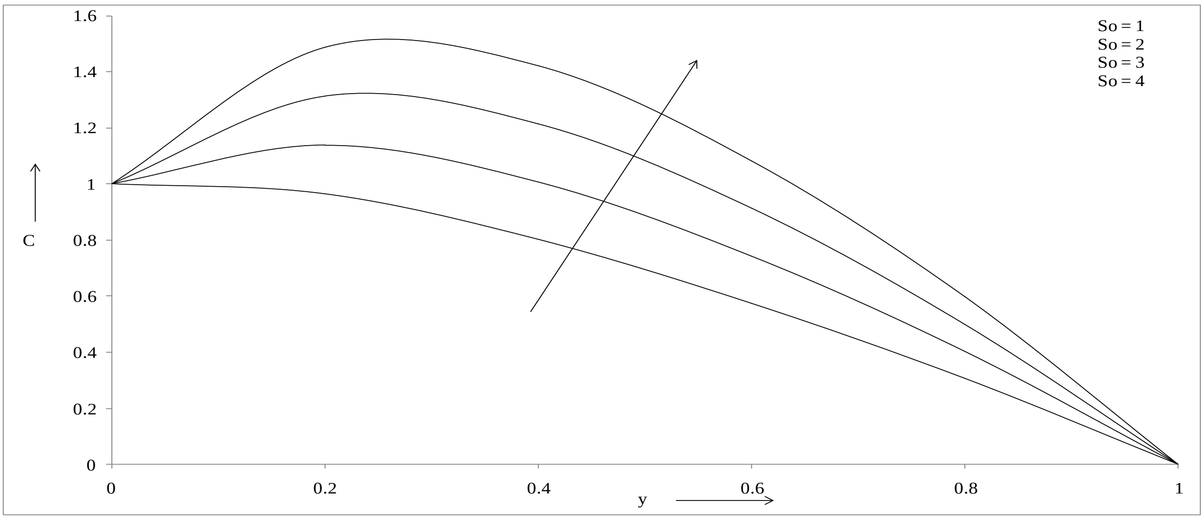

Figure 18. Concentration profiles versus y for different values of So when Gr=5, Gc=5, Sc=0.6, Pr=0.71, R=2, S=5, Ha=1, K=0.2, Kr=2, Re =1, h=0.2, ε=0.001, t=0.1, n=1.

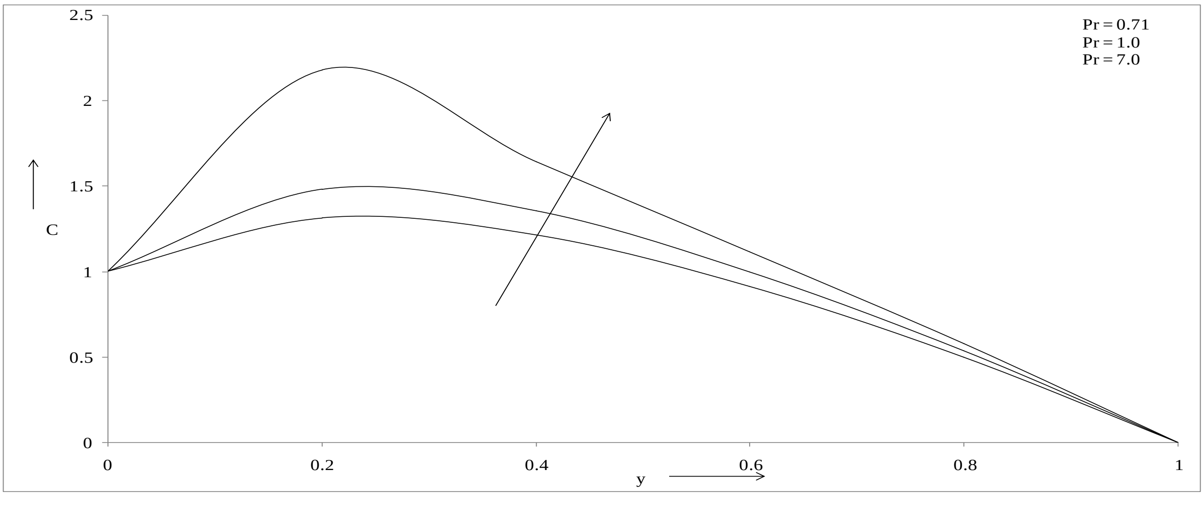

Figure 19. Concentration profiles versus y for different values of $\Pr$ when Gr=5, Gc=5, Sc=0.6, So=3, R=2, S=5, Ha=1, K=0.2, Kr=2, Re =1, h=0.2, ε=0.001, t=0.1, n=1.

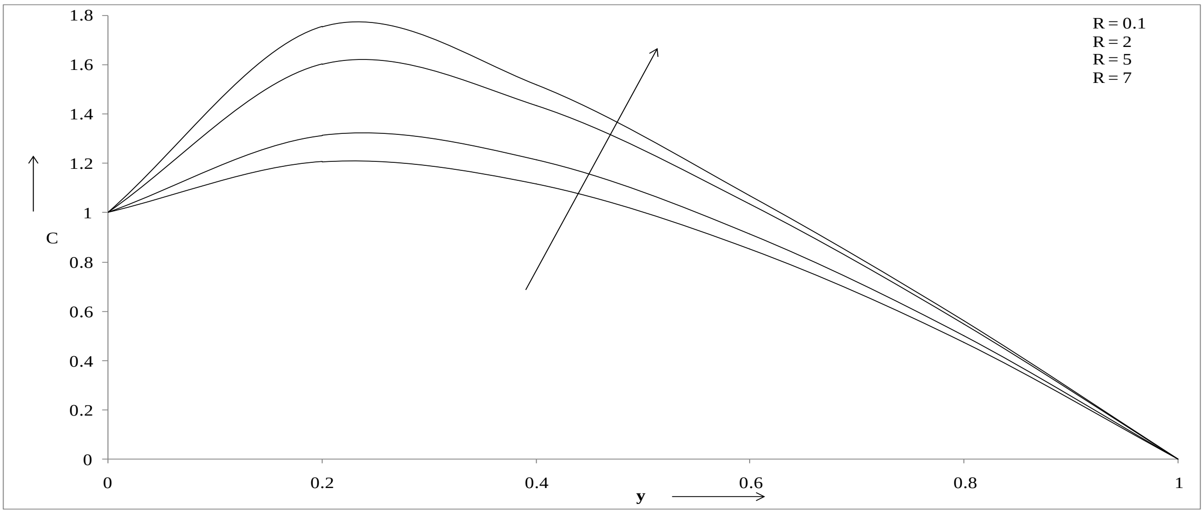

Figure 20. Concentration profiles versus y for different values of R when Gr=5, Gc=5, Sc=0.6, So=3, Pr=0.71, S=5, Ha=1, K=0.2, Kr=2, Re =1, h=0.2, ε=0.001, t=0.1, n=1.

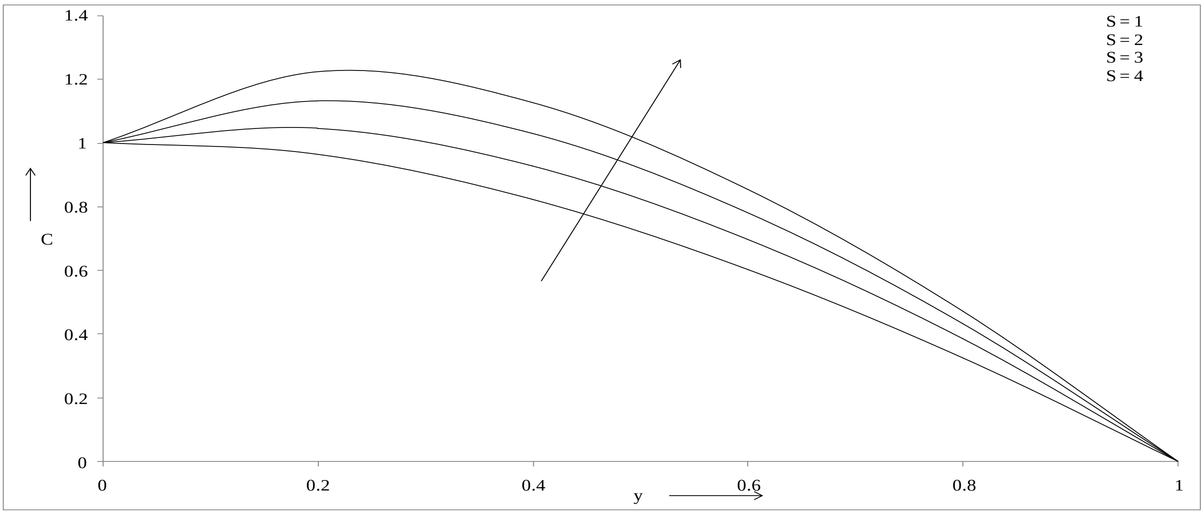

Figure 21. Concentration profiles versus y for different values of S when Gr=5, Gc=5, Sc=0.6, So=3, Pr=0.71, R=2, Ha=1, K=0.2, Kr=2, Re =1, h=0.2, ε=0.001, t=0.1, n=1.

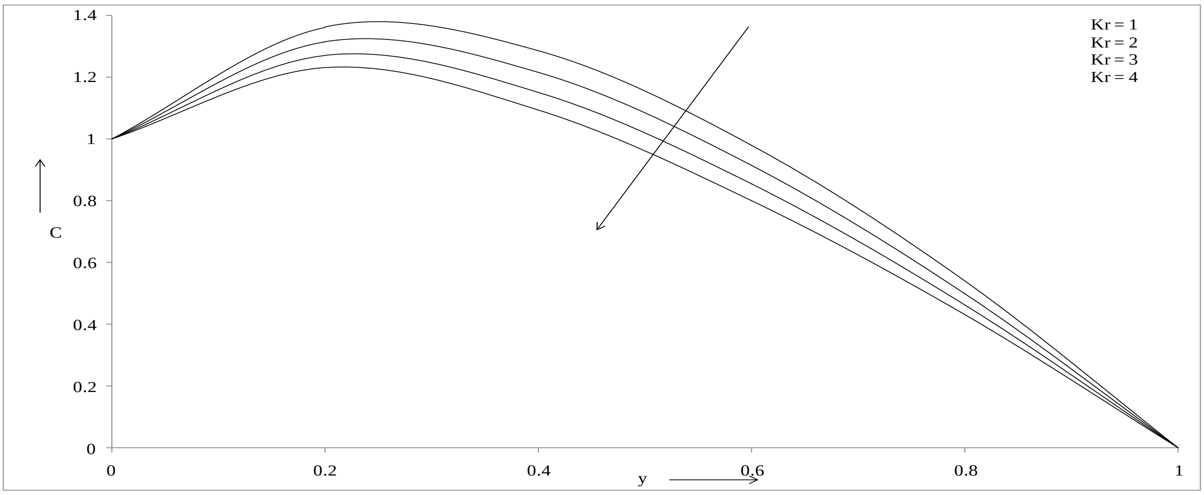

Figure 22. Concentration profiles versus y for different values of Kr when Gr=5, Gc=5, Sc=0.6, So=3, Pr=0.71, R=2, S=5, Ha=1, K=0.2, Re =1, h=0.2, ε=0.001, t=0.1, n=1.

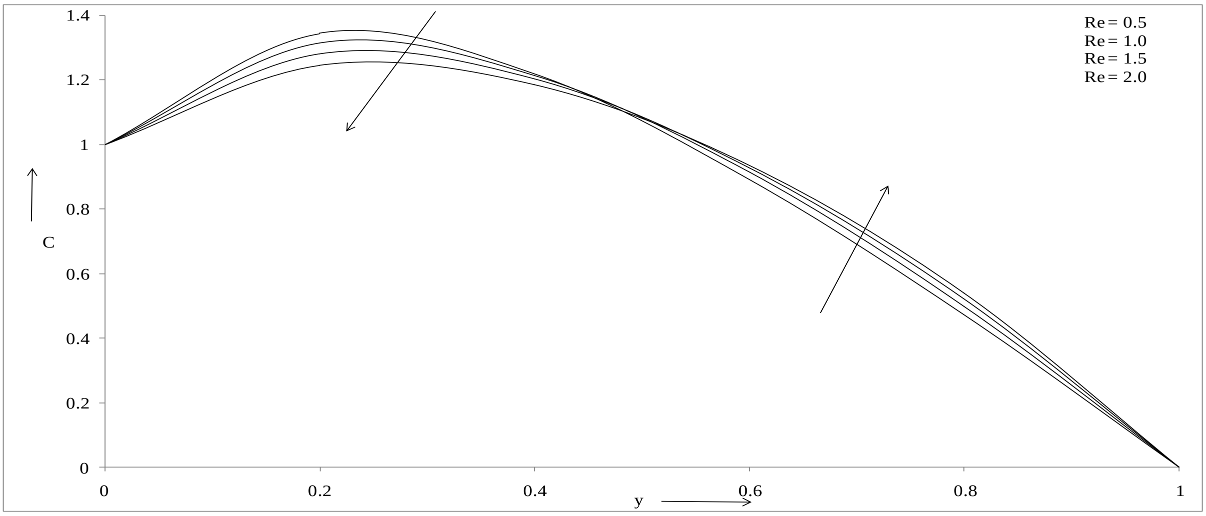

Figure 23. Concentration profiles versus y for different values of Re when Gr=5, Gc=5, Sc=0.6, So=3, Pr=0.71, R=2, S=5, Ha=1, K=0.2, Kr=2, h=0.2, ε=0.001, t=0.1, n=1.

Table 1. Numerical values of skin friction coefficient at the plates for various values of physical parameters

Gr	Gc	Sc	So	Pr	R	S	Ha	K	Kr	Re	h	Cf_0	Cf_1
5	5	0.6	3	0.71	2	5	1	0.2	2	1	0.2	2.598	-2.968
1	5	0.6	3	0.71	2	5	1	0.2	2	1	0.2	2.02	-2.549
3	5	0.6	3	0.71	2	5	1	0.2	2	1	0.2	2.309	-2.759
5	1	0.6	3	0.71	2	5	1	0.2	2	1	0.2	1.098	-1.012
5	3	0.6	3	0.71	2	5	1	0.2	2	1	0.2	1.848	-1.99
5	5	0.16	3	0.71	2	5	1	0.2	2	1	0.2	2.091	-2.105
5	5	0.22	3	0.71	2	5	1	0.2	2	1	0.2	2.164	-2.229
5	5	0.6	1	0.71	2	5	1	0.2	2	1	0.2	2.119	-2.169
5	5	0.6	2	0.71	2	5	1	0.2	2	1	0.2	2.358	-2.568
5	5	0.6	3	1	2	5	1	0.2	2	1	0.2	2.679	-3.071
5	5	0.6	3	7	2	5	1	0.2	2	1	0.2	3.092	-3.412
5	5	0.6	3	0.71	0.2	5	1	0.2	2	1	0.2	2.546	-2.896
5	5	0.6	3	0.71	5	5	1	0.2	2	1	0.2	2.736	-3.134
5	5	0.6	3	0.71	2	1	1	0.2	2	1	0.2	2.416	-2.667
5	5	0.6	3	0.71	2	3	1	0.2	2	1	0.2	2.507	-2.83
5	5	0.6	3	0.71	2	5	2	0.2	2	1	0.2	1.994	-2.059
5	5	0.6	3	0.71	2	5	3	0.2	2	1	0.2	1.469	-1.326
5	5	0.6	3	0.71	2	5	1	1	2	1	0.2	2.364	-2.609
5	5	0.6	3	0.71	2	5	1	1.5	2	1	0.2	2.121	-2.245
5	5	0.6	3	0.71	2	5	1	0.2	1	1	0.2	2.673	-3.111
5	5	0.6	3	0.71	2	5	1	0.2	3	1	0.2	2.531	-2.839
5	5	0.6	3	0.71	2	5	1	0.2	2	0.5	0.2	2.684	-2.566
5	5	0.6	3	0.71	2	5	1	0.2	2	1.5	0.2	2.486	-3.365
5	5	0.6	3	0.71	2	5	1	0.2	2	1	0.1	2.812	-2.648
5	5	0.6	3	0.71	2	5	1	0.2	2	1	0.3	2.415	-3.243

Table 2. Numerical values of Nusselt number at the plates for various values of physical parameters

Pr	R	S	Re	Nu_0	Nu_1
0.71	2	5	1	3.68067	0.11924
1	2	5	1	4.85184	0.04548
7	2	5	1	28.9919	7.9E-12
0.71	0.2	5	1	3.02561	0.17576
0.71	5	5	1	5.87186	0.02524
0.71	2	1	1	1.80635	0.64803
0.71	2	3	1	2.63729	0.29691
0.71	2	5	0.5	3.9675	0.09322
0.71	2	5	1.5	3.40314	0.15148

Table 3. Numerical values of Sherwood number at the walls for various values of physical parameters

Sc	So	Pr	R	S	Kr	Re	Sh_0	Sh_1
0.6	3	0.71	2	5	2	1	-3.211	2.711
0.16	3	0.71	2	5	2	1	-0.22	1.453
0.22	3	0.71	2	5	2	1	-0.658	1.623
0.6	1	0.71	2	5	2	1	-0.338	1.637
0.6	2	0.71	2	5	2	1	-1.775	2.174
0.6	3	1	2	5	2	1	-5.195	2.858
0.6	3	7	2	5	2	1	-48.16	3.041
0.6	3	0.71	0.2	5	2	1	-2.115	2.601
0.6	3	0.71	5	5	2	1	-6.951	2.907
0.6	3	0.71	2	1	2	1	-0.165	1.741
0.6	3	0.71	2	3	2	1	-1.488	2.379
0.6	3	0.71	2	5	1	1	-3.502	2.937
0.6	3	0.71	2	5	3	1	-2.941	2.511
0.6	3	0.71	2	5	2	0.5	-3.695	2.492
0.6	3	0.71	2	5	2	1.5	-2.749	2.919

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