

An Application of Interval Valued Fuzzy Soft Matrix In Medical Diagnosis

Dr.N.Sarala¹ and M.prabhavathi²

¹Department of mathematics, A.D.M.college for women (Auto),Nagai, India.

²Department of mathematics,E.G.S.PillayArts&sciencecollege,Nagai,India.

Abstract: Today is a world of uncertainty with its associated problems, which can be well handled by soft set theory. In this paper, we extend sanchez's approach for medical diagnosis using the representation of an interval valued fuzzy soft matrices. we introduce the definition of union and intersection of Interval valued fuzzy soft matrices with examples.Finally, we extend our approach in application of these matrices in medical Diagnosis.

Keywords: Soft set, fuzzy soft set, Interval valued fuzzy soft matrix, Union and intersection of Interval valued fuzzy soft matrix, Interval valued fuzzy soft matrix medical diagnosis.

I. Introduction

The concept of interval valued fuzzy matrix(IVFM) is one of the recent topics developed for dealing with the uncertainties present in most of our real life situations, the parameterization tool of interval valued fuzzy matrixenhances the flexibility of its applications. Most of our real life problems in medical sciences,engineering,management environment and social sciences often involve data which are not necessarily crisp. Precise and deterministic in character due to various uncertainties associated with these problems. Such uncertainties are usually being handled with the help of the topics like probability, fuzzy set, intuitionistic fuzzy sets, interval mathematics and rough sets etc. The concept of IVFM as a generalization of fuzzy matrix was introduced and developed by shyamal and pal [8], by extending the max. min operations on fuzzy algebra $\mathcal{F}=[0,1]$, for elements $a, b \in \mathcal{F}, a+b = \max \{ a,b\}$ and $a \cdot b = \min \{ a,b\}$. Let \mathcal{F}_{mn} be the set of all $m \times n$ Fuzzy Matrices over the Fuzzy algebra with support $[0,1]$, that is matrices whose entries are intervals and all the intervals are subintervals of the interval $[0,1]$.

De et.al. [2]have studied sanchez's [5,6] method of medical diagnosis using intuitionistic fuzzy set. Saikia et.al.[7]have extended the method in [2] using intuitionistic fuzzy soft set theory. In [1],Chetia and Das have studied sanchez's approach of medical diagnosis through IVFSS obtaining an improvement of the same presented in De et .al.[2 and 7]. In our earlier work [3], we have represented an IVFM $A = (a_{ij}) = (a_{ijL}, a_{ijU})$ where each a_{ij} is a subinterval of interval $[0,1]$, as the Interval matrix $A = [A_L, A_U]$ whose ij^{th} entry is the interval $[a_{ijL}, a_{ijU}]$, where the lower limit $A_L = (a_{ijL})$ and the upper limit $A_U = (a_{ijU})$ are fuzzy matrices such that $A_L \leq A_U$. By using this representation we have discussed the consistency of Interval valued fuzzy relational equations in [4]. In[17] P.Rajarajeswari and P.Dhanalakshmi have introduced interval valued fuzzy soft matrix, its types with examples and some new operations on the basis of weights.

In this paper, we extend sanchez's approach for medical diagnosis is using the representation of an interval valued fuzzy soft matrix. We introduce the definition of union and intersection of an interval valued fuzzy soft matrix with examples. Finally, we extend are approach is application of these matrices in medical Diagnosis.

II. Preliminaries

Soft set 2.1 [9]

Suppose that U is an initial Universe set and E is a set of parameters, let $P(U)$ denotes the power set of U . A pair (F, E) is called a soft set over U where F is a mapping given by $F : E \rightarrow P(U)$. Clearly a soft set is a mapping from parameters to $P(U)$ and it is not a set, but a parameterized family of subsets of the Universe.

Fuzzy soft set 2.2 [10]

Let U be an initial Universe set and E be the set of parameters, let $A \subseteq E$. A pair (F, A) is called fuzzy soft set over U where F is a mapping given by $F: A \rightarrow I^U$, where I^U denotes the collection of all fuzzy subsets of U .

Fuzzy soft Matrices 2.3 [12]

Let $U = \{c_1, c_2, c_3, \dots, c_m\}$ be the Universe set and E be the set of parameters given by $E = \{e_1, e_2, e_3, \dots, e_n\}$. Let $A \subseteq E$ and (F, A) be a fuzzy soft set in the fuzzy soft class (U, E) . Then fuzzy soft set (F, A) in a matrix form as $A_{m \times n} = [a_{ij}]_{m \times n}$ or $A = [a_{ij}]$ $i=1, 2, \dots, m, j=1, 2, 3, \dots, n$

$$F(e_j) = \begin{cases} \mu_j(c_i) & \text{if } e_j \in A \text{ Where } a_{ij} = \mu_j(c_i) \text{ represents the membership of } c_i \text{ in the fuzzy set} \\ 0 & \text{if } e_j \notin A \end{cases}$$

Interval valued fuzzy soft set 2.4 [11]

Let U be an initial Universe set and E be the set of parameters, let $A \subseteq E$. A pair (F,A) is called Interval valued fuzzy soft set over U where F is a mapping given by $F: A \rightarrow I^U$, where I^U denotes the collection of all Interval valued fuzzy subsets of U.

Interval valued fuzzy soft matrix 2.5[13]

Let $U = \{c_1, c_2, c_3, \dots, c_m\}$ be the Universe set and E be the set of parameters given by $E = \{e_1, e_2, e_3, \dots, e_n\}$. Let $A \subseteq E$ and (F,A) be a interval valued fuzzy soft set over U, where F is a mapping given by $F: A \rightarrow I^U$, where I^U denotes the collection of all Interval valued fuzzy subsets of U. Then the Interval valued fuzzy soft set can expressed in matrix form as

$$\tilde{A}_{m \times n} = [a_{ij}]_{m \times n} \text{ or } \tilde{A} = [a_{ij}] \quad i=1,2,\dots,m, j=1,2,\dots,n$$

$$\text{Where } a_{ij} = \begin{cases} [\mu_{jL}(c_i), \mu_{jU}(c_i)] & \text{if } e_j \in A \\ [0,0] & \text{if } e_j \notin A \end{cases}$$

$[\mu_{jL}(c_i), \mu_{jU}(c_i)]$ represents the membership of c_i in the Interval valued fuzzy set $F(e_j)$.

Note that if $\mu_{jU}(c_i) = \mu_{jL}(c_i)$ then the Interval- valued fuzzy soft matrix (IVFSM) reduces to an FSM

Example: 2.1

Suppose that there are four houses under consideration, namely the universes $U = \{h_1, h_2, h_3, h_4\}$, and the parameter set $E = \{e_1, e_2, e_3, e_4\}$ where e_i stands for “beautiful”, ”large”, ”cheap”, and “in green surroundings” respectively. Consider the mapping F from parameter set $A = \{e_1, e_2\} \subseteq E$ to all interval valued fuzzy subsets of power set U. Consider an interval valued fuzzy soft set (F,A) which describes the “attractiveness of houses” that is considering for purchase. Then interval valued fuzzy soft set (F,A) is

$$(F,A) = \{ F(e_1) = \{(h_1, [0.6,0.8]), (h_2, [0.8,0.9]), (h_3, [0.6,0.7]), (h_4, [0.5,0.6])\} \\ F(e_2) = \{(h_1, [0.7,0.8]), (h_2, [0.6,0.7]), (h_3, [0.5,0.7]), (h_4, [0.8,0.9])\}$$

We would represent this Interval valued fuzzy soft set in matrix form as

$$\begin{bmatrix} [0.6,0.8] & [0.7,0.8] & [0.0,0.0] & [0.0,0.0] \\ [0.8,0.9] & [0.6,0.7] & [0.0,0.0] & [0.0,0.0] \\ [0.6,0.7] & [0.5,0.7] & [0.0,0.0] & [0.0,0.0] \\ [0.5,0.6] & [0.8,0.9] & [0.0,0.0] & [0.0,0.0] \end{bmatrix}$$

Addition of interval valued fuzzy soft matrices 2.6 [13]

If $\tilde{A} = [a_{ij}] \in \text{IVFSM}_{m \times n}$, $\tilde{B} = [b_{ij}] \in \text{IVFSM}_{m \times n}$, then we define $\tilde{A} + \tilde{B}$, addition of \tilde{A} and \tilde{B} as $\tilde{A} + \tilde{B} = [c_{ij}]_{m \times n} = [\max(\mu_{AL}, \mu_{BL}), \max(\mu_{AU}, \mu_{BU})]$ for all i and j.

Example: 2.2

Consider

$$\tilde{A} = \begin{bmatrix} [0.6,0.8] & [0.7,0.8] \\ [0.5,0.6] & [0.8,0.9] \end{bmatrix}_{2 \times 2} \text{ and } \tilde{B} = \begin{bmatrix} [0.8,0.9] & [0.6,0.7] \\ [0.6,0.7] & [0.5,0.7] \end{bmatrix}_{2 \times 2}$$

are two interval valued fuzzy soft matrices then sum of these two is

$$\tilde{A} + \tilde{B} = \begin{bmatrix} [0.8,0.9] & [0.7,0.8] \\ [0.6,0.7] & [0.8,0.9] \end{bmatrix}_{2 \times 2}$$

Multiplication of interval valued fuzzy soft matrices 2.7 [13]

If $\tilde{A} = [a_{ij}] \in \text{IVFSM}_{m \times n}$, $\tilde{B} = [b_{jk}] \in \text{IVFSM}_{n \times p}$, then we define $\tilde{A} * \tilde{B}$, multiplication of \tilde{A} and \tilde{B} as $\tilde{A} * \tilde{B} = [c_{ik}]_{m \times p} = [\max \min(\mu_{AL_j}, \mu_{BL_j}), \max \min(\mu_{AU_j}, \mu_{BU_j})], \forall i, j, k$

Example: 2.3

Consider

$$\tilde{A} = \begin{bmatrix} [0.6,0.8] & [0.7,0.8] \\ [0.5,0.6] & [0.8,0.9] \end{bmatrix}_{2 \times 2} \text{ and } \tilde{B} = \begin{bmatrix} [0.8,0.9] & [0.6,0.7] \\ [0.6,0.7] & [0.5,0.7] \end{bmatrix}_{2 \times 2}$$

are two interval valued fuzzy soft matrices then product of these two matrices is

$$\tilde{A} * \tilde{B} = \begin{bmatrix} [0.6,0.8] & [0.6,0.7] \\ [0.6,0.7] & [0.5,0.7] \end{bmatrix}_{2 \times 2}$$

Remark: $\tilde{A} * \tilde{B} \neq \tilde{B} * \tilde{A}$

Interval valued fuzzy soft complement matrix 2.8 [13]

Let $\tilde{A} = [a_{ij}] \in \text{IVFSM}_{m \times n}$, when $a_{ij} = [\mu_{jL}(c_i), \mu_{jU}(c_i)]$ then \tilde{A}^C is called interval valued fuzzy soft complement if $\tilde{A}^C = [b_{ij}]_{m \times n}$ where $b_{ij} = [1 - \mu_{jU}(c_i), 1 - \mu_{jL}(c_i)]$, $\forall ij$.

Example: 2.4

$$\text{Let } \tilde{A} = \begin{bmatrix} [0.6, 0.8] & [0.7, 0.8] \\ [0.5, 0.6] & [0.8, 0.9] \end{bmatrix}_{2 \times 2}$$

Be interval valued fuzzy soft matrix then complement of this matrix is

$$\tilde{A}^C = \begin{bmatrix} [0.2, 0.4] & [0.2, 0.3] \\ [0.4, 0.5] & [0.1, 0.2] \end{bmatrix}_{2 \times 2}$$

III. Union and intersection of Interval valued fuzzy soft matrices

In this section, we introduce the definition of union and intersection of Interval valued fuzzy soft matrices with examples and its properties

Definition: 3.1

Let $\tilde{A} = [a_{ij}]$, $\tilde{B} = [b_{ij}] \in \text{IVFSM}_{m \times n}$. Then union of \tilde{A}, \tilde{B} is defined by $\tilde{A} \cup \tilde{B} = \tilde{C}_{m \times n} = [c_{ij}]_{m \times n}$, where $\tilde{C}_{ij} = [a_{ij}] \cup [b_{ij}] = [\mu_{AL}, \mu_{AU}] \cup [\mu_{BL}, \mu_{BU}] = [\mu_{AL} + \mu_{BL} - \mu_{AL} \times \mu_{BL}, \mu_{AU} + \mu_{BU} - \mu_{AU} \times \mu_{BU}]$. For all i and j .

Example: 3.1

$$\text{Let } \tilde{A} = \begin{bmatrix} [0.6, 0.8] & [0.7, 0.8] \\ [0.5, 0.6] & [0.8, 0.9] \end{bmatrix}_{2 \times 2} \quad \text{and } \tilde{B} = \begin{bmatrix} [0.8, 0.9] & [0.6, 0.7] \\ [0.6, 0.7] & [0.5, 0.7] \end{bmatrix}_{2 \times 2}$$

are two interval valued fuzzy soft matrices then union of these two is

$$\tilde{A} \cup \tilde{B} = \begin{bmatrix} [0.92, 0.98] & [0.88, 0.94] \\ [0.80, 0.88] & [0.90, 0.97] \end{bmatrix}$$

Proposition: 3.1

Let $A = [a_{ij}]$, $B = [b_{ij}] \in \text{IVFSM}_{m \times n}$,

Then

- (i) $\tilde{A} \cup \tilde{A} = \tilde{A}$
- (ii) $\tilde{A} \cup \tilde{U} = \tilde{U}$
- (iii) $\tilde{A} \cup \tilde{B} = \tilde{B} \cup \tilde{A}$
- (iv) $(\tilde{A} \cup \tilde{B}) \cup \tilde{C} = \tilde{A} \cup (\tilde{B} \cup \tilde{C})$.

Proof:

(i) Let $A = [a_{ij}] = [\mu_{AL}, \mu_{AU}]$

$$\begin{aligned} \tilde{A} \cup \tilde{A} &= [\mu_{AL}, \mu_{AU}] \cup \tilde{A} \\ &= [(\mu_{AL} + 0 - \mu_{AL} \times 0), (\mu_{AU} + 0 - \mu_{AU} \times 0)] \\ &= [\mu_{AL}, \mu_{AU}] \\ &= \tilde{A} \end{aligned}$$

$$\begin{aligned} \text{(ii) } \tilde{A} \cup \tilde{U} &= [\mu_{AL}, \mu_{AU}] \cup [1, 1] \\ &= [(\mu_{AL} + 1 - \mu_{AL} \times 1), (\mu_{AU} + 1 - \mu_{AU} \times 1)] \\ &= [1, 1] = \tilde{U}. \end{aligned}$$

$$\begin{aligned} \text{(iii) } \tilde{A} \cup \tilde{B} &= \tilde{B} \cup \tilde{A} \\ \text{Let } \tilde{A} &= [\mu_{AL}, \mu_{AU}], \tilde{B} = [\mu_{BL}, \mu_{BU}]. \\ \tilde{A} \cup \tilde{B} &= [(\mu_{AL} + \mu_{BL} - \mu_{AL} \times \mu_{BL}), (\mu_{AU} + \mu_{BU} - \mu_{AU} \times \mu_{BU})] \\ &= [(\mu_{BL} + \mu_{AL} - \mu_{BU} \times \mu_{AU}), (\mu_{BU} + \mu_{AU} - \mu_{BU} \times \mu_{AU})] \\ &= \tilde{B} \cup \tilde{A}. \end{aligned}$$

$$\begin{aligned} \text{(iv) } (\tilde{A} \cup \tilde{B}) \cup \tilde{C} &= [(\mu_{AL} + \mu_{BL} - \mu_{AL} \times \mu_{BL}), (\mu_{AU} + \mu_{BU} - \mu_{AL} \times \mu_{BU})] \cup [\mu_{CL}, \mu_{CU}] \\ &= [(\mu_{AL} + \mu_{BL} - \mu_{AL} \times \mu_{BL} + \mu_{CL} - (\mu_{AL} + \mu_{BL} - \mu_{AL} \times \mu_{BL}) \times (\mu_{CL})), \\ &\quad (\mu_{AU} + \mu_{BU} - \mu_{AL} \times \mu_{BU} + \mu_{CU} - (\mu_{AU} + \mu_{BU} - \mu_{AL} \times \mu_{BU}) \times (\mu_{CU}))] \\ &= [(\mu_{AL} + \mu_{BU} + \mu_{CL} - \mu_{AL} \times \mu_{BL} - \mu_{AL} \times \mu_{CL} - \mu_{BL} \times \mu_{CL} + \mu_{AL} \times \mu_{BL} \times \mu_{CL}), \\ &\quad (\mu_{AU} + \mu_{BU} + \mu_{CU} - \mu_{AU} \times \mu_{BU} - \mu_{AU} \times \mu_{CU} - \mu_{BU} \times \mu_{CU} + \mu_{AU} \times \mu_{BU} \times \mu_{CU})] \end{aligned}$$

$$\begin{aligned} \tilde{A} \cup (\tilde{B} \cup \tilde{C}) &= [\mu_{AL}, \mu_{AU}] \cup [\mu_{BL} + \mu_{CL} - \mu_{BL} \times \mu_{CL}, \mu_{BU} + \mu_{CU} - \mu_{BU} \times \mu_{CU}]. \\ &= [(\mu_{AL} + \mu_{BL} + \mu_{CL} - \mu_{BL} \times \mu_{CL} - (\mu_{AL}) \times (\mu_{BL} + \mu_{CL} - \mu_{BL} \times \mu_{CL})), \\ &\quad (\mu_{AU} + \mu_{BU} + \mu_{CU} - \mu_{BU} \times \mu_{CU} - (\mu_{AU}) \times (\mu_{BU} + \mu_{CU} - \mu_{BU} \times \mu_{CU}))] \\ &= [(\mu_{AL} + \mu_{BL} + \mu_{CL} - \mu_{AL} \times \mu_{BL} - \mu_{BL} \times \mu_{CL} - \mu_{AL} \times \mu_{CL} - \mu_{AL} \times \mu_{BL} \times \mu_{CL}), \\ &\quad (\mu_{AU} + \mu_{BU} + \mu_{CU} - \mu_{AU} \times \mu_{BU} - \mu_{BL} \times \mu_{CU} - \mu_{AU} \times \mu_{CU} - \mu_{AL} \times \mu_{BL} \times \mu_{CL})] \end{aligned}$$

$$\therefore \tilde{A} \cup (\tilde{B} \cup \tilde{C}) = (\tilde{A} \cup \tilde{B}) \cup \tilde{C}.$$

Definition: 3.2

Let $A=[a_{ij}], B=[b_{ij}] \in IVFSM_{m \times n}$ then intersection of A,B is defined by $\tilde{A}_{m \times n} \cap \tilde{B}_{m \times n} = \tilde{C}_{m \times n} = [c_{ij}]_{m \times n}$, where $c_{ij} = [a_{ij}] \cap [b_{ij}] = [\mu_{AL}, \mu_{AU}] \cap [\mu_{BL}, \mu_{BU}] = [\mu_{AL} \times \mu_{BL}, \mu_{AU} \times \mu_{BU}]$

Example: 3.2

Let $\tilde{A} = \begin{bmatrix} [0.6, 0.8] & [0.7, 0.8] \\ [0.5, 0.6] & [0.8, 0.9] \end{bmatrix}_{2 \times 2}$ and $\tilde{B} = \begin{bmatrix} [0.8, 0.9] & [0.6, 0.7] \\ [0.6, 0.7] & [0.5, 0.7] \end{bmatrix}_{2 \times 2}$

are two interval valued fuzzy soft matrices then the intersection of these two is.

$$\tilde{A} \cap \tilde{B} = \begin{bmatrix} [0.48, 0.72] & [0.42, 0.56] \\ [0.30, 0.42] & [0.40, 0.63] \end{bmatrix}_{2 \times 2}$$

Proposition: 3.2

Let A,B, and C $\in IVFSM_{m \times n}$,

Then

- (i) $\tilde{A} \cap \tilde{O} = \tilde{O}$
- (ii) $\tilde{A} \cap \tilde{U} = \tilde{A}$
- (iii) $\tilde{A} \cap \tilde{B} = \tilde{B} \cap \tilde{A}$
- (iv) $(\tilde{A} \cap \tilde{B}) \cap \tilde{C} = \tilde{A} \cap (\tilde{B} \cap \tilde{C})$.

Proof:

Let $A = [a_{ij}]_{m \times n}$, $B = [b_{ij}]_{m \times n}$, $C = [c_{ij}]_{m \times n}$.

Where $[a_{ij}] = [\mu_{AL}, \mu_{AU}]$, $[b_{ij}] = [\mu_{BL}, \mu_{BU}]$ and $[c_{ij}] = [\mu_{CL}, \mu_{CU}]$.

$$\begin{aligned} \text{(i) } \tilde{A} \cap \tilde{O} &= [\mu_{AL}, \mu_{AU}] \cap [0] \\ &= [\mu_{AL} \times 0, \mu_{AU} \times 0] \\ &= [0, 0] \\ &= \tilde{O}. \end{aligned}$$

$$\begin{aligned} \text{(ii) } \tilde{A} \cap \tilde{U} &= \tilde{A} \\ \tilde{A} \cap \tilde{U} &= [\mu_{AL}, \mu_{AU}] \cap [1, 1] \\ &= [\mu_{AL} \times 1, \mu_{AU} \times 1] \\ &= [\mu_{AL}, \mu_{AU}] \\ &= \tilde{A}. \end{aligned}$$

$$\begin{aligned} \text{(iii) } \tilde{A} \cap \tilde{B} &= [\mu_{AL}, \mu_{AU}] \cap [\mu_{BL}, \mu_{BU}] \\ &= [\mu_{AL} \times \mu_{BL}, \mu_{AU} \times \mu_{BU}] \\ &= [\mu_{BL} \times \mu_{AL}, \mu_{BU} \times \mu_{AU}] \\ &= \tilde{B} \cap \tilde{A}. \end{aligned}$$

$$\begin{aligned} \text{(iv) } (\tilde{A} \cap \tilde{B}) \cap \tilde{C} &= [\mu_{AL} \times \mu_{BL}, \mu_{AU} \times \mu_{BU}] \cap [\mu_{CL}, \mu_{CU}] \\ &= [\mu_{AL} \times \mu_{BL} \times \mu_{CL}, \mu_{AU} \times \mu_{BU} \times \mu_{CU}] \\ &= [\mu_{AL} \times (\mu_{BL} \times \mu_{CL}), \mu_{AU} \times (\mu_{BU} \times \mu_{CU})] \\ &= [\mu_{AL}, \mu_{AU}] \cap [\mu_{BL} \times \mu_{CL}, \mu_{BU} \times \mu_{CU}] \\ &= \tilde{A} \cap (\tilde{B} \cap \tilde{C}) \end{aligned}$$

IV. Application of interval valued fuzzy soft matrix in medical diagnosis.

Suppose S is a set of symptoms of certain diseases, D is a set of diseases and P is a set of patients. Construct an interval - valued fuzzy soft set (F,D) over S, where F is a mapping $F:D \rightarrow F(S)$. A relation matrix say , R_1 is constructed from the interval - valued fuzzy soft set (F,D) and called symptom – diseases matrix. Similarly its complement $(F,D)^C$ gives another relation matrix, say R_2 , called non symptom – diseases matrix. Analogous to sanchez’s notion of ‘Medical Knowledge’ we refer to each of the matrices R_1 and R_2 as ‘interval – valued soft Medical Knowledge’. Again we construct another interval – valued fuzzy soft set (F_1, S) over P, Where F_1 is a mapping given by $F_1: S \rightarrow \tilde{F}(P)$. This interval – valued fuzzy soft set gives another relation matrix Q called patient- symptom matrix. Then we obtain two new relation matrices $T_1 = Q.R_1$ and $T_2 = Q.R_2$, called symptom-patient matrix and non - symptom patient matrix respectively, in which the membership values are given by

$$\begin{aligned} \mu_{T_1}(p_i, d_k) &= [\max\{\min(\mu_Q^L(p_i, e_j), \mu_{R_1}^L(p_j, d_k)), \max\{\min(\mu_Q^U(p_i, e_j), \mu_{R_1}^U(p_j, d_k))\}\}] \\ \mu_{T_2}(p_i, -d_k) &= [\max\{\min(\mu_Q^U(p_i, e_j), \mu_{R_2}^L(p_j, d_k)), \max\{\min(\mu_Q^U(p_i, e_j), \mu_{R_2}^U(p_j, d_k))\}\}] \end{aligned}$$

We calculate

$$\begin{aligned} S_{T_1} &= \max_{ij} \{(\mu_{T_1}^L(p_i, d_j), \mu_{T_1}^L(p_j, d_i)), \{(\mu_{T_1}^U(p_i, d_j) - \mu_{T_1}^U(p_j, d_i))\} \text{ and} \\ S_{T_2} &= \max_{ij} \{(\mu_{T_2}^L(p_i, d_j), \mu_{T_2}^L(p_j, d_i)), \{(\mu_{T_2}^U(p_i, d_j) - \mu_{T_2}^U(p_j, d_i))\}, \end{aligned}$$

Which we call as diagnosis score for and against the disease respectively.

Now, if $\max\{^S T_1(p_i, d_j) - ^S T_2(p_i, -d_j)\}$ occurs for exactly (p_i, d_k) only, then we conclude that the acceptable diagnostic hypothesis for patient p_i is the disease d_k . In case there is a tie, the process has to be repeated patient P_i by reassessing the symptoms.

V. Algorithm

1. Input the interval valued fuzzy soft sets (F, D) and $(F, D)^C$ over the sets S of symptoms, where D is the set of diseases. Also write the soft medical Knowledge R_1 and R_2 reassessing the relation matrices of the IVFSS (F, D) and $(F, D)^C$ respectively.
2. Input the IVFSS (F_1, S) over the set P of patients and write its relation matrix Q .
3. Compute the relation matrices $T_1 = Q.R_1$ and $T_2 = Q.R_2$.
4. Compute the diagnosis scores $^S T_1$ and $^S T_2$.
5. Find $S_K = \max\{^S T_1(p_i, d_j) - ^S T_2(p_i, -d_j)\}$.
Then we conclude that the patient p_i is suffering from the disease d_k .
6. If S_K has more than one value then go to step one and repeat the process by reassessing the symptoms for the patients.

VI. Case study

Suppose that there are three patients P_1, P_2 and P_3 in a hospital with symptoms fever, headache, generalized body pain (especially in the joint muscles) and rash problem. Let the possible diseases relating to the above symptoms be Dengue and Chikangunya. We consider the set $S = \{e_1, e_2, e_3, e_4\}$ as a universal set, where e_1, e_2, e_3 and e_4 represent the symptoms fever, headache, generalized body pain (especially in the joint muscles) and rash problem respectively and the set $D = \{d_1, d_2\}$ where d_1 and d_2 represent parameterized Dengue and Chikangunya respectively.

Suppose that $F(d_1) = \{\langle e_1, [0.5, 0.6] \rangle, \langle e_2, [0.2, 0.3] \rangle, \langle e_3, [0.8, 0.9] \rangle, \langle e_4, [0.3, 0.4] \rangle\}$,

$F(d_2) = \{\langle e_1, [0.8, 0.9] \rangle, \langle e_2, [0.6, 0.7] \rangle, \langle e_3, [0.7, 0.8] \rangle, \langle e_4, [0.5, 0.6] \rangle\}$,

The interval valued fuzzy soft set (F, D) is a parameterized family $\{F(d_1), F(d_2)\}$ of all interval valued fuzzy set over the set S and are determined from expert medical documentation. Thus the fuzzy soft set (F, D) gives an approximate description of interval valued fuzzy soft medical Knowledge of the two diseases and their symptoms. This interval valued fuzzy soft set (F, D) and its complement $(F, D)^C$ are represented by two relation matrices R_1 and R_2 , called symptom – disease matrix respectively, given by

$$R_1 = \begin{matrix} & d_1 & d_2 \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} & \begin{bmatrix} [0.5, 0.6] & [0.8, 0.9] \\ [0.2, 0.3] & [0.6, 0.7] \\ [0.8, 0.9] & [0.7, 0.8] \\ [0.3, 0.4] & [0.5, 0.6] \end{bmatrix} \end{matrix} \text{ and } R_2 = \begin{matrix} & d_1 & d_2 \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} & \begin{bmatrix} [0.4, 0.5] & [0.1, 0.2] \\ [0.7, 0.8] & [0.3, 0.4] \\ [0.1, 0.2] & [0.2, 0.3] \\ [0.6, 0.7] & [0.4, 0.5] \end{bmatrix} \end{matrix}$$

Again we take $P = \{P_1, P_2, P_3\}$ as the universal set where P_1, P_2 and P_3 represent patients respectively and $S = \{e_1, e_2, e_3, e_4\}$ as the set of parameters. Suppose that,

$F_1(e_1) = \{\langle p_1, [0.8, 0.9] \rangle, \langle p_2, [0.2, 0.3] \rangle, \langle p_3, [0.4, 0.5] \rangle\}$

$F_1(e_2) = \{\langle p_1, [0.6, 0.8] \rangle, \langle p_2, [0.3, 0.5] \rangle, \langle p_3, [0.5, 0.6] \rangle\}$

$F_1(e_3) = \{\langle p_1, [0.4, 0.6] \rangle, \langle p_2, [0.5, 0.7] \rangle, \langle p_3, [0.6, 0.8] \rangle\}$

$F_1(e_4) = \{\langle p_1, [0.7, 0.9] \rangle, \langle p_2, [0.6, 0.9] \rangle, \langle p_3, [0.3, 0.6] \rangle\}$

The interval - valued fuzzy soft set (F_1, S) is another parameterized family of all interval - valued fuzzy set and gives a collection of approximate description of the patient – symptoms in the hospital. This interval - valued fuzzy soft set (F_1, S) represents a relation matrix Q called patient – symptom matrix given by

$$Q = \begin{matrix} & e_1 & e_2 & e_3 & e_4 \\ \begin{matrix} P_1 \\ P_2 \\ P_3 \end{matrix} & \begin{bmatrix} [0.8, 0.9] & [0.6, 0.8] & [0.4, 0.6] & [0.7, 0.9] \\ [0.2, 0.3] & [0.3, 0.5] & [0.5, 0.7] & [0.6, 0.9] \\ [0.4, 0.5] & [0.5, 0.6] & [0.6, 0.8] & [0.3, 0.6] \end{bmatrix} \end{matrix}$$

Then combining the relation matrices R_1 and R_2 separately with Q we get two matrices T_1 and T_2 called patient – disease and patient – non disease matrices respectively, given by

$$T_1 = Q . R_1 = \begin{matrix} & d_1 & d_2 \\ \begin{matrix} P_1 \\ P_2 \\ P_3 \end{matrix} & \begin{bmatrix} [0.5, 0.6] & [0.8, 0.9] \\ [0.5, 0.7] & [0.5, 0.7] \\ [0.6, 0.8] & [0.6, 0.8] \end{bmatrix} \end{matrix}$$

$$T_2 = Q \cdot R_2 = \begin{matrix} & d_1 & d_2 \\ P_1 & [0.6,0.8] & [0.4,0.5] \\ P_2 & [0.6,0.7] & [0.4,0.5] \\ P_3 & [0.5,0.6] & [0.3,0.5] \end{matrix}$$

Now we calculate

${}^sT_1 - {}^sT_2$	d_1	d_2
P_1	0.2	-0.1
P_2	-0.1	0.0
P_3	0.1	0.2

Now, it clear that the patient P_1 is suffering from disease d_1 and patients P_2 and P_3 are both suffering from disease d_2 .

VII. Conclusion

We have applied the notion of interval valued fuzzy soft matrices in Sanchez's method of medical diagnosis. A case study have been taken to exhibit the simplicity of the technique.

References

- [1]. Chetia, B., and Das, P.K. (2010). An Application of Interval valued fuzzy soft set in medical diagnosis, Int.J. contempt.math., science, vol. 5, 38, 1887-1894.
- [2]. De, S. K., Biswas, R., and Roy, A.R. (2001), An Application Intuitionistic fuzzy set medical diagnosis, Fuzzy sets and systems, 117, 209-213.
- [3]. Meenakshi, A.R., and Kaliraja, M. (2010) . Regular Interval valued fuzzy matrices, Advances in Fuzzy Mathematics, Vol.5 (1), 7-15.
- [4]. Meenakshi, A.R., and Kaliraja, M. Regular Interval valued fuzzy relational equations, Int.J.comp.cognition (accepted).
- [5]. Sanchez, E. (1976). \ Resolution of composite Fuzzy Relational equations, Information and control, 30, 38 -48.
- [6]. Sanchez, E. (1976).Inverse ofFuzzy Relational, Application to possibility distributions and medical diagnosis, Fuzzy set and systems, 2 (1), 75 – 86.
- [7]. Saikia, B.K., Das, p.k., and Borkakati, A.K.(2003). An Application Intuitionistic fuzzy soft set medical diagnosis, Bio science Research Bulletin, 19(2), 121-127.
- [8]. Shyamal, A.K., and Pal. (2006).Interval valued fuzzy matrices, Journal of Fuzzy Mathematics, Vol.14(3), 582 -592.
- [9]. P.K. Maji, R. Biswas and A.R. Roy, fuzzy soft set, TheJournal of Fuzzy Mathematics, 9(3) (2001) 589 -602.
- [10]. P.K. Maji, R. Biswas and A.R. Roy, Intuitionistic fuzzy soft set, The Journal of Fuzzy Mathematics, 12 (2004) 669 – 683 .
- [11]. Y. Yang and J. Chenli, Fuzzy soft matrices and their applications, Part I, Lecture Notes in computer science, 7002, 2011, pp: 618 – 627.
- [12]. L.A. Zadeh, Fuzzy sets, Information and control, 8 (1985) 338 – 353.
- [13]. P. Rajarajeswari andP.Dhanalakshmi Interval - valued fuzzy soft matrix theory, Annals of Pure and Applied Mathematics, Vol. 7 (2014) 61 – 72.