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A Nash Equilibrium Game and Pareto Efficient Solution to A Cooperative Advertising Model Where Demand Is Dependent On Price and Advertising Expenditure

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Abstract: *We consider a manufacturer retailer channel co ordination in a scenario where the demand is dependent on the price and the advertising expenditures by the manufacturer and the retailer, the channel members. We extend a previous model developed by the authors to a non cooperative simultaneous move nash game frame work. We also derive pareto efficient schemes in case of a cooperative problem.*
Keywords- *Cooperative advertising, Game theory, Nash equilibrium, Stackelberg equilibrium, Supply chain coordination*

I. Introduction

We develop co operative model in this paper where demand is modeled as a multiplicative effect of price and an additive sales response function. Kuehn, 1962; Thompson and Teng, 1984; Jorgensen and Zaccour, 1999, 2003 used multiplicative effect of the price (using an appropriate function) and advertising (using a sales response function) in their papers. In recent market retailer has equal or more power then manufacturer (Buzzell al., 1990 and Fulop, 1988). We develop the classic relationship between manufacturer and retailer named "leader-follower" two stage games. An equilibrium of this game is called the stackelberg equilibrium.

In a sequential move game the manufacturer as a leader specifies the brand name investments and the retailer as a follower then decides on the local advertising level. Solanki and Gor (2013) developed two game theoretic models; a cooperative model and a non cooperative model in a stackelberg game framework. This paper follows some of the part with a different sales response function.

In next section we consider a simultaneous move game called Nash equilibrium. The manufacturer's brand name investment is higher at Nash equilibrium then at stackelberg equilibrium. If the profits ratio of the retailer and the manufacturer is relatively low, then the local advertising expenditure is lower at Nash equilibrium than at stackelberg equilibrium; otherwise it is higher at Nash equilibrium than at stackelberg equilibrium.

The system profit is higher with cooperation then with non-cooperation and maximized for every Pareto efficient co op advertising scheme, but not for any other schemes. Also, if the marginal profits ratio of the retailer and manufacturer is relatively high, then the manufacturer's brand name investment is higher at any Pareto efficient scheme than at both noncooperative equilibriums; otherwise the manufacturer's brand name investment at any Pareto efficient scheme is higher than at Stackelberg equilibrium and is lower than at Nash equilibrium. The local advertising expenditure is higher at any Pareto efficient scheme than at both noncooperative equilibriums.

Assumptions

Co operative advertising is used to attract customers at the time of actual purchase. Customers can be aware about the product by manufacturer's national advertising and local advertising by retailer bring potential customers to the stage of desire and action. It gives customers the reason such as low price and high quality to buy and also aware about when and where to obtain the product. We assume that one manufacturer sells through one retailer i.e. Single-manufacturer-single retailer channel in which the retailer sells only the manufacturer's brand within the product class. It can be further extended thereafter.

Notations:

p: retailer's selling (retail) price
w: manufacturer's selling (wholesale) price
a: retailer's local advertising expenditure, $a \geq 0$
A: manufacturer's national advertising expenditure, $A \geq 0$
t: the manufacturer participation rate, the percentage that the manufacturer agrees to pay the retailer to subsidize the local advertising cost

The demand function is,

$$\mathrm{D(p,a,A) = g(p).h(a,A)} \tag{1}$$

Where g(p) reflects the impact of the retail price on the demand and h(a , A), the sales response function reflects the impact of the advertising expenditures on the demand. Following the well known structure in demand literature, g(p) is a linearly decreasing function of p and is specifically taken as

$$\mathrm{g(p)} = \gamma_0 - \gamma_1 \mathrm{p} \tag{2}$$

where $\gamma_0 >> \gamma_1$ and to ensure g(p) > 0, we need to restrict p < $\left({}^{\gamma_0}/_{\gamma_1}\right)$

Modeling the sale response function: Simon and Arndt (1980) concluded that diminishing returns characterize the shape of the advertising-sales response function. Similar approaches of relating demand and advertising expenditure were used in Kim and Staelin (1999) and Karray and Zaccour (2006). Assuming that both the types of advertising efforts; national and local, could influence sales and that their effects should be assessed separately (Jorgensen et al., 2000; Huang et al., 2002), we model advertising effects on consumer demand as

$$\mathrm{h(a,A)} = \mathrm{k_r}\sqrt{\mathrm{a}} + \mathrm{k_m}\sqrt{\mathrm{A}} \tag{3}$$

where $\mathrm{k_r}$, $\mathrm{k_m}$ are positive constants reflecting the efficacy of each type of advertising in generating sales. The above equation captures both types of advertising effects which usually are not substitutes. The demand h is an increasing and concave function with respect to a and A, and has the property that is consistent with the commonly observed ‘‘advertising saturation effect”, i.e., additional advertising spending generates continuously diminishing returns.

$$\mathrm{D(p,a,A)} = (\gamma_0 - \gamma_1 \mathrm{p})\,(\mathrm{k_r}\sqrt{\mathrm{a}} + \mathrm{k_m}\sqrt{\mathrm{A}}) \tag{4}$$

The manufacturer’s, retailer’s and system’s expected profit functions are as follows.

$$\pi_\mathrm{m} = \mathrm{w}\,(\gamma_0 - \mathrm{p}\gamma_1)\left(\mathrm{k_r}\sqrt{\mathrm{a}} + \mathrm{k_m}\sqrt{\mathrm{A}}\right) - \mathrm{ta} - \mathrm{A} \tag{5}$$

$$\pi_\mathrm{r} = (\mathrm{p} - \mathrm{w})\,(\gamma_0 - \mathrm{p}\gamma_1)\left(\mathrm{k_r}\sqrt{\mathrm{a}} + \mathrm{k_m}\sqrt{\mathrm{A}}\right) - (1-\mathrm{t})\mathrm{a} \tag{6}$$

$$\pi_\mathrm{m+r} = \mathrm{p}\,(\gamma_0 - \mathrm{p}\gamma_1)\left(\mathrm{k_r}\sqrt{\mathrm{a}} + \mathrm{k_m}\sqrt{\mathrm{A}}\right) - \mathrm{a} - \mathrm{A} \tag{7}$$

II. Stackelberg Equilibrium

We model the relationship between the manufacturer and retailer in which manufacturer is a leader and retailer a follower. It is a sequential noncooperative game.

Since the π_r is a concave function we can set its first derivative with respect to a to be zero.

$$\frac{\partial \pi_r}{\partial a} = (p - w)\,(\gamma_0 - p\gamma_1)\frac{k_r}{2\sqrt{a}} - (1-t) = 0 \tag{8}$$

Then, we have

$$a = \left(\frac{(p-w)(\gamma_0 - p\gamma_1)k_r}{2(1-t)}\right)^2 \tag{9}$$

Equation (9) shows positive change in manufacturer’s co – op advertising reimbursement policy and brand name investments. We can observe that

$$\frac{\partial a}{\partial t} = \frac{((p-w)(\gamma_0 - p\gamma_1)k_r)^2}{2(1-t)^3} > 0 \tag{10}$$

$$\frac{\partial a}{\partial A} = 0 \tag{11}$$

Equation (10) shows that the more the manufacturer is willing to share the cost of local advertising, the more the retailer will spend on the local advertising.

Also, the optimal value of A and t are determined by maximizing the manufacturer’s profit subject to the constraint imposed by eq. (9). The manufacturer’s problem is as follow.

$$max_{A,t}\, \pi_m = w\,(\gamma_0 - p\gamma_1)\left(k_r\sqrt{a} + k_m\sqrt{A}\right) - ta - A \tag{12}$$

s.t. $0 \le t \le 1, A \ge 0$,

where, $a = \left(\frac{(p-w)(\gamma_0 - p\gamma_1)k_r}{2(1-t)}\right)^2$

Substituting the value of a in to (12) , we get

$$max_{A,t}\, \pi_m = w\,(\gamma_0 - p\gamma_1)\left(k_r\left(\frac{(p-w)(\gamma_0-p\gamma_1)k_r}{2(1-t)}\right) + k_m\sqrt{A}\right) - t\left(\frac{(p-w)(\gamma_0-p\gamma_1)k_r}{2(1-t)}\right)^2 - A \tag{13}$$

s.t. $0 \le t \le 1, A \ge 0.$
Solving equation (13) for t and A and substituting then in eq. (9) we get unique equilibrium point, (a^*, t^*, A^*) of the two – stage game as follows.

$$a^* = \left(\frac{(p+w)(\gamma_0-p\gamma_1)k_r}{4}\right)^2 \tag{14}$$

$$t^* = \begin{cases} \frac{3w-p}{p+w}, & 3w > p \\ 0, & otherwise \end{cases} \tag{15}$$

$$A^* = \left(\frac{w(\gamma_0-p\gamma_1)k_m}{2}\right)^2 \tag{16}$$

We have following important proposition regarding manufacturer – retailer relation.

Proposition1. If $3w > p$, then (i) the manufacturer offers positive advertising allowance to the retailer, otherwise he will offer nothing, and (ii) the manufacturer's advertising allowance for the retailer is positively and negatively correlated to changes in the manufacturer's marginal profit and retailer's marginal profit, respectively.

III. Nash Equilibrium

In above section, we deal with two – stage noncooperative game structure where manufacturer as a leader holds extreme power and has complete control over the behavior of the retailer. It is the relationship of employer and employee.

Let $max_{A,t}\, \pi_m = w\,(\gamma_0 - p\gamma_1)(k_r\sqrt{a} + k_m\sqrt{A}) - ta - A$ and

$max_{A,t}\, \pi_r = p\,(\gamma_0 - p\gamma_1)(k_r\sqrt{a} + k_m\sqrt{A}) - (1-t)a$

Solving by taking first derivative of π_m with respect to A and π_r with respect to a equal to zero and substituting t = 0 we get,

$$A^{**} = \left(\frac{w(\gamma_0 - p\gamma_1)k_m}{2}\right)^2$$

$$a^{**} = \left(\frac{(p-w)\,(\gamma_0 - p\gamma_1)k_r}{2}\right)^2$$

$t^{**} = 0$

In recent market studies we found that now retailers have increased their power relative to manufacturers. In this section we assume a symmetric relationship between the manufacturer and the retailer. It is assumed that the manufacturer and the retailer simultaneously and noncooperatively maximize their profits with respect to any possible strategies set by other member.

The following two propositions give the detailed comparisons among two different noncooperative game structures.

Proposition 2. (i) The manufacturer always prefers the leader-follower structure rather than the simultaneous move structure. (ii) If $p \ge 3w$, the retailer prefers the simultaneous move game structure; otherwise he prefers the leader – follower game structure.

Proposition 3. (i) The manufacturer's brand name investment is same at Nash and Stackelberg. (ii) If $p \ge 3w$, then the retailer's local advertising expenditure is higher at Nash than at Stackelberg; otherwise, it is lower at Nash than at Stackelberg. (iii) The manufacturer's advertising allowance for the retailer is zero.

IV. An Efficiency Co – Op Advertising Model

We discussed two noncooperative game structures above for a sequential move and a simultaneous move. The manufacturer who can promote its brand by national advertisement may not know about local market and retailer's advertising behavior .So, the retailer knows how much if any, of the manufacturer's money is spent on local advertising. But many retailers use the manufacturer's money and co operative advertising programs for their own purposes and reduce their dependence on the manufacturers. Co – op advertising should not be used as a offer from a manufacturer to pay part of a retailer's local advertising costs on the product, but should be used as a tool to enhance manufacturer's brand name and retailer's store reputation.

Here, we will consider the symmetric relationship between the manufacturer and retailer and discuss the efficiency of manufacturer and retailer transactions in a vertical co – op advertising agreements.

Pareto efficient scheme (a_0, t_0, A_0) is that if we cannot find any other scheme (a,t,A) such that neither the manufacturer's nor the retailer's profit is less at (a,t,A) but atleast one of them has profit higher at (a,t,A) that at (a_0, t_0, A_0) . So, (a_0, t_0, A_0) is Pareto efficient iff $\pi_m(a,t,A) \geq \pi_m(a_0, t_0, A_0)$ and $\pi_r(a,t,A) \geq \pi_r(a_0, t_0, A_0)$ for some (a,t,A) implies that $\pi_m(a,t,A) = \pi_m(a_0, t_0, A_0)$ and $\pi_r(a,t,A) = \pi_r(a_0, t_0, A_0)$.

Here, π_m and π_r are quasi – concave. So, the set of Pareto efficient schemes consists of those points where the manufacturer's and retailer's iso – profit surfaces are tangent to each other. i.e.

$$\nabla \pi_m (a,t,A) + \mu \nabla \pi_r(a,t,A) = 0 \tag{17}$$

For some $\mu \geq 0$, where $\nabla \pi_m = \left(\frac{\partial \pi_m}{\partial a}, \frac{\partial \pi_m}{\partial t}, \frac{\partial \pi_m}{\partial A}\right)$ stands for the gradient of π_m .

Proposition 4. The collection of Pareto efficient schemes is described by the set Y = $\{(\bar{a}^*, t, \bar{A}^*): 0 \leq t \leq 1\}$ where $\bar{a}^* = \left(\frac{p\,(\gamma_0 - p\gamma_1)k_r}{2}\right)^2$ and $\bar{A}^* = \left(\frac{p\,(\gamma_0 - p\gamma_1)k_m}{2}\right)^2$. (18)

This proposition shows that Pareto efficient schemes are associated with a single local advertising expenditure $\bar{a}^*$ and a single manufacturer's brand name investment $\bar{A}^*$ and with manufacturer's share of local advertising expenditure between o and 1.

Proposition 5. An advertising scheme is Pareto efficient iff it is an optimal solution of the joint system profit maximization problem.

Appendix A. Proof of results

Proof of Proposition 2

(i) Since the retailer's response to every strategy of the manufacturer is unique in the two-stage game, the leader's payoff will not be less than that at Nash equilibrium. Therefore, ${\pi_m}^{**} \leq {\pi_m}^{*}$.

(ii) $${\pi_r}^{**} - {\pi_r}^{*} = \frac{{k_r}^2}{8}(\gamma_0 - p\gamma_1)^2(p - 3w)(p - w) \geq 0, \text{ if } p \geq 3w$$
$$< 0, \text{ otherwise}$$

Proof of Proposition 3

(i) $$A^{**} - A^* = \left(\frac{w(\gamma_0 - p\gamma_1)k_m}{2}\right)^2 - \left(\frac{w(\gamma_0 - p\gamma_1)k_m}{2}\right)^2 = 0$$

(ii) $$a^{**} - a^* = \left(\frac{(p-w)(\gamma_0 - p\gamma_1)k_r}{2}\right)^2 - \left(\frac{(p+w)(\gamma_0 - p\gamma_1)k_r}{4}\right)^2$$
$$= \left((\gamma_0 - p\gamma_1)k_r\right)^2 \left\{\frac{(p-3w)(3p-w)}{16}\right\} \geq 0, \text{ if } p \geq 3w$$
$$< 0, \text{ otherwise}$$

(iii) $t^{**} = 0$

Proof of Proposition 4

Since $\nabla \pi_m (a,t,A) = \left(\frac{w(\gamma_0 - p\gamma_1)k_r}{2\sqrt{a}} - t\,, -a\,, \frac{w(\gamma_0 - p\gamma_1)k_m}{2\sqrt{A}} - 1\right)$

$\nabla \pi_r (a,t,A) = \left(\frac{(p-w)(\gamma_0 - p\gamma_1)k_r}{2\sqrt{a}} - (1-t)\,, a\,, \frac{(p-w)(\gamma_0 - p\gamma_1)k_m}{2\sqrt{A}}\right)$

Utilizing eq. (17) we can get $\mu = 1$, $\bar{a}^*$ and $\bar{A}^*$ in eq. (18) and with t between o and 1.

Proof of Proposition 5

Let the joint system maximization problem be as follows:

$$\bar{\pi}^* = \max_{a,A,t} \boldsymbol{\pi} = \pi_{m+r} = p\,(\gamma_0 - p\gamma_1)\left(k_r\sqrt{a} + k_m\sqrt{A}\right) - a - A \tag{19}$$

s.t. $0 \leq t \leq 1, A \geq 0\,, a \geq 0$

It does not contain the variable t, any value of t between 0 and 1 can be component for any optimal solution of (19).

Taking the first derivatives of π with respect to a ad A, and setting them to zero, we have

$$\bar{a}^* = \left(\frac{p\,(\gamma_0 - p\gamma_1)k_r}{2}\right)^2$$

and

$$\bar{A}^* = \left(\frac{p\,(\gamma_0 - p\gamma_1)k_m}{2}\right)^2$$

Therefore, $(\bar{a}^*, t, \bar{A}^*)$ for any t in [0, 1] is an optimal solution of (19).

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