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Unified Contact Riemannian Manifold Admitting Semi-Symmetric Metric S-Connection

Shalini Singh[1]
Department of Applied Mathematics JSS Academy of Technical Education Noida-201301, India

Abstract: *The present paper deals with the different geometrical properties of an unified contact Riemannian manifold [7] equipped with semi-symmetric metric S-connection. Also the form of curvature tensor $\tilde{R}$ of the manifold relative to this connection has been derived. It has been shown that if an unified contact Riemannian manifold admits a semi-symmetric metric S-connection whose curvature tensor is locally isometric to the unit sphere $S^{(n)}(1)$, then the Conformal and Con-harmonic curvature tensors with respect to the Riemannian connection are identical iff $n+\frac{4a^2}{c}=0$. Also it has been shown that if an unified contact Riemannian manifold admits a semi-symmetric metric S-connection whose curvature tensor is locally isometric to the unit sphere $S^{(n)}(1)$, then the Con-circular curvature tensor coincides with the Riemannian connection if $\frac{4a^2}{c}+n=0$. Some other useful results and theorem have been obtained, which are of great geometrical importance.*

Keywords: *C^∞-manifold, Unified contact Riemannian manifold, Riemannian connexion, Semi-symmetric metric S-connexion.*

I. Introduction

We consider a differentiable manifold M_n of differentiability class C^∞. Let there exist in M_n a vector valued C^∞ - linear function Φ, a C^∞ - vector field η and a C^∞ -one form ξ such that

(1.1) $$\Phi^2(X)=a^2X+c\xi(X)\eta$$

(1.2) $$\bar{\eta}=0$$

(1.3) $$\xi(\eta)=-\frac{a^2}{c}$$

(1.4) $$\xi(\bar{X})=0$$

Where $\Phi(X)=\bar{X}$, a is a nonzero complex number and c is an integer.

Let us agree to say that Φ gives to M_n a differentiable structure define by algebraic equation (1.1). We shall call (Φ,η,a,c,ξ) as an unified contact structure. It may be noted that an unified contact structure (Φ,η,a,c,ξ) gives an almost contact structure [5], almost Para-contact structure [6] or hyperbolic contact structure [1] according as $(a=\pm i,\ c=1),(a=\pm 1,\ c=-1)$ or $(a=\pm 1, c=1)$ respectively. The manifold M_n equipped with an unified contact structure will be called an unified contact structure manifold.

Let us define the metric G in M_n by

(1.5) $$G(\bar{X},\bar{Y})=-a^2G(X,Y)-c\xi(X)\xi(Y)$$

where

(1.6) $$G(X,\eta)\ \underline{\underline{def}}\ \xi(X)$$

Then the unified contact structure M_n will be called an unified contact Riemannian manifold.

Remark 1.1: An unified contact Riemannian manifold gives an almost contact metric manifold, an almost hyperbolic paracontact metric manifold or an almost hyperbolic contact metric manifold according as $(a=\pm i,\ c=1),(a=\pm 1,\ c=-1)$ or $(a=\pm 1, c=1)$

Definition 1.1: A C^∞ -manifold M_n, satisfying

(1.7) $$D_X\eta=\Phi(X)$$

will be denoted by M_n^*

In M_n^*, we can easily shown that

(1.8) $$(D_X\xi)(Y)={}'\Phi(X,Y)=-(D_Y\xi)(X)$$

where

(1.9) $${}'\Phi(X,Y)\ \underline{\underline{def}}\ G(\bar{X},Y)=-G(X,\bar{Y})={}'\Phi(Y,X)$$

Definition 1.2: An affine connection $\tilde{\nabla}$ is said to be metric if

(1.10) $$\tilde{B}_X G=0$$

The metric connection $\tilde{\nabla}$ satisfying

(1.11) $$(\tilde{\nabla}_X\Phi)(Y)=\xi(Y)X-G(X,Y)\eta$$

is called S -connection.

A metric S -connection $\tilde{\nabla}$ is called semi-symmetric metric S -connection [4] If

(1.12) $$\tilde{\nabla}_X Y=D_X Y-\xi(X)\bar{Y}$$

Where D is the Riemannian connection. Also equation (1.12) implies

(1.13) $$S(X,Y)=\xi(Y)\bar{X}-\xi(X)\bar{Y}$$

where S is the torsion tensor of connection $\tilde{\nabla}$.

Replacing Y by η in (1.11), we have

$$(\tilde{\nabla}_X\Phi)(\eta)=\xi(\eta)X-G(X,\eta)\eta$$

Using (1.1), (1.3) and (1.6) in the above equation, we get

(1.14) $$(\tilde{\nabla}_X\Phi)(\eta)=-\frac{\bar{\bar{X}}}{c}$$

From (1.2), we have

$$\Phi\eta=0$$

Differentiating covariantly above equation with respect to X, we get

$$(\tilde{\nabla}_X\Phi)(\eta)+\Phi(\tilde{\nabla}_X\eta)=0$$

Using (1.14) in the above equation, we get

(1.15) $$\tilde{\nabla}_X\eta=\frac{\bar{X}}{c}$$

Now, from (1.6), we have

$$G(Y,\eta)=\xi(Y)$$

Differentiating covariantly above equation with respect to X and using (1.9), (1.11) and (1.15), we get

(1.16) $${}'\Phi(X,Y)=c(\tilde{\nabla}_X\xi)(Y)$$

We know that

$$\Phi Z=\bar{Z}$$

Differentiating covariantly above equation with respect to X and using (1.12), we get

(1.17) $$(D_X\Phi)(Z)=(\tilde{\nabla}_X\Phi)(Z)$$

Let $\tilde{R}$ and K be the curvature tensors with respect to the connection $\tilde{\nabla}$ and D respectively then

(1.18) $\tilde{R}(X,Y,Z) \underline{\underline{def}} \tilde{\nabla}_X \tilde{\nabla}_Y Z - \tilde{\nabla}_Y \tilde{\nabla}_X Z - \tilde{\nabla}_{[X,Y]} Z$

and

(1.19) $K(X,Y,Z) \underline{\underline{def}} D_X D_Y Z - D_Y D_X Z - D_{[X,Y]} Z$

Using (1.12) and (1.19) in (1.18), we get

$$\tilde{R}(X,Y,Z) = K(X,Y,Z) + \xi(Y)(D_X\Phi)(Z) + \left\{(\tilde{\nabla}_Y \xi)(X) - (\tilde{\nabla}_X \xi)(Y)\right\}\bar{Z}$$
$$+\left\{\xi(\tilde{\nabla}_Y X) - \xi(\tilde{\nabla}_X Y)\right\}\bar{Z} + \xi(D_X Y - D_Y X)\bar{Z} - \xi(X)(D_Y\Phi)(Z)$$

Using (1.6), (1.8), (1.12), (1.13), (1.17) and (1.18) in the above equation, we get

(1.20) $$\tilde{R}(X,Y,Z) = K(X,Y,Z) + \xi(Y)\xi(Z)X - \xi(Y)G(X,Z)\eta$$
$$-\xi(X)\xi(Z)Y + \xi(X)G(Y,Z)\eta - \frac{2`\Phi(X,Y)\bar{Z}}{c}$$

Let us consider that $\tilde{R}(X,Y,Z) = 0$ then above equation implies

(1.21) $$K(X,Y,Z) = -\xi(Y)\xi(Z)X + \xi(Y)G(X,Z)\eta + \xi(X)\xi(Z)Y$$
$$-\xi(X)G(Y,Z)\eta + \frac{2`\Phi(X,Y)\bar{Z}}{c}$$

Contracting X in the above equation, we get

(1.22) $$Ric(Y,Z) = -(n-4)\xi(Y)\xi(Z) + \frac{3a^2}{c}G(Y,Z)$$

Contracting with respect to Z in the above equation, we get

(1.23) $$rY = -(n-4)\xi(Y)\eta + \frac{3a^2}{c}Y$$

Contracting Y in the above equation, we get

(1.24) $$\tilde{R} = \frac{4a^2}{c}(n-1)$$

Where Ric and $\tilde{R}$ are Ricci tensor and scalar curvature respectively.

The Conformal curvature tensor V, Conharmonic curvature tensor L, Projective curvature tensor W and Concircular curvature tensor C in a Riemannian manifold are given by [2], [3].

(1.25) $$V(X,Y,Z) = K(X,Y,Z) - \frac{1}{(n-2)}\left[Ric(Y,Z)X - Ric(X,Z)Y\right.$$
$$\left.+G(Y,Z)r(X) - G(X,Z)r(Y)\right] + \frac{\tilde{R}}{(n-1)(n-2)}\left[G(Y,Z)X - G(X,Z)Y\right]$$

(1.26) $$L(X,Y,Z) = K(X,Y,Z) - \frac{1}{(n-2)}\left[Ric(Y,Z)X - Ric(X,Z)Y\right.$$
$$\left.+G(Y,Z)r(X) - G(X,Z)r(Y)\right]$$

(1.27) $$W(X,Y,Z) = K(X,Y,Z) - \frac{1}{(n-1)}\left[Ric(Y,Z)X - Ric(X,Z)Y\right]$$

(1.28) $$C(X,Y,Z) = K(X,Y,Z) - \frac{\tilde{R}}{n(n-2)}\left[G(Y,Z)X - G(X,Z)Y\right]$$

where

(1.29) $'V(X,Y,Z,T) \underline{\underline{def}} G(V(X,Y,Z),T)$

(1.30) $$`L(X,Y,Z,T) \underline{\underline{def}}\; G(L(X,Y,Z),T)$$

(1.31) $$`W(X,Y,Z,T) \underline{\underline{def}}\; G(W(X,Y,Z),T)$$

(1.32) $$`C(X,Y,Z,T) \underline{\underline{def}}\; G(C(X,Y,Z),T)$$

II. Curvature Tensors

Theorem 2.1: If an unified contact Riemannian manifold admits a semi-symmetric metric S -connection whose curvature tensor is locally isometric to the unit sphere $S^{(n)}(1)$, then the Conformal and Con-harmonic curvature tensors with respect to the Riemannian connection are identical iff $n+\frac{4a^2}{c}=0$

Proof: If the curvature tensor with respect to the semi-symmetric non metric S - connection is locally isometric to the unit sphere $S^{(n)}(1)$, then

(2.1) $$\tilde{R}(X,Y,Z)=G(Y,Z)X-G(X,Z)Y$$

Using (2.1) in (1.20), we get

$$G(Y,Z)X-G(X,Z)Y=K(X,Y,Z)+\xi(Y)\xi(Z)X-\xi(Y)G(X,Z)\eta$$
$$-\xi(X)\xi(Z)Y+\xi(X)G(Y,Z)\eta-\frac{2}{c}`\Phi(X,Y)\bar{Z}$$

Contracting above with respect to X , we get

(2.2) $$Ric(Y,Z)=\left(\frac{3a^2}{c}+n-1\right)G(Y,Z)-(n-4)\xi(Y)\xi(Z)$$

Contracting above equation with respect to Z , we get

$$rY=\left(\frac{3a^2}{c}+n-1\right)Y-(n-4)\xi(Y)\eta$$

Contracting above equation with respect to Y , we get

(2.3) $$\tilde{R}=(n-1)\left(\frac{4a^2}{c}+n\right)$$

Where Ric and $\tilde{R}$ are Ricci tensor and scalar curvature of the manifold respectively.
From (2.3), (1.25) and (1.26), we obtain the necessary part of the theorem. Converse part is obvious from (1.25) and (1.26).

Theorem 2.2: If an unified contact Riemannian manifold M_n admits a semi-symmetric metric S -connection whose curvature tensor is locally isometric to the unit sphere $S^{(n)}(1)$, then the Con-circular curvature tensor coincides with the Riemannian connection if $\frac{4a^2}{c}+n=0$

Proof: Using (2.3) in (1.28), we get

(2.4) $$C(X,Y,Z)=K(X,Y,Z)-\left(\frac{\frac{4a^2}{c}+n}{n}\right)\left[G(Y,Z)X-G(X,Z)Y\right]$$

which is the required proves of the theorem.
Now, let us consider that the curvature tensor of the semi-symmetric metric S -connection has the form

(2.5) $$\tilde{R}(X,Y,Z)=`\Phi(X,Z)\bar{Y}-`\Phi(Y,Z)\bar{X}$$

Using above equation in (1.20), we get

(2.6) $$K(X,Y,Z)=-\xi(Y)\xi(Z)X+\xi(X)\xi(Z)Y-\xi(X)G(Y,Z)\eta$$
$$+\xi(Y)G(X,Z)\eta+`\Phi(X,Z)\bar{Y}-`\Phi(Y,Z)\bar{X}-\frac{2}{c}`\Phi(X,Y)\bar{Z}$$

Contracting X in the above equation and using (1.1), (1.3) and (1.6), we get

(2.7) $$Ric(Y,Z)=(c-n)\xi(Y)\xi(Z)+\frac{a^2(c-1)}{c}G(Y,Z)$$

Contracting above equation with respect to Z, we get

(2.8) $$rY=(c-n)\xi(Y)\eta+\frac{a^2(c-1)}{c}Y$$

Contracting above equation with respect to Y, we get

(2.9) $$\tilde{R}=a^2(n-1)$$

Using (2.6), (2.7), (2.8) and (2.9) in (1.25), we get

(2.10) $$V(X,Y,Z)=`\Phi(X,Z)\bar{Y}-`\Phi(Y,Z)\bar{X}-\frac{2}{c}`\Phi(X,Y)\bar{Z}$$
$$+a^2\left(\frac{\frac{2}{c}-1}{n-2}\right)\left[G(Y,Z)X-G(X,Z)Y\right]$$
$$+\left(\frac{c-2}{n-2}\right)\left[\xi(X)\xi(Z)Y-\xi(Y)\xi(Z)X+\xi(Y)G(X,Z)\eta-\xi(X)G(Y,Z)\eta\right]$$

Operating G both sides of above equation and using (1.6), (1.9) and (1.29), we get

(2.11) $$V(X,Y,Z,T)=`\Phi(X,Z)`\Phi(Y,T)-`\Phi(Y,Z)`\Phi(X,T)-\frac{2}{c}`\Phi(X,Y)`\Phi(Z,T)$$
$$+a^2\left(\frac{\frac{2}{c}-1}{n-2}\right)\left[G(Y,Z)G(X,T)-G(X,Z)G(Y,T)\right]$$
$$+\left(\frac{c-2}{n-2}\right)\left[\xi(X)\xi(Z)G(Y,T)-\xi(Y)\xi(Z)G(X,T)\right.$$
$$\left.+\xi(Y)\xi(T)G(X,Z)-\xi(X)\xi(T)G(Y,Z)\right]$$

Now, using (2.6), (2.7) and (2.8) in (1.26), we get

(2.12) $$L(X,Y,Z)=`\Phi(X,Z)\bar{Y}-`\Phi(Y,Z)\bar{X}-\frac{2}{c}`\Phi(X,Y)\bar{Z}$$
$$-\frac{2a^2}{c}\left(\frac{n-1}{n-2}\right)\left[G(Y,Z)X-G(X,Z)Y\right]$$
$$\left(\frac{c-2}{n-2}\right)\left[\xi(Y)G(X,Z)\eta-\xi(X)G(Y,Z)\eta+\xi(X)\xi(Z)Y-\xi(Y)\xi(Z)X\right]$$

Operating G on both sides of the above equation and using (1.6), (1.9) and (1.30), we get

(2.13) $$`L(X,Y,Z,T)=`\Phi(X,Z)`\Phi(Y,T)-`\Phi(Y,Z)`\Phi(X,T)-\frac{2}{c}`\Phi(X,Y)`\Phi(Z,T)$$
$$-\frac{2a^2}{c}\left(\frac{n-1}{n-2}\right)\left[G(Y,Z)G(X,T)-G(X,Z)G(Y,T)\right]$$

$$+\left(\frac{c-2}{n-2}\right)\left[\xi(Y)\xi(T)G(X,Z)-\xi(X)\xi(T)G(Y,Z)+\xi(X)\xi(Z)G(Y,T)-\xi(Y)\xi(Z)G(X,T)\right]$$

Using (2.6) and (2.7) in (1.27), we get

(2.14)
$$W(X,Y,Z)={}'\Phi(X,Z)\bar{Y}-{}'\Phi(Y,Z)\bar{X}-\xi(X)G(Y,Z)\eta+\xi(Y)G(X,Z)\eta+\xi(X)\xi(Z)Y-\xi(Y)\xi(Z)X-\frac{2}{c}{}'\Phi(X,Y)\bar{Z}+\left(\frac{c-n}{n-1}\right)\left[\xi(X)\xi(Z)Y-\xi(Y)\xi(Z)X\right]+\frac{a^2}{c}\left(\frac{c-1}{n-1}\right)\left[G(X,Z)Y-G(Y,Z)X\right]$$

Now operating G on both sides of the above equation and using (1.6), (1.9) and (1.31), we get

(2.15)
$${}'W(X,Y,Z,T)={}'\Phi(X,Z){}'\Phi(Y,T)-{}'\Phi(Y,Z){}'\Phi(X,T)-\frac{2}{c}{}'\Phi(X,Y){}'\Phi(Z,T)-\left(\frac{c-1}{n-1}\right)\left[\xi(Y)\xi(Z)G(X,T)-\xi(X)\xi(Z)G(Y,T)\right]+\left[\xi(Y)\xi(T)G(X,Z)-\xi(X)\xi(T)G(Y,Z)\right]+\frac{a^2}{c}\left(\frac{c-1}{n-1}\right)\left[G(Y,Z)G(X,T)-G(X,Z)G(Y,T)\right]$$

Using (2.6) and (2.9) in (1.28), we get

(2.16)
$$C(X,Y,Z)={}'\Phi(X,Z)\bar{Y}-{}'\Phi(Y,Z)\bar{X}-\xi(Y)\xi(Z)X+\xi(Y)G(X,Z)\eta+\xi(X)\xi(Z)Y-G(Y,Z)\xi(X)\eta-\frac{2}{c}{}'\Phi(X,Y)\bar{Z}-\frac{a^2}{n}\left[G(Y,Z)X-G(X,Z)Y\right]$$

Operating G on both sides of the above equation and using (1.6), (1.9) and (1.32), we get

(2.17)
$${}'C(X,Y,Z,T)={}'\Phi(X,Z){}'\Phi(Y,T)-{}'\Phi(Y,Z){}'\Phi(X,T)-\frac{2}{c}{}'\Phi(X,Y){}'\Phi(Z,T)-\xi(Y)\xi(Z)G(X,T)+\xi(Y)\xi(T)G(X,Z)+\xi(X)\xi(Z)G(Y,T)-\xi(T)\xi(X)G(Y,Z)-\frac{a^2}{n}\left[G(Y,Z)G(X,T)-G(X,Z)G(Y,T)\right]$$

Theorem 2.3: On a manifold M_n , we have

(2.18a) $V(\eta,Y,Z,T)=0$

(2.18b) $V(X,Y,Z,\eta)=0$

(2.18c) $V(\eta,Y,Z,\eta)=0$

(2.18d) $V(X,Y,\eta,\eta)=0$

(2.18e) $V(\bar{X},\bar{Y},Z,\eta)=0$

(2.18f) $V(\eta,Y,\bar{Z},\bar{T})=0$

(2.18g) $V(X,Y,\eta)=0$

(2.18h) $V(\eta,Y,\eta)=0$

Proof: Replacing X by η in (2.11) and using (1.2), (1.3), (1.6) and (1.9), we get (2.18a).
Replacing T by η in (2.11) and using (1.2), (1.3), (1.6) and (1.9), we get (2.18b).

Replacing T by η in (2.18a), we get (2.18c).
Replacing Z by η in (2.18b), we get (2.18d).
Replacing X by $\bar{X}$ and Y by $\bar{Y}$ in (2.18b), we get (2.18e).
Replacing Z by $\bar{Z}$ and T by $\bar{T}$ in (2.18a), we get (2.18f).
Replacing Z by η in (2.10) and using (1.3), (1.6) and (1.9), we get (2.18g).
Replacing X by η in (2.18g), we get (2.18h).

Theorem 2.4: On a manifold M_n, we have

(2.19a) $${}^{\backprime}L(X,Y,Z,\eta)=\frac{a^2}{c}\left(\frac{c-2n}{n-2}\right)\left[G(Y,Z)\xi(X)-G(X,Z)\xi(Y)\right]$$

(2.19b) $${}^{\backprime}L(\eta,Y,Z,T)=\frac{a^2}{c}\left(\frac{c-2n}{n-2}\right)\left[G(Y,Z)\xi(T)-G(Y,T)\xi(Z)\right]$$

(2.19c) $${}^{\backprime}L(\eta,Y,Z,\eta)=\frac{a^2}{c^2}\left(\frac{c-2n}{n-2}\right)G(\bar{Y},\bar{Z})$$

(2.19d) $${}^{\backprime}L(X,Y,\eta,\eta)=0$$

(2.19e) $${}^{\backprime}L(\bar{X},\bar{Y},Z,\eta)=0$$

(2.19f) $${}^{\backprime}L(\eta,Y,\bar{Z},\bar{T})=0$$

Proof: Replacing T by η in (2.13) and using (1.2), (1.3), (1.6) and (1.9), we get (2.19a).
Replacing X by η in (2.13) and using (1.2), (1.3), (1.6) and (1.9), we get (2.19b).
Replacing T by η in (2.19b) and using (1.3) and (1.6), we get (2.19c).
Replacing Z by η in (2.19a) and using (1.6), we get (2.19d).
Replacing X by $\bar{X}$ and Y by $\bar{Y}$ in (2.19a) and using (1.3), we get (2.19e).
Replacing Z by $\bar{Z}$ and T by $\bar{T}$ in (2.19b) and using (1.3), we get (2.19f).

Theorem 2.5: On a manifold M_n, we have

(2.20a) $${}^{\backprime}W(\eta,Y,Z,T)=\left(\frac{n-c}{n-1}\right)\left[\frac{a^2}{c}G(Y,Z)\xi(T)+\xi(Y)\xi(Z)\xi(T)\right]$$

(2.20b) $${}^{\backprime}W(\eta,Y,\bar{Z},\bar{T})=0$$

(2.20c) $${}^{\backprime}W(X,Y,Z,\eta)=\frac{a^2}{c}\left(\frac{n-c}{n-1}\right)\left[G(Y,Z)\xi(X)-G(X,Z)\xi(Y)\right]$$

(2.20d) $${}^{\backprime}W(\eta,Y,Z,\eta)=\frac{a^2}{c}\left(\frac{n-c}{n-1}\right)\left[-\frac{a^2}{c}G(Y,Z)-\xi(Y)\xi(Z)\right]$$

(2.20e) $${}^{\backprime}W(\bar{X},\bar{Y},Z,\eta)=0$$

(2.20f) $${}^{\backprime}W(X,Y,\eta,\eta)=0$$

Proof: Replacing X by η in (2.15) and using (1.2), (1.3), (1.6) and (1.9), we get (2.20a).
Replacing Z by $\bar{Z}$ and T by $\bar{T}$ in (2.20a) and using (1.3), we get (2.20b).
Replacing T by η in (2.15) and using (1.2), (1.3), (1.6) and (1.9), we get (2.20c).
Replacing X by η in (2.20c) and using (1.3) and (1.6) we get (2.20d).
Replacing X by $\bar{X}$ and Y by $\bar{Y}$ in (2.19c) and using (1.3), we get (2.20e).
Replacing Z by η in (2.20c) and using (1.6), we get (2.20f).

Theorem 2.6: On a manifold M_n, we have

(2.21a) $$\breve{C}(\eta, Y, Z, T) = \frac{a^2}{c} G(Y, Z)\left[\xi(T) - \xi(Z)\right]$$

(2.21b) $$\breve{C}(X, Y, Z, \eta) = a^2\left(\frac{1}{n} - \frac{1}{c}\right)\left[G(X, Z)\xi(Y) - G(Y, Z)\xi(X)\right]$$

(2.21c) $$\breve{C}(\eta, Y, Z, \eta) = -\frac{a^4}{c^2} G(Y, Z) - \frac{a^2}{c}\xi(Z) G(Y, Z)$$

(2.21d) $$\breve{C}(X, Y, \eta, \eta) = 0$$

(2.21e) $$\breve{C}(\bar{X}, \bar{Y}, Z, \eta) = 0$$

(2.21f) $$\breve{C}(\eta, Y, \bar{Z}, \bar{T}) = 0$$

Proof: Replacing X by η in (2.17) and using (1.2), (1.3), (1.6) and (1.9), we get (2.21a).
Replacing T by η in (2.17) and using (1.2), (1.3), (1.6) and (1.9), we get (2.21b).
Replacing T by η in (2.21a) and using (1.3) and (1.6), we get (2.21c).
Replacing Z by η in (2.21b) and using (1.6), we get (2.21d).
Replacing X by $\bar{X}$ and Y by $\bar{Y}$ in (2.21b) and using (1.3), we get (2.21e).
Replacing Z by $\bar{Z}$ and T by $\bar{T}$ in (2.21a) and using (1.3), we get (2.21f).

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