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Determination of Energy Involved In a Stepwise Size Reduction of Maize, Using a Numerical Approach.

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Abstract: *This work described the determination of energy involved in a stepwise size reduction of maize, using a numerical approach. It adopts an implicit one-step third-derivative method of the form;*

$$E_{n+1} = E_n + \frac{h}{2}\left(E_{n+1}^{(1)} + E_n^{(1)}\right) - \frac{h^2}{10}\left(E_{n+1}^{(11)} - E_n^{(11)}\right) + \frac{h^3}{120}\left(E_{n+1}^{(111)} + E_n^{(111)}\right).$$

The size to which maize grain is to be reduced relative to its initial size affects the energy required of the milling machine. The smaller the size of the milled product the higher the energy required of the machine. The energy required is evaluated by the size reduction formula:

$$\frac{dE}{dL} = GL^2 .$$

Comparison of the numerical result with the exact solution showed that the third- derivative method is accurate and thus recommended for solving engineering problems on rate of change.

Keywords: *Energy, Maize, Size reduction, One-step method, Milling.*

I. Introduction

Maize is an economic crop. Primary processing methods of this food in dry state requires size reduction, an energy driven unit operation. Maize is an important agricultural crop that plays significant roles in the diet of the people all over the world particularly in the developing nations. FAO (2012) respectively ranked maize as third, important cereal in the world. According to Kent and Evers (1994), maize grain contains about 10.8% moisture, 10.0% protein, 4.3% fat, 1.7% fiber, 1.5% ash, and 71.7% starch

Size reduction is the unit operation in which the average size of solid pieces of food is reduced by the application of grinding, compression or impact forces (Famurewa, 2007). Size reduction into flour is an important food processing operation because a large proportion of food materials are reduced into flour before conversion into finished products. One measure of the efficiency of the milling operation is based on energy required to create new surfaces. Size reduction is one of the least energy-efficient of all the unit operation and the cost of power is a major expense in crushing and grinding, so the factors that control this cost are important (McCabe et al., 2005).

During milling, Kinetic Energy is dissipated on the material in excess of its internal strength which causes rupture along the line of cleavages resulting into smaller particles (Famurewa, 2007). Some benefits of size reduction in food processing include; increase in the surface-area-to-volume ratio of the food which increases the rate of drying, heating or cooling, improvement in the eating quality or suitability of foods for further processing and increase in the range of products available.

The relationship between the comminution energy and the product size obtained for a given feed size has been researched extensively over the last century. Theoretical and empirical energy-size reduction equations were proposed by Rittinger (1867), Kick (1885) and Bond (1952), known as the three theories of comminution; and their general formulation by Walker et al. (1937). Finally, Hukki (1962) proposed the revised form of the general form of comminution and suggested that the energy-size relation is a combined form of these three laws. Walker et al. (1937) proposed the following equation, for a general form of comminution.

$$dE = -C\frac{dx}{x^n} \quad [1]$$

Where E is the net specific energy; x is the characteristic dimension of the product; n is the exponent; and C is a constant related to the material. Equation [1] states that the required energy for a differential decrease in size is proportional to the size change (dx) and inversely proportional to the size to some power n. If the exponent n in Equation [1] is replaced by the values of 2, 1 and 1.5 and then integrated, the well-known equations of Rittinger, Kick and Bond, are obtained respectively. Rittinger (1867) stated that the energy required for size reduction is proportional to the new surface area generated. Since the specific surface area is inversely proportional to the particle size, Rittinger's hypothesis can be written in the following form:

$$E = K_1\left(\frac{1}{x_p} - \frac{1}{x_f}\right) \quad [2]$$

where E is the net specific energy; x_f and x_p are the feed and product size indices, respectively; and K_1 is a constant. Kick (1885) proposed the theory that the equivalent relative reductions in sizes require equal energy. Kick's equation is as follows:

$$E = K_2 \ln\left(\frac{x_f}{x_p}\right) \quad [3]$$

where E is the net specific energy; x_f and x_p are the feed and product size indices, respectively, and K_2 is a constant. Bond (1952) proposed the 'Third Law' of grinding. The Third Law states that the net energy required in comminution is proportional to the total length of the new cracksformed. The resulting equation is:

According to Fellows (2010), combined impact and shearing forces are necessary for fibrous foods like maize, therefore Rittinger's equation was adopted for this study.

A differential equation of the form: $y^1 = f(x, y)$, $y(x_0) = y_0$, where f is assumed to be Lipschiz continuous, can be solved using various approximation methods, among which is the Implicit one- step third derivative numerical method of order 6, which is an improvement on the conventional implicit family of linear multistep methods. This method includes more derivative properties of the differential equation into the existing method which makes it more accurate and of more practical values.

II. Methodology

Maize grains were milled from 6 mm to 0.0012 mm using a 10 hp (7.5 KW) motorised plate milling machine. The energy required in a stepwise size reduction is evaluated by the size reduction formula:

$\frac{dE}{dL} = GL^2$, where

$\frac{dE}{dL}$ is the rate of change of energy with length (Jm^{-1}).

G is the Rittinger's constant (Jm^{-2}).

L is the length of the maize grain (m).

Implicit one- step third derivative numerical method of order 6, of the form;

$$E_{n+1} = E_n + \frac{h}{2}\left(E_{n+1}^{(1)} + E_n^{(1)}\right) - \frac{h^2}{10}\left(E_{n+1}^{(11)} - E_n^{(11)}\right) + \frac{h^3}{120}\left(E_{n+1}^{(111)} + E_n^{(111)}\right) \quad [4]$$

was adopted for the measurement of the energy required for the size reduction.

III. Derivation of the Numerical Method.

Theorem

Setting k=1, l = 3 in the generalized implicit multi-derivative linear multistep method of the form:

$$\sum_{j=0}^{k} \alpha_j E_{n+j} = \sum_{i=1}^{k} h^i \sum_{j=0}^{k} \beta_{ij} E^i{}_{n+j}, \quad \alpha_k = +1 \quad [5]$$

(Famurewa and Olorunsola, 2013)

gives a one-step third derivative method of the form;

$$E_{n+1} = E_n + \frac{h}{2}\left(E_{n+1}^{(1)} + E_n^{(1)}\right) - \frac{h^2}{10}\left(E_{n+1}^{(11)} - E_n^{(11)}\right) + \frac{h^3}{120}\left(E_{n+1}^{(111)} + E_n^{(111)}\right) \quad [6]$$

Proof

Setting k=1, l=3 in (5) gives

$$\alpha_0 E_n + \alpha_1 E_{n+1} = h\left(\beta_{10}E_n^{(1)} + \beta_{11}E_{n+1}^{(1)}\right) + h^2\left(\beta_{20}E_n^{(11)} + \beta_{21}E_{n+1}^{(11)}\right) + h^3\left(\beta_{30}E_n^{(111)} + \beta_{31}E_{n+1}^{(111)}\right)$$

[7]

with local truncation error

$$T_{n+1} = \alpha_0 E_n + \alpha_1 E_{n+1} - h(\beta_{10} E_n^{(1)} + \beta_{11} E_{n+1}^{(1)}) - \mathrm{h}^2(\beta_{20} \mathrm{E}_n^{(11)} + \beta_{21} E_{n+1}^{(11)}) - \mathrm{h}^3(\beta_{30} \mathrm{E}_n^{(111)} + \beta_{31} E_{n+1}^{(111)}) \qquad [8]$$

Adopting Taylor's series expansion of E_{n+1}, $E_{n+1}^{(n)}$, $E_{n+1}^{(11)}$ and $E_{n+1}^{(111)}$ in (8) and combining terms in equal powers of h gives

$$T_{n+1} = C_0 E_n + C_1 h E_n^{(1)} + C_2 h^2 E_n^{(11)} + C_3 h^3 E_n^{(111)} + \ldots + 0(h^8) \qquad [9]$$

where

$$\left.\begin{cases} C_0 = \alpha_0 + \alpha_1 \\ C_1 = \alpha_1 - \beta_{10} - \beta_{11} \\ C_2 = \dfrac{\alpha_1}{2} - \beta_{11} - \beta_{20} - \beta_{21} \\ C_3 = \dfrac{\alpha_1}{3!} - \dfrac{\beta_{11}}{2} - \beta_{21} - \beta_{30} - \beta_{31} \\ C_4 = \dfrac{\alpha_1}{4!} - \dfrac{\beta_{11}}{3!} - \dfrac{\beta_{21}}{2} - \beta_{31} \\ C_5 = \dfrac{\alpha_1}{5!} - \dfrac{\beta_{11}}{4!} - \dfrac{\beta_{21}}{3!} - \dfrac{\beta_{31}}{2} \\ C_6 = \dfrac{\alpha_1}{6!} - \dfrac{\beta_{11}}{5!} - \dfrac{\beta_{21}}{4!} - \dfrac{\beta_{31}}{3!} \\ C_7 = \dfrac{\alpha_1}{7!} - \dfrac{\beta_{11}}{6!} - \dfrac{\beta_{21}}{5!} - \dfrac{\beta_{31}}{4!} \end{cases}\right\} - - - (8) \qquad [10]$$

Imposing accuracy of order 7 on T_{n+1} to have

$C_0 = C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = 0$ and $T_{n+1} = 0(h^7)$

which gives

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & -1 & -1 & -1 \\ 0 & 0 & -\frac{1}{3!} & 0 & -\frac{1}{2} & 0 & -1 \\ 0 & 0 & -\frac{1}{4!} & 0 & -\frac{1}{3!} & 0 & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{5!} & 0 & -\frac{1}{4!} & 0 & -\frac{1}{3!} \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \beta_{10} \\ \beta_{11} \\ \beta_{20} \\ \beta_{21} \\ \beta_{30} \\ \beta_{31} \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -\frac{1}{2} \\ -\frac{1}{3!} \\ -\frac{1}{4!} \\ -\frac{1}{5!} \\ -\frac{1}{6!} \end{pmatrix}$$

Solving gives;

$\alpha_0 = -1, \beta_{10} = 0.5, \beta_{11} = 0.5, \beta_{20} = 0.1, \beta_{21} = -0.1, \beta_{30} = 0.0083$ and $\beta_{31} = -0.0083$

putting these values into equation (7) gives the one – step third derivative method of equation (6).

Implementation: To access the accuracy of the scheme, the method was re-written in FORTRAN programming language and implemented on a digital computer. The program was used to solve the size reduction equation. The results and errors are shown in Table 1.

Problem; $\frac{dE}{dL} = GL^2$, $E(0.006\ \mathrm{mm}) = 50\ \mathrm{J}$ (when the machine is operating empty),

Exact solution; $E = \frac{28}{3} L^3 + 50$

Numerical method;

$$E_{n+1} = E_n + \frac{h}{2}\left(E_{n+1}^{(1)} + E_n^{(1)}\right) - \frac{h^2}{10}\left(E_{n+1}^{(11)} - E_n^{(11)}\right) + \frac{h^3}{120}\left(E_{n+1}^{(111)} + E_n^{(111)}\right)$$

IV. Results And Discussion

Table 1: The Result Obtained for the Solutions.

x-value	exact-solution	computed-solution	error
0.0059	2.249999881443072200	2.249999881443071700	4.440892e-16
0.0058	2.249999767627776300	2.249999767627775400	8.881784e-16
0.0056	2.249999658457344100	2.249999658457343200	8.881784e-16
0.0055	2.249999553835008400	2.249999553835007000	1.332268e-15
0.0054	2.249999453664000000	2.249999453663998700	1.332268e-15
0.0053	2.249999357847552300	2.249999357847550600	1.776357e-15
0.0052	2.249999266288896100	2.249999266288894400	1.776357e-15
0.0050	2.249999178891264300	2.249999178891262100	2.220446e-15
0.0049	2.249999095557888100	2.249999095557885900	2.220446e-15
0.0048	2.249999016192000000	2.249999016191997800	2.220446e-15
0.0047	2.249998940696832200	2.249998940696829500	2.664535e-15
0.0046	2.249998868975616100	2.249998868975613400	2.664535e-15
0.0044	2.249998800931584300	2.249998800931581200	3.108624e-15
0.0043	2.249998736467968100	2.249998736467965000	3.108624e-15
0.0042	2.249998675488000400	2.249998675487996800	3.552714e-15
0.0041	2.249998617894912300	2.249998617894908700	3.552714e-15
0.0040	2.249998563591936000	2.249998563591932400	3.552714e-15
0.0038	2.249998512482304300	2.249998512482300300	3.996803e-15
0.0037	2.249998464469248100	2.249998464469244100	3.996803e-15
0.0036	2.249998419456000300	2.249998419455995900	4.440892e-15
0.0035	2.249998377345792100	2.249998377345787700	4.440892e-15
0.0034	2.249998338041856000	2.249998338041851600	4.440892e-15
0.0032	2.249998301447424100	2.249998301447419200	4.884981e-15
0.0031	2.249998267465728000	2.249998267465723100	4.884981e-15
0.0030	2.249998236000000200	2.249998235999994800	5.329071e-15
0.0029	2.249998206953472300	2.249998206953466500	5.773160e-15
0.0028	2.249998180229376000	2.249998180229370300	5.773160e-15
0.0026	2.249998155730944300	2.249998155730938100	6.217249e-15
0.0025	2.249998133361408300	2.249998133361401600	6.661338e-15
0.0024	2.249998113024000100	2.249998113023993400	6.661338e-15
0.0023	2.249998094621952200	2.249998094621945100	7.105427e-15
0.0022	2.249998078058496300	2.249998078058488700	7.549517e-15
0.0020	2.249998063236864400	2.249998063236856400	7.993606e-15
0.0019	2.249998050060288100	2.249998050060280100	7.993606e-15
0.0018	2.249998038432000000	2.249998038431992000	7.993606e-15
0.0017	2.249998028255232100	2.249998028255223700	8.437695e-15
0.0016	2.249998019433216100	2.249998019433207700	8.437695e-15
0.0014	2.249998011869184000	2.249998011869175600	8.437695e-15
0.0013	2.249998005466368000	2.249998005466359500	8.437695e-15
0.0012	2.249998000128000000	2.249998000127991100	8.881784e-15
0.0011	2.249997995757312200	2.249997995757302900	9.325873e-15
0.0010	2.249997992257536200	2.249997992257526800	9.325873e-15
0.0008	2.249997989531904000	2.249997989531894700	9.325873e-15
0.0007	2.249997987483648200	2.249997987483638400	9.769963e-15
0.0006	2.249997986016000000	2.249997986015990200	9.769963e-15
0.0005	2.249997985032192300	2.249997985032182100	1.021405e-14
0.0004	2.249997984435456300	2.249997984435445600	1.065814e-14
0.0002	2.249997984129024100	2.249997984129013400	1.065814e-14
0.0001	2.249997984016128200	2.249997984016117100	1.110223e-14

It was observed that the computed result is very close to the exact solution as shown in Table 1. Hence, the method is recommended for solving practical problems of Science and Engineering.

V. Conclusion

In this study, one – step third derivative method was used to determine the energy required for the stepwise size reduction of maize grains. The result obtained showed that the numerical method is accurate, effective and efficient, hence, can be adopted in solving practical problems on rate of change.

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