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# An inventory model for variable demand, constant holding cost and without shortages

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## I. Introduction

Deterioration is defined as decay, change, damage, spoilage or obsolescence that results in decreasing usefulness from its original purpose. Some kinds of inventory products (e.g., vegetables, fruit, milk, and others) are subject to deterioration. **Ghare and Schrader (1963)** first established an economic order quantity model having a constant deterioration. Chang gives a Partial Backlogging Inventory Model rate of deterioration and constant rate of demand over a finite planning horizon. **Covert and Philip (1973)** extended Ghare and Schrader's constant deterioration rate to a two parameter Weibull distribution. **Dave and Patel (1981)** discussed an inventory model for deteriorating items with time-proportional demand when shortages were not allowed. The related analysis on inventory systems with deterioration have been performed by **Balkhi and Benkherouf (1996), Wee (1997), Mukhopadhyay et al. (2004**),etc.

In reality, not all kinds of inventory items deteriorated as soon as they received by the retailer. In the fresh product time, the product has no deterioration and keeps their original quality. **Ouyang et al. (2006**) named this phenomenon as "non-instantaneous deterioration", and they established an inventory model for non-instantaneous deteriorating items with permissible delay in payments. In some fashionable products, some customers would like to wait for backlogging during the shortage period. But the willingness is diminishing with the length of the waiting time for the next replenishment. The longer the waiting time is, the smaller the backlogging rate would be. The opportunity cost due to lost sales should be considered. Chang and Dye (1999) developed an inventory model in which the demand rate is a time-continuous function and items deteriorate at a constant rate with partial backlogging rate which is the reciprocal of a linear function of the waiting time**.** **Papachristos and Skouri (2000)** developed an EOQ inventory model with time-dependent partial backlogging. They supposed the rate of backlogged demand increases exponentially with the waiting time for the next replenishment decreases. **Teng et al. (2002)** then extended the backlogged demand to any decreasing function of the waiting time up to the next replenishment. The related analysis on inventory systems with artial backlogging have been performed by **Teng and Yang (2004), Yang (2005), Dye et al. (2006),** San **José et al. (2006), Teng et al. (2007),** etc.

Many articles assume that the demand is constant during the sale period. It needs to be discussed. In real life, the requirements may be stimulated if there is a large pile of goods displayed on shelf.**Gupta and Vrat (1986)** presented an inventory model for stock-dependent consumption rate on initial stock level rather than instantaneous inventory level**. Baker and Urban (1988)** established a deterministic inventory system in which the demand rate depended on the inventory level is described by a polynomial function**. Wu et al. (2006)** presented an inventory model for non-instantaneous deteriorating items with stock-dependent. The related analysis on inventory systems with stock-dependent consumption rate have been performed **by Datta and Paul (2001), Balkhi and Benkherouf (2004), Chang et al. (2007),**etc.

In all of the above mentioned models, the influences of the inflation and time value of money were not discussed**. Buzacott (1975)** first established an EOQ model with inflation subject to different types of pricing policies. **Chung and Lin (2001)** followed the discounted cash flow approach to investigate inventory model with constant demand rate for deteriorating items taking account of time value of money. **Hou (2006)** established an inventory model with stock-dependent consumption rate simultaneously considered the inflation and time value of money when shortages are allowed over a finite planning horizon.

In this article, inventory model for deteriorating items with stock-dependent demand rate, along with the effects of inflation are considered.

The rest of this paper is organized as follows. In Section 2, we described the assumptions and notations used through out this paper. In Section 3, we establish the mathematical model and boundary condition to find the minimum total relevant cost and the optimal order quantity. In Section 6, we use numerical examples to illustrate the results we proposed and we make a sensitivity analysis to study the effects of changes in the system parameters on the inventory model.

## II. Notation and Assumptions

The mathematical model is based on the following notations and assumptions:

**Notations:-**

$C_o$: the ordering cost per order;
$C_P$: the purchase cost per unit;
$\theta$ : the deterioration rate; $0<\theta<1$.
$C_H(t)$: the inventory holding cost per unit per time unit;
$C_B$: the backordered cost per unit short per time unit;
$C_{L:}$ the cost of lost sales per unit;
T: the time at which the inventory level reaches zero .
$I_{max}$: the maximum inventory level during [0, T];
$Q_o$: $I_{max}$ the order quantity during a cycle of length T;
$I_P(t)$: the positive inventory level at time t;
$I_N(t)$: the negative inventory level at time t;
$T_C(T)$: the total cost per time unit.

**Assumptions:-**

1. The demand rate is time dependent that is if 'a' is fix fraction of demand and 'b' is that fraction of demand which is vary with time then demand function is $D(t) = (a + bt)\sqrt{I}$, where $a > 0$, $b > 0$.
2. Holding cost is $h(t) = h$ is constant
3. Shortages are not allowed.
4. The lead time is zero.
5. The replenishment rate is infinite.
6. The planning horizon is finite.
7. The deterioration rate is $\theta$ is constant.
8. Inflation rate r

## III. Mathematical Formulation of the Model

The rate of change of inventory during positive stock period [0, T] is governed by the differential equations.

$$\frac{dI_P}{dt} + \theta I_P = -(a+bt)\sqrt{I_P} \ , \quad 0 \le t \le T \qquad \text{....(1)}$$

With boundary conditions $I_P(t) = I_N(t) = 0$ at $t = T$ and $I_P(t) = I_{max}$ at $t = 0$.

## IV. Analytical Solution and Graphical Representation of Inventory System

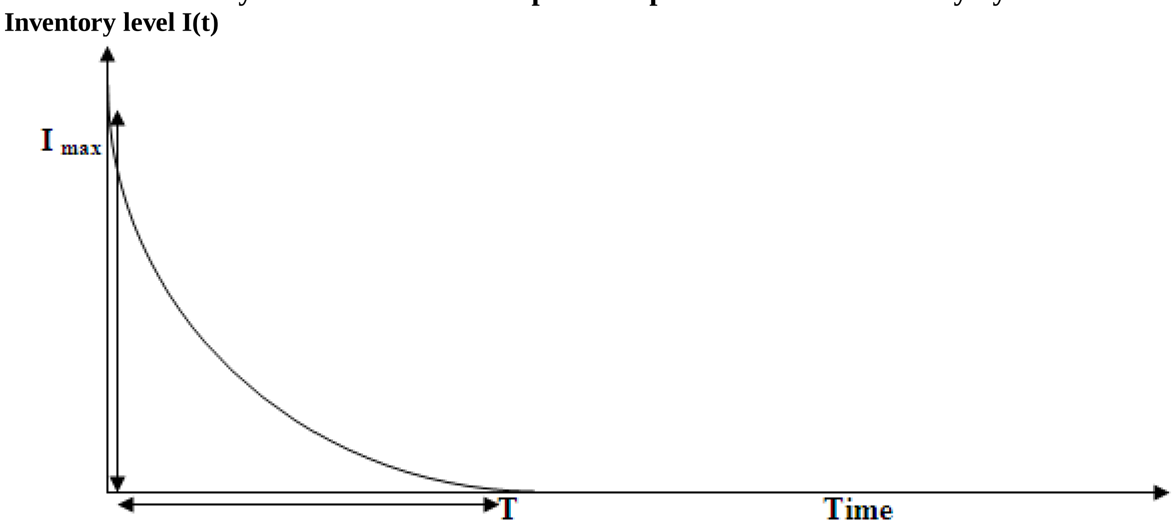


**Fig 1**

The Inventory level have not shortages,in this period [0, T], the inventory depletes due to the deterioration and demand. So the inventory level at the time during $[0, t_1]$ forms the differential equation:

$$\frac{dI_P}{dt}+\theta I_P=-(a+bt)\sqrt{I_P}\ , \qquad 0\le t\le T \qquad \ldots(2)$$

Where θ is the constant detritions rate with $\theta$ with boundary conditions $I_P(t_1)=0$ and $I_P(0)=Q$
With boundary conditions $I_P(t)=0$ at t = T.
The system gives the solution:

$$I_P=\frac{1}{\theta^4}\left[\theta(a+bt)-2b\right]^2+\left[\theta(a+bt_1)-2b\right]^2 e^{\theta(t-t_1)}+2\left[\theta(a+bt)-2b\right]\left[\theta(a+bt_1)-2b\right]e^{\frac{\theta}{2}(t-t_1)} \qquad \ldots(3)$$

Or

$$I_P=\left\{\begin{matrix}\frac{1}{\theta^2}(a+bt)^2+(a+bt)\left(2\theta^2(a+bt_1)-2b\theta-\frac{4b}{\theta^3}\right)+(a+bt)(t-t_1)\\ \left[\theta^3(a+bt_1)-2b\theta^2\right]+(t-t_1)\left[\theta^3\left(a+bt_1\right)^2-6b\theta^2(a+bt_1)+4b\theta+4b^2\theta\right]\\ +\left[\theta(a+bt_1)^2-8b\theta(a+bt_1)+8b+\frac{4b^2}{\theta^4}\right]\end{matrix}\right\} \qquad \ldots(4)$$

Now of we consider a = 4, b = 5, θ = 0.3 in the respective proper units. Then we get inventory label as given in graph and table computed by silab.

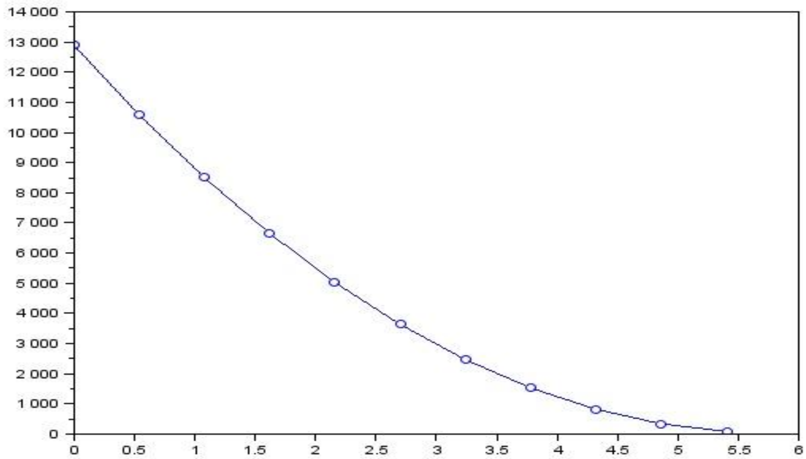


**Fig 2. Representation of inventory**

**Table 1. Representation of inventory**

| T | IP |
|---|---|
| 0 | 12893.86 |
| 0.54 | 10585.12 |
| 1.08 | 8504.85 |
| 1.62 | 6653.05 |
| 2.16 | 5029.72 |
| 2.7 | 3634.86 |
| 3.24 | 2468.47 |
| 3.78 | 1530.54 |
| 4.32 | 821.09 |
| 4.86 | 340.11 |
| 5.4 | 87.59 |

## V. Cost Associated in Inventory

Putting $I_P(0)=Q$ in (5), then

$$Q=\begin{Bmatrix} \frac{1}{\theta^2}a^2+a\left(2\theta^2(a+bT)-2b\theta-\frac{4b}{\theta^3}\right)-\left[\theta^3(a+bT)-2b\theta^2\right] \\ -T\left[\theta^3\left(a+bT\right)^2-6b\theta^2(a+bT)+4b\theta+4b^2\theta\right]+ \\ \left[\theta(a+bT)^2-8b\theta(a+bT)+8b+\frac{4b^2}{\theta^4}\right] \end{Bmatrix}$$

…(5)

**Purchase cost per cycle :-**

PC = (Purchase cost per unit) * (Order quantity in one cycle)

$=C_P*Q$

$$PC=C_P\begin{Bmatrix} \frac{1}{\theta^2}a^2+a\left(2\theta^2(a+bt_1)-2b\theta-\frac{4b}{\theta^3}\right)-at_1\left[\theta^3(a+bt_1)-2b\theta^2\right] \\ -t_1\left[\theta^3\left(a+bt_1\right)^2-6b\theta^2(a+bt_1)+4b\theta+4b^2\theta\right] \\ +\left[\theta(a+bt_1)^2-8b\theta(a+bt_1)+8b+\frac{4b^2}{\theta^4}\right] \end{Bmatrix} \quad \ldots (6)$$

**Sales revenue**$=\frac{\text{selling cost}}{T}\left[T^2\left(2a+bT\right)^2\right]=pT(2a+bT)^2$ … (7)

**Cost of placing order** = $\frac{C_o}{T}$

... (8)

**Inventory holding cost per cycle:-**

$IC_H(t)=\int_0^{t_1}h(t)I_P(t)dt$

$$=\int_0^{t_1}he^{-rt}I_P(t)dt$$

$$=h\int_0^{t_1}e^{-rt}I_P(t)dt$$

$$IC_H=\frac{h}{T}\begin{Bmatrix} \frac{1}{\theta^3}\left[\frac{a^2}{r}+\frac{2a}{r^2}+\frac{2b}{r^3}-(a+bt_1)^2\frac{e^{-rt_1}}{r}-2(a+bt_1)\frac{e^{-rt_1}}{r^2}-\frac{2be^{-rt_1}}{r^3}\right] \\ -\left(2\theta^2(a+b)t_1-2b\theta-\frac{4b}{\theta^3}\right)\left((a+bt_1)\frac{e^{-rt_1}}{r}+\frac{be^{-rt_1}}{r^2}-\frac{a}{r}-\frac{b}{r^2}\right) \\ -\left[\theta^3(a+bt_1)-2b\theta^2\right]\begin{Bmatrix} (a-bt_1)\frac{t_1e^{-rt_1}}{r}+(a-bt_1)\frac{e^{-rt_1}}{r^2}+\frac{bt_1^2e^{-rt_1}}{r}-\frac{at_1e^{-rt_1}}{r} \\ +\frac{2bt_1e^{-rt_1}}{r^2}-\frac{2be^{-rt_1}}{r^3}+\frac{(a-bt_1)}{r^2}-\frac{2b}{r^3}+\frac{at_1}{r} \end{Bmatrix} \\ +\left[\theta(a+bt_1)^2-6b\theta^2(a+bt_1)+4b\theta+4b^2\theta\right]\left[\frac{e^{-rt_1}}{r^2}+\frac{t_1}{r}-\frac{1}{r^2}\right] \\ -\left[\theta(a+bt_1)^2-8b\theta(a+bt_1)+8b+\frac{4b^2}{\theta^4}\right]\left[\frac{e^{-rt_1}}{r}-\frac{1}{r}\right] \end{Bmatrix}$$

… (9)

The total annual profit is given by
$T_C$ (T, p) =Sales revenue-Cost of placing order –Cost of purchasing – holding Cost

$$= -\frac{C_P}{T}\left\{\begin{array}{l}\frac{1}{\theta^2}a^2 + a\left(2\theta^2(a+bT) - 2b\theta - \frac{4b}{\theta^3}\right) - aT\left[\theta^3(a+bT) - 2b\theta^2\right] \\ -T\left[\theta^3(a+bT)^2 - 6b\theta^2(a+bT) + 4b\theta + 4b^2\theta\right] \\ +\left[\theta(a+bT)^2 - 8b\theta(a+bT) + 8b + \frac{4b^2}{\theta^4}\right]\end{array}\right\} + pT(2a+bT)^2$$

$$-\frac{h}{T}\left\{\begin{array}{l}\frac{1}{\theta^3}\left[\frac{a^2}{r} + \frac{2a}{r^2} + \frac{2b}{r^3} - (a+bT)^2\frac{e^{-rT}}{r} - 2(a+bT)\frac{e^{-rT}}{r^2} - \frac{2be^{-rT}}{r^3}\right] \\ -\left(2\theta^2(a+bT) - 2b\theta - \frac{4b}{\theta^3}\right)\left((a+bT)\frac{e^{-rT}}{r} + \frac{be^{-rT}}{r^2} - \frac{a}{r} - \frac{b}{r^2}\right) \\ -\left[\theta^3(a+bT) - 2b\theta^2\right]\left\{\begin{array}{l}(a-bT)\frac{Te^{-rT}}{r} + (a-bT)\frac{e^{-rT}}{r^2} + \frac{bT^2e^{-rT}}{r} - \frac{aTe^{-rT}}{r} \\ +\frac{2bTe^{-rT}}{r^2} - \frac{2be^{-rT}}{r^3} + \frac{(a-bT)}{r^2} - \frac{2b}{r^3} + \frac{aT}{r}\end{array}\right\} \\ +\left[\theta(a+bT)^2 - 6b\theta^2(a+bT) + 4b\theta + 4b^2\theta\right]\left[\frac{e^{-rT}}{r^2} + \frac{T}{r} - \frac{1}{r^2}\right] \\ -\left[\theta(a+bT)^2 - 8b\theta(a+bT) + 8b + \frac{4b^2}{\theta^4}\right]\left[\frac{e^{-rT}}{r} - \frac{1}{r}\right]\end{array}\right\}$$

$$-\frac{C_o}{T} \qquad \text{... (10)}$$

We have $e^{-rT}=1-rT+r^2T^2-$……………….and r is very small which is inflation, so we use only $e^{-rT}=1-rT$.

For profit maximization we calculate $\frac{dT_C(T,p)}{dT}$

$$\frac{dT_C(T,p)}{dT} = 2pT(2a+bT) \quad + bp(2a+bT)^2 + \frac{C_o}{T^2} + \frac{C_P}{T^2}\left[\frac{a^2}{\theta^2} + a(2\theta^2(a+bT) - 2b\theta - \frac{4b}{\theta})\right]$$

$$-\frac{C_P}{T}(2ab\theta^2) - a[\theta^2(a+bT-2b\theta^2)] - aTb\theta^2\,]$$

$$-\left\{\left[\theta^3(a+bT)^2 - 6b\theta^2(a+bT) + 4b\theta + 4b^2\theta\right]\right\} - T[2\theta^3b(a+bT) - 6b^2\theta^2] + 2\theta T(a+bT) + 8b^2\theta$$

$$+\frac{h}{T^2}\left\{\begin{array}{l}\frac{1}{\theta^3}\left[\frac{a^2}{r}+\frac{2a}{r^2}+\frac{2b}{r^3}-(a+bT)^2\left(\frac{1}{r}-T\right)-2(a+bT)\left(\frac{1}{r^2}-\frac{T}{r}\right)-2b\left(\frac{1}{r^3}-\frac{T}{r^2}\right)\right]\\ -\left(2\theta^2(a+bT)-2b\theta-\frac{4b}{\theta^3}\right)\left((a+bT)\left(\frac{1}{r}-T\right)+b\left(\frac{1}{r^2}-\frac{T}{r}\right)-\frac{a}{r}-\frac{b}{r^2}\right)\\ -\left[\theta^3(a+bT)-2b\theta^2\right]\left\{\begin{array}{l}(a-bT)T\left(\frac{1}{r}-T\right)+(a-bT)\left(\frac{1}{r^2}-\frac{T}{r}\right)+bT^2\left(\frac{1}{r}-T\right)-aT\left(\frac{1}{r}-T\right)\\ +2bT\left(\frac{1}{r^2}-\frac{T}{r}\right)-2b\left(\frac{1}{r^3}-\frac{T}{r^2}\right)+\frac{(a-bT)}{r^2}-\frac{2b}{r^3}+\frac{aT}{r}\end{array}\right\}\\ -T\left[\theta(a+bT)^2-8b\theta(a+bT)+8b+\frac{4b^2}{\theta^4}\right]\end{array}\right\}$$

$$-\frac{h}{T}\left\{\begin{array}{l}\frac{1}{\theta^3}\left[-2b(a+bT)\left(\frac{1}{r}-T\right)+(a+bT)^2-2b\left(\frac{1}{r^2}-\frac{T}{r}\right)+\frac{2}{r}(a+bT)+2b\left(\frac{1}{r^2}\right)\right]\\ -\left(2\theta^2(a+bT)-2b\theta-\frac{4b}{\theta^3}\right)\left(b\left(\frac{1}{r}-T\right)-(a+bT)-\frac{b}{r}\right)-2\theta^2 b\\ \left((a+bT)\left(\frac{1}{r}-T\right)+b\left(\frac{1}{r^2}-\frac{T}{r}\right)-\frac{a}{r}-\frac{b}{r^2}\right)\\ -\theta^3 b\left\{\begin{array}{l}(a-bT)T\left(\frac{1}{r}-T\right)+(a-bT)\left(\frac{1}{r^2}-\frac{T}{r}\right)+bT^2\left(\frac{1}{r}-T\right)-aT\left(\frac{1}{r}-T\right)\\ +2bT\left(\frac{1}{r^2}-\frac{T}{r}\right)-2b\left(\frac{1}{r^3}-\frac{T}{r^2}\right)+\frac{(a-bT)}{r^2}-\frac{2b}{r^3}+\frac{aT}{r}\end{array}\right\}\\ -[\theta^3(a+bT)-2b\theta^2]\left\{\begin{array}{l}(a-bT)\left(\frac{1}{r}-T\right)-bT\left(\frac{1}{r}-T\right)-T(a-bT)-b\left(\frac{1}{r^2}-\frac{T}{r}\right)-\\ \frac{(a-bT)}{r}+2bT\left(\frac{1}{r}-T\right)\\ -bT^2+aT-a\left(\frac{1}{r}-T\right)+2b\left(\frac{1}{r^2}-\frac{T}{r}\right)-\frac{2bT}{r}+\frac{b}{r^2}+\frac{a}{r}\end{array}\right\}\\ -\left[\theta(a+bT)^2-8b\theta(a+bT)+8b+\frac{4b^2}{\theta^4}\right]-T\left[2b\theta(a+bT)-8b^2\theta\right]\end{array}\right\}$$

… (11)

Putting, $\frac{dT_C(T,p)}{dT}=0.$ for the condition $\frac{d^2T_C(T,p)}{dT^2}$<0 we can find the optimal replenishment cycle.

## VI. Numerical Example And Sensitivity Analysis

We consider example for computation no shortages that is $t_1$=T and $t_2$=0 , Co = 25, a = 4, b = 5, $C_P$=10, p = 100, r = 0.5, θ = 0.3 in the respective proper units. Then we get T=5.9. Then Q = 14356.34 and total profit TC = 823547.40

**Table 2**

| Constant Parameter | Changing Parameter | Variation | T | Q | TC |
|---|---|---|---|---|---|
| a = 4 | r | 0.1 | 0.54 | 12423.62 | 378457.66 |
| b =5 | | 0.1 | 1.08 | 12465.4 | 243975.86 |
| p=100 | | 0.2 | 2.75 | 12650.68 | 176002.58 |
| | | 0.3 | 3.78 | 12801.04 | 287549.40 |
| θ = 0.3 | | 0.4 | 4.32 | 12889.03 | 366557.07 |
| a = 4 | All variables are fixed | | 1.62 | 12516.51 | 10717.87 |
| b =5 | | | 2.75 | 12650.68 | 123957.99 |
| p=100<br>r = 0.5 | | | 3.27 | 12723.54 | 187167.44 |
| | | | 3.78 | 12801.04 | 261297.57 |
| θ = 0.3 | | | 4.32 | 12889.03 | 355846.46 |
| | | | 4.86 | 12912.77 | 383395.77 |
| | | | 5.4 | 13080.76 | 605362.30 |

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