IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 1 Ver. IV (Jan - Feb. 2015), PP 38-41
www.iosrjournals.org

Modeling Of an Inventory System for Non-Instantaneous decaying Items with Partial Backlogging and Time Dependent Demand Rate under Permissible Delay

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Abstract: *We will discuss an inventory model is investigates with variable demand rate and time dependent deteriorating items.In this study, we have taken shortages in inventory are allowed and fully backlogged. This model is studied under the condition for decaying items of permissible delay in payments which is most important and an outcome of interaction between product and financial markets which arises. This model based on time-dependent, holding cost, shortages cost and the combination of model is unique and practical.*

I. Introduction

Trade credit would play an important role in the conduct of business for many reasons. For a supplier who offers trade credit, it is also an efficient method to stimulate the demand of the product. For a retailer, it is an efficient method of bonding a supplier when the retailer is at the risk of receiving inferior quality goods or service and is also an effective means of reducing the cost of holding stocks.

Teng and Yang (2004) developed a deterministic economic order quantity models with partial backlogging when demand and cost are fluctuating with time. Chang, et al. (2001) developed an inventory model for deteriorating items with linear trend demand under the condition of permissible delay in payments. Teng et al. (2002) has discussed inventory model for deteriorating items with time varying demand and partially backlogging.

II. Formulation And Solution Of The Model

To discussan inventory model with the same assumptions as adopted by Vashistha (17), except the time dependent demand, the inflation and time-discounting.

The inventory system is governed by the following differential equations in the interval (0, T) are

$q'(t) + \alpha\beta t^{\beta-1} q(t) = -(a + bt + ct^2), \qquad 0 \le t \le \mu \ldots(1)$

$q'(t) + \alpha\beta t^{\beta-1} q(t) = -[a + (b + c\mu)t], \mu \le t \le t_1 \ldots(2)$

And

$q'(t) = -a\theta, \qquad t_1 \le t \le T \ldots(3)$

With the condition q (0) =S and q (t_1) =0 …(4)

Solution of equations (1) and (2) by using (4) are giving by $q(t) = \Big(at + b\frac{t^2}{2} + c\frac{t^3}{3} +$

aαtβ+1β+1+bαtβ+2β+2+cαtβ+3β+3e−αtβ+Se−αtβ0≤t≤μ

…(5)

$$q(t) = \Big(at_1 + m\frac{{t_1}^2}{2} + a\alpha\frac{{t_1}^{\beta+1}}{(\beta+1)} + m\alpha\frac{{t_1}^{\beta+2}}{(\beta+2)} - at - m\frac{t^2}{2} - a\alpha\frac{t^{\beta+1}}{(\beta+1)} - m\alpha\frac{t^{\beta+2}}{(\beta+2)}\Big)e^{-\alpha t^\beta}$$

$$\mu \le t \le t_1 \qquad \ldots(6)$$

From (5) and (6), we get

S= $\Big(at_1 + m\frac{{t_1}^2}{2} + a\alpha\frac{{t_1}^{\beta+1}}{(\beta+1)} + m\alpha\frac{{t_1}^{\beta+2}}{(\beta+2)} - c\frac{\mu^3}{6} - c\alpha\frac{\mu^{\beta+3}}{(\beta+2)(\beta+3)}\Big)\ldots(7)$

Using (7) and (5) becomes

$q(t) = \Big(at_1 + m\frac{{t_1}^2}{2} + a\alpha\frac{{t_1}^{\beta+1}}{(\beta+1)} + m\alpha\frac{{t_1}^{\beta+2}}{(\beta+2)} - c\frac{\mu^3}{6} - c\alpha\frac{\mu^{\beta+3}}{(\beta+2)(\beta+3)} - at - \frac{b}{t^2} - c\frac{t^3}{3} - \quad a\alpha\frac{t^{\beta+1}}{(\beta+1)} -$

$b\alpha\frac{t^{\beta+2}}{(\beta+2)} - c\alpha\frac{t^{\beta+3}}{(\beta+3)}\Big)e^{-\alpha t^\beta}\, 0 \le t \le \mu \ldots(8)$

Also the solution of equation (3) by using (4) is given by
$q(t) = -\theta a(t-t_1) t_1 \leq t \leq T$…(9)

The holding cost during the period $(0,t_2)$ is given by

$HC = h[\int_0^{\mu} q(t)\, e^{-rt} dt + \int_{\mu}^{t_1} q(t) e^{-rt} dt]$

$= h\ [\frac{m\mu^3}{3} - \frac{c\mu^4}{4} - \frac{2c\alpha\mu^{\beta+4}}{(\beta+2)(\beta+4)} - \frac{b\mu^3}{6} + \frac{b\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)} + \frac{rc\mu^5}{12} + \frac{3c\alpha r\mu^{\beta+5}}{2(\beta+2)(\beta+5)} + \frac{br\mu^4}{8} + \frac{c\mu^5}{15} + \frac{(b-m)\alpha r\mu^{\beta+4}}{(\beta+2)(\beta+4)} - \frac{\alpha m\mu^{\beta+3}}{2(\beta+3)} + \frac{(\beta+2)\alpha c\mu^{\beta+4}}{2(\beta+1)(\beta+4)} + \frac{at_1^2}{2} + \frac{mt_1^3}{3} - \frac{m\mu t_1^2 2m\alpha\beta t_1^{\beta+3}}{2(\beta+1)(\beta+3)} + \frac{a\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{m\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)} - \frac{art_1^3}{6} - \frac{mrt_1^4}{2} - \frac{ar\alpha t_1^{\beta+3}}{2(\beta+3)} - \frac{(\beta+6)m\alpha rt_1^{\beta+4}}{2(\beta+2)(\beta+4)} - \frac{mr\mu^4}{8} + \frac{mrt_1^2\mu^{\beta+1}}{(\beta+1)}]$…(10)

The deterioration cost during the period $(0,t_1)$ is given by

$DC = d[\int_0^{\mu} \alpha\beta t^{\beta-1} e^{-rt} q(t) dt + \int_{\mu}^{t_1} \alpha\beta t^{\beta-1} e^{-rt}\, q(t) dt]$

$= d\alpha\beta[\frac{rc\mu^{\beta+4}}{6(\beta+1)} - \frac{b\mu^{\beta+2}}{2(\beta+2)} - \frac{c\mu^{\beta+3}(\beta+1)}{2\beta(\beta+3)} - \frac{rmt_1^2\mu^{\beta+1}}{2(\beta+1)} + \frac{br\mu^{\beta+3}}{2(\beta+3)} + \frac{c\mu^{\beta+4}}{3(\beta+4)} + \frac{at_1^{\beta+1}}{\beta(\beta+1)} + \frac{mt_1^{\beta+2}}{\beta(\beta+2)} + \frac{m\mu^{\beta+2}}{2(\beta+2)} - \frac{art_1^{\beta+2}}{(\beta+2)(\beta+1)} - \frac{mrt_1^{\beta+2}}{2(\beta+1)} + \frac{mrt_1\mu^{\beta+1}}{2(\beta+1)} + \frac{mrt_1^{\beta+3}}{2(\beta+3)} - \frac{mr\mu^{\beta+3}}{2(\beta+3)}]$…(11)

The shortage cost during the period (t_1, T) is given by

$SC = -s[\int_{t_1}^{T} q(t) e^{-rt} dt]$

$= \frac{s\theta a}{r^2}\left\{e^{-rt_1} - e^{-rT}[rT\left(1 - \frac{t_1}{T}\right) + 1]\right\}$…(12)

The opportunity cost due to lost sales during the period (t_1, T) is given by

$LSC = l\int_{t_1}^{T} a(1-\theta)\, e^{-rt} dt$

$= \frac{la(1-\theta)}{r}[e^{-rt_1} - e^{-rT}]$…(13)

The ordering cost is given by

OC=A …(14)

The total cost of the system is given by

The total average cost of the inventory system per unit time is given by

$$C_A(t_1, T) = \frac{OC + HC + DC + SC + LSC}{T}$$

$$= \frac{A}{T} + \frac{h}{T}[\frac{m\mu^3}{3} - \frac{c\mu^4}{4} - \frac{2c\alpha\mu^{\beta+4}}{(\beta+2)(\beta+4)} - \frac{b\mu^3}{6} + \frac{b\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)} + \frac{rc\mu^5}{12} + \frac{3c\alpha r\mu^{\beta+5}}{2(\beta+2)(\beta+5)} + \frac{br\mu^4}{8} + \frac{c\mu^5}{15} + \frac{(b-m)\alpha r\mu^{\beta+4}}{(\beta+2)(\beta+4)} - \frac{\alpha m\mu^{\beta+3}}{2(\beta+3)} + \frac{(\beta+2)\alpha c\mu^{\beta+4}}{2(\beta+1)(\beta+4)} + \frac{at_1^2}{2} + \frac{mt_1^3}{3} - \frac{m\mu t_1^2 2m\alpha\beta t_1^{\beta+3}}{2(\beta+1)(\beta+3)} + \frac{a\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{m\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)} - \frac{art_1^3}{6} - \frac{mrt_1^4}{2} - \frac{ar\alpha t_1^{\beta+3}}{2(\beta+3)} - \frac{(\beta+6)m\alpha rt_1^{\beta+4}}{2(\beta+2)(\beta+4)} - \frac{mr\mu^4}{8} + \frac{mrt_1^2\mu^{\beta+1}}{(\beta+1)}]$$
$$+ \frac{d\alpha\beta}{T}[\frac{rc\mu^{\beta+4}}{6(\beta+1)} - \frac{b\mu^{\beta+2}}{2(\beta+2)} - \frac{c\mu^{\beta+3}(\beta+1)}{2\beta(\beta+3)} - \frac{rmt_1^2\mu^{\beta+1}}{2(\beta+1)} + \frac{br\mu^{\beta+3}}{2(\beta+3)} + \frac{c\mu^{\beta+4}}{3(\beta+4)} + \frac{at_1^{\beta+1}}{\beta(\beta+1)} + \frac{mt_1^{\beta+2}}{\beta(\beta+2)} + \frac{m\mu^{\beta+2}}{2(\beta+2)} - \frac{art_1^{\beta+2}}{(\beta+2)(\beta+1)} - \frac{mrt_1^{\beta+2}}{2(\beta+1)} + \frac{mrt_1\mu^{\beta+1}}{2(\beta+1)} + \frac{mrt_1^{\beta+3}}{2(\beta+3)} - \frac{mr\mu^{\beta+3}}{2(\beta+3)}]$$
$$+ \frac{s\theta a}{r^2 T}\left\{e^{-rt_1} - e^{-rT}[rT\left(1 - \frac{t_1}{T}\right) + 1]\right\}$$
$$+ \frac{la(1-\theta)}{rT}[e^{-rt_1} - e^{-rT}] \quad \text{….(15)}$$

III. Approximation Solution Procedure

The total average cost has the two variables t_1 and T. To minimize the total average cost , the optimal values of t_1 and T can be obtained by solving the following equations simultaneously

$$\frac{\partial C_A(t_1, T)}{\partial t_1} = 0 \quad \text{… … . (16)}$$

$$\frac{\partial C_A(t_1, T)}{\partial T} = 0 \qquad \ldots\ldots(17)$$

Provided, they satisfy the following conditions

$\frac{\partial^2 C_A(t_1,T)}{\partial t_1^2} > 0$, $\frac{\partial^2 C_A(t_1,T)}{\partial T^2} > 0$

$$\left(\frac{\partial^2 C_A(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 C_A(t_1,T)}{\partial T^2}\right) - \left(\frac{\partial^2 C_A(t_1,T)}{\partial t_1 \partial T}\right) > 0 \qquad \ldots\ldots(18)$$

The numerical solution of the equations (16), (17) and (18) can be obtained by using suitable computational numerical method.

Numerical Example

a = 50, s =25, b = 20, h = 1, c = 10, d = 20, C = 200, l =10, μ = 1, α = .10, β = 3, r = .2,θ = .56, H = 10 in appropriate units.

IV. Results

In this study one of the main objectives a volume flexible production inventory model is developed for perishable items having linear demand. Then with the help of this model demand rate is taken as quadratic function of time and production rate is decision variable and total inventory cost is obtained. The total cost obtained than can be used to obtain a total inventory average cost, using by calculus techniques.

V. Discussion

The inventory model is developed for deteriorating items with time dependent demand Rate. The unit inventory cost is depending upon material cost, labor cost and tool or die Cost. Shortages with partially backlogged are allowed a very natural phenomenon in Inventory model. The suggested model and study will help retailers in deciding their Optimal ordering quantity to have minimum inventory cost.

VI. Conclusion

This model is very most useful in their retail business. The result is our model, the fresh product time increases the order quantity and total cost are decreases. It can we develop an inventory model with time dependent deterioration, the effect of inflation and time value of money in formulating the inventory replenishment policy.We have taken shortages in inventory are allowed and fully backlogged. It can be used for electronic components, fashion apparel etc. Furtherwe have considered demand rate is an exponential increasing function of time, the small change in time, demand is increasing a lot.

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