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Boundary Layer Flow in the Vicinity of the Forward Stagnation Point of the Spinning and Translating Sphere

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Abstract: *Exact solutions are important not only in its own right as solution of particular flows, but also serve as accuracy check for numerical solution. Exact solution of the Navier-Strokes equation are, for example, those of steady and unsteady flows near a stagnation point, Stagnation point flows can either be viscous or inviscid, steady or unsteady, two dimensional or three dimensional, normal or oblique and forward or reverse. The classic problems of two dimensional and three dimensional stagnation point flow are associated with the names of Hiemenz and Homan A novel radial stagnation point flow impinging axi symmetrically on a circular cylinder was reported by Wang. The present paper deals with the laminar boundary layer flow and heat transfer in the stagnation region of a rotating and translating sphere with uniform magnetic fields. The governing equations of flow are derived for* $\xi = 0$ ($t^*=0$) *and* $\xi=1$ ($t^*\to\infty$) *and solutions in the closed form are obtained. The temperature and velocity fields for* $\xi = 0$ *are numerically computed. This shows that the thermal boundary layer thickness decreases as Prandtl number Princreases.The surface heat transfer (28) increases with the Prandtl number Pr. The surface heat transfer (28) at the starting of motion is found to be strangely dependent on the Prandtl number Pr. But it is dependent of magnetic field, buoyancy force Bp and Rotation Parameter Ro.*
Keywords:_Temperature field, velocity field,uniform magnetic field, buoyancy force, Rotation Parameter_.

I. Introduction

Exact solutions are important not only in its own right as solution of particular flows, but also serve as accuracy check for numerical solution.

Exact solution of the Navier-Strokes equation are, for example, those of steady and unsteady flows near a stagnation point, Stagnation point flows can either be viscous or inviscid, steady or unsteady, two dimensional or three dimensional, normal or oblique and forward or reverse. The classic problems of two dimensional and three dimensional stagnation point flow are associated with the names of Hiemenz and Homan A novel radial stagnation point flow impinging axi symmetrically on a circular cylinder was reported by Wang

Luthander and Rydberg measured drag coefficient on a rofating sphere in axial flow. Homan and Frossling first obtained the exact solution of the Navier – Strokes equations for rotationally symmetrical stagnation point flow and found that the boundary layer thickness was independent of the distance along the wall and the velocity profiles were similar. Mishra and Choudhary studied axi-symmetric stagnation point flow with uniform suction. Rott and Crabtree simplified the boundary layer calculations for bodies of revolution. Lok et al. studied the growth of the boundary layer of micropolar fluid started implusively from rest near the forward stagnation point of a two dimensional plane surface.

We discussed axi-symmetric stagnation flow of a viscous and electrically conducting fluid near the blunt nose of a spinning body with pressure of magnetic field. Sparrow et. al investigated the effect of transpiration cooling in MHD stagnation point flow. Ece has investigated the initial boundary layer flow past an impulsively started translating and spinning body of revolution. Rajasekaran and Palekar studied the influence of buoyancy force on the steady forced convection flow over a spinning sphere. Lee et. al discussed heat transfer over rotating bodies in forced flows. Hatrikonstantinou studied the effects of a mixed convection and viscous dissipation on heat transfer about porous rotating sphere.

Bush analyzed the stagnation point boundary layer in the presence of an applied magnetic field. Ozturk and Ece investigated into unsteady force convection heat transfer from a translating and spinning body. Thakur et. al investigated hydromagnetic boundary layer flow and heat transfer in the stagnation region of a spinning and translating sphere in the presence of buoyancy forces.

The present paper deals with the laminar boundary layer flow and heat transfer in the stagnation region of a rotating and translating sphere with uniform magnetic fields. The governing equations of flow are derived for $\xi = 0$ ($t^*=0$) and $\xi=1$ ($t^*\to\infty$) and solutions in the closed form are obtained. The temperature and velocity fields for $\xi = 0$ are numerically computed.

Notations

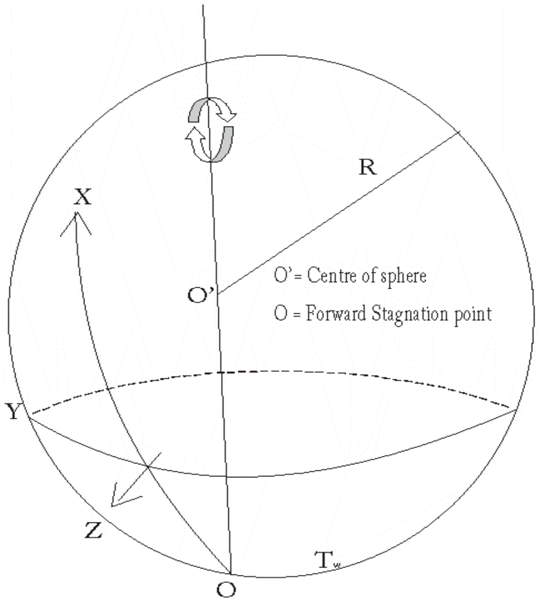
u, v, w	:	velocity components in the direction of X- axis, and Y-axis and Z-axis respectively
V	:	Characteristic velocity
L	:	Characteristic length
σ	:	electrical conductivity
μ_0	:	magnetic permeability
$Rm = \mu_0 \sigma vL << 1$	:	magnetic Reynold number
B	:	constant magnetic field applied in the z-direction
T	:	temperature
t	:	time
ρ	:	density
μ	:	coefficient of visconsity
ν	:	μ/ρ = kinenaticviscousity
K	:	thermal conductivity
Ω	:	angular velocity of the sphere
G	:	acceleration due to gravity
β	:	coefficient of thermal expansion
R	:	radius of the sphere
Cp	:	specific heat at a constant pressure Subscripts
e,w,∞	:	denote conditions of the edge of the boundary layer on the surface and in the free stream
$T\omega$	:	temperature on the surface
$T\infty$	:	temperature in the free stream
$Pr = \mu Cp / K$	:	Prandtl number
$M = \sigma B^2 / Pr$	:	magnetic parameter
$Bp = \mu Gr\ R / Re2R$	:	Buoyancy parameter
$GrR = g\ \beta\ (T\omega - T\infty)\ R3/ V^3$	:	Grashof number
$\Theta = T - T\infty / T\omega - T\infty$	:	dimensionless temperature
b	:	velocity gradient at the edge of the boundary layer.
$ReR = b\ R^2 / \nu$	:	Reynolds number
$Ro = (\Omega/b)^2$	:	rotation parameter

ue	:	ax, velocity on the edge of the boundary layer, a>0
$\eta = (2b/\nu)^{1/2} Z/\xi^{1/2}$	:	dimensionless variable
$\bar{t}$	:	dimensionless time
f^1	:	dimensionless velocity component along x-direction
s	:	dimensionless velocity component along y-direction

II. Formulation of the problems, assumptions and governing equations

Formulation

Suppose a sphere is at rest in an abient fluid with surface temperature T∞ at t< 0 (i.e. prior to the time t = 0). The sphere is suddenly spinning with the constant angular velocity Ω. When at t=0 an impulsive motion is imposed to the fluid, and T∞ is suddenly raised to Tω (Tω> T∞). The unsteadiness is caused by the impulsive motion of the fluid and the impulsive motion of sphere.



$U_O \; U_\infty$
↑ ↑

Flow Model

Consider the unsteady laminar boundary layer flow of a viscous, incompressible fluid of small electrical conductivity in the front stagnation region of this spinning sphere in the presence of uniform magnetic field and a buoyancy force. Take x the distance along a meridian from the front stagnation point, y the distance in the direction of spinning and z the distance normal to the surface.

Assumptions

Following assumptions are made.

i. A uniform magnetic field B is imposed in the direction of z-axis.
ii. The boundary layer flow under uniform magnetic field is axi-symmetric.
iii. The magnetic Reynolds number Rm is very small. i.e. Rm<< 1.
iv. As Rm<<1, the effect of the induced magnetic field as compared to B is neglected.
v. The dissipation terms, Ohmic heating and surface curvature are neglected in the region of front stagnation point of the surface.
vi. The fluid has constant properties except the density changes which produce buoyancy forces.
vii. The effect of the buoyancy induced stream wise pressure gradient terms on the flow and temperature profile is negligible.
viii. T_w and T_∞ are taken as constants.

Governing Equations

Under the above assumptions the boundary layer equations governing the flow of the present problem after lee et. al, Ozturk et. al and Bush are

$$\frac{\partial}{\partial x}(ux) + \frac{\partial}{\partial z}(wx) = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} - \frac{v^2}{x} = \mu_e\frac{du_e}{dx} + v\frac{\partial^2 u}{\partial z^2} + g\,\beta\,(T - T_\infty)\frac{x}{R} - \frac{\sigma B^2}{\rho}(u - u_e) \quad (2)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + w\frac{\partial v}{\partial z} - \frac{uv}{x} = v\frac{\partial^2 w}{\partial z^2} - \frac{\sigma B^2}{\rho}v \quad (3)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z} = \frac{K}{\rho C_p}\frac{\partial^2 T}{\partial t^2} \quad (4)$$

With initial conditions

$$t<0:\quad \left.\begin{aligned} u(x,z,t) &= 0\\ v(x,z,t) &= 0\\ w(x,z,t) &= 0\\ T(x,z,t) &= T_\infty \end{aligned}\right\} \quad (5)$$

And the boundary layer conditions

$$t\geq 0\quad \left.\begin{aligned} u(x,0,t) &= 0\\ v(x,0,t) &= \Omega x\\ w(x,0,t) &= Tw\\ u(x,\infty,t) &= u_e(x)\\ v(x,\infty,t) &= 0\\ T(x,\infty,t) &= T_\infty \end{aligned}\right\} \quad (6)$$

Application of Transformation

Following William and Rhyme, we apple the transformation given below for making the region of time integration finite:

$$\left.\begin{aligned} \bar{t} &= b\,t,\ b>0\\ \xi &= 1 - e^{-t}\\ \eta &= \left(\frac{2b}{v}\right)^{1/2}\xi^{-1/2} z\\ R_0 &= \left(\frac{\Omega}{b}\right)^2\\ B_p &= \frac{Gr_R}{R^2\,eR}\\ Gr_R &= \frac{g\beta\,(T_\omega - T_\infty)R^3}{v^2}\\ R_{eR} &= \frac{bR^2}{v}\\ M &= \frac{\sigma\,B^2}{\rho\,b}\\ \mu_e &= bx \end{aligned}\right\} \quad (7)$$

$$\left.\begin{aligned} V_W &= \Omega\,x\\ f'(\xi,\eta) &= \frac{u(x,z,t)}{b\,x}\\ S(\xi,\eta) &= \frac{u(x,z,t)}{\Omega\,x}\\ f(\xi,\eta) &= \frac{-w(x,z,t)}{\xi^{1/2}(2b^2)^{1/2}}\\ \Theta(\xi,\eta) &= \frac{T(x,z,t) - T_\infty}{T_\omega - T_\infty}\\ P_r &= \frac{\mu\,C_p}{K} \end{aligned}\right\} \quad (7)$$

These transformations (7) are used in the governing equations. Equation (1) is identically satisfied and equations (2), (3) and (4) are transformed into equations.

$$f''' + \frac{\eta}{4}(1-\xi)f'' + \xi f f'' + \frac{\xi}{2}[1 - f(f')^2 R_0 s^2] + \frac{\xi}{2}\eta M(1 - f') + \frac{1}{2}\xi B_p\theta = \xi(1-\xi)\frac{\partial f'}{\partial \xi} \quad (8)$$

$$s'' + \frac{\eta}{4}(1-\xi)s' + \xi(fs' - f's) - \frac{\xi M s}{2} = \frac{1}{2}\xi(1-\xi)\left(\frac{\partial s}{\partial \xi}\right) \quad (9)$$

$$\frac{\theta''}{P_r} + \frac{\eta}{4}(1-\xi)\theta' + \xi f\theta' = \frac{1}{2}\xi(1-\xi)\left(\frac{\partial \theta}{\partial \xi}\right) \quad (10)$$

The boundary condition (6) become

$$f(\xi,0) = f'(\xi,0) = 0;\ s(\xi,0) = \theta(\xi,0) = 1$$

$f'(\xi,\infty) = 1; s(\xi,\infty) = \theta(\xi,\infty) = 0$ (11)

Special forms of governing equation at time infinity and at time zero

When ξ =1(t →∞) equation (8), (9) and (10) reduce to

$f''' + ff'' + \frac{1}{2}[1-(f')^2 + R_0 S^2] + \frac{M}{2}(1-f') + \frac{B_p\theta}{2} = 0$ (12)

$s'' + fs' - f's - \frac{Ms}{2} = 0$ (13)

$\frac{\theta''}{P_r} + f\theta' = 0$ (14)

When ξ = 0 (t = 0 i.e at the start of the motion), equations (8), (9) and (10) becomes.

$f''' + \frac{\eta}{4} f'' = 0$ (15)

$s'' + \frac{\eta}{4} s' = 0$ (16)

$\frac{1}{P_r}\theta'' + \frac{\eta}{4}\theta' = 0$ (17)

The boundary conditions (11), for the equations (12) – (17) changed to

$$\left.\begin{array}{l} f(0) = f'(0) = 0 \\ s(0) = \theta(0) = 1 \\ f'(\infty) = 1,\ s(\infty) = \theta(\infty) = 0 \end{array}\right\} \quad (18)$$

Closed from solutions for the case t = 0 (ξ = 0)

From (17)

$$\frac{1}{P_r}\theta'' = -\frac{\eta}{4}\theta'$$

or, $\int \frac{\theta''}{\theta} d\eta = -\frac{P_r}{4}\int \eta\, d\eta$

or $\log\left(\theta'/_C\right) = -\frac{P_r}{8}\eta^2$

$$\frac{\theta'}{C} = e^{-\frac{P_r}{8}\eta^2}$$

or, $\theta'(\eta) = C\,.\,e^{-\frac{P_r}{8}\eta^2}$ (17a)

Or $\frac{d\theta}{d\eta} = C\,.\,e^{-\frac{P_r}{8}\eta^2}$

$\int d\theta = C\int e^{-\frac{P_r}{8}\eta^2} d\eta$

$\theta(\eta) = C\int_0^\eta e^{-\frac{P_r}{8}\eta^2} d\eta + D$, D is constant

$\theta(\eta) = C\int_0^\eta e^{-\left(\frac{\sqrt{P_r}}{2\sqrt{2}}\eta\right)^2} d\eta + D$ (18a)

Now, $erf(\eta) = \frac{2}{\sqrt{\pi}}\int_0^n e^{-x^2} dx$

$erfc(\eta) = 1 - erf(\eta)$ (19)

(18) can be written as

$$\left.\begin{array}{l} \eta = 0 : f = f' = 0,\ s = 1,\ \theta = 1 \\ \eta \to \infty : f' = 1,\ s = \theta = 0 \end{array}\right\} \quad (20)$$

Using first condition of (20) in (18), we get

$\theta(0) = 0 + D$

Or $1 = D$

Let $x = \sqrt{\frac{P_r}{8}}\,\eta$

Then $dx = \sqrt{\frac{P_r}{8}}\, d\eta$

$\eta \to 0,\ x \to 0$

$\eta \to \infty,\ x \to \infty$

So using second condition of (20) in (18), we get

$O = C\int_0^\infty e^{-x^2}\sqrt{\frac{8}{P_r}}dx + 1$

or $\quad -1 = C\int_0^{\infty} e^{-x^2}\sqrt{\frac{8}{P_r}}dx$

$$-\sqrt{\frac{P_r}{8}} = C\int_0^{\infty} e^{-x^2}dx$$
$$= C\frac{\sqrt{\pi}}{2}$$

or, $\quad C = -\sqrt{\frac{P_r}{8}\frac{4}{\pi}}$

$$C = -\sqrt{\frac{P_r}{2\pi}}$$

Putting C and D in (18), we get

$$\theta(\eta) = 1 - \sqrt{\frac{P_r}{2\pi}}\int_0^{n} e^{-\left(\frac{\sqrt{P_r}}{8}\eta\right)^2} d\eta$$

$$\theta(\eta) = 1 - \sqrt{\frac{P_r}{2\pi}}\int_0^{n} e^{-\left(\frac{\sqrt{P_r}}{8}y\right)^2} dy$$

Put $\qquad \sqrt{\frac{P_r}{8}}\ y = t$

Then $\qquad \sqrt{\frac{P_r}{8}}\ dy = dt$

So, $\quad \theta(\eta) = 1 - \sqrt{\frac{P_r}{2\pi}}\ \int_0^{\sqrt{\frac{P_r}{8}}\eta} e^{-t^2}\sqrt{\frac{8}{P_r}}\ dt$

$$= 1 - \sqrt{\frac{P_r}{2\pi}}\frac{2\sqrt{2}}{\sqrt{P_r}}\int_0^{\sqrt{\frac{P_r}{8}}\eta} e^{-t^2}\ dt$$

$$\theta(\eta) = 1 - erf\left(\sqrt{\frac{P_r}{8}}\ \eta\right)$$

or, $\qquad \theta(\eta) = erfc\left(\sqrt{\frac{P_r}{8}}\ \eta\right) \qquad$ (20a)

From (15), $f'''(\eta) + \frac{\eta}{4}f''(\eta) = 0$

or, $\int \frac{f'''}{f''(\eta)}\,d\eta = -\int \frac{\eta}{4}d\eta$

On integration

$\log_e \frac{f''(\eta)}{C_1} = -\frac{\eta^2}{8}$

$$f''(\eta) = C_1 e^{-\frac{\eta^2}{8}}\ , C_1 \text{ is constant of integration}$$

Again on, integration

$f'(\eta) = C_1\int_0^{\eta} e^{-\frac{\eta^2}{8}}d\eta + D_1, D_1$ is constant

Using first condition of (18), D1= 0

Then $\quad f'(\eta) = C_1\int_0^{\eta} e^{-\frac{\eta^2}{8}}d\eta \qquad$ (20b)

put $\qquad \frac{\eta}{2\sqrt{2}} = x \qquad$ (20c)

Then $\frac{d\eta}{2\sqrt{2}} = dx$

$\eta \to \infty,\ x \to \infty$
$\eta \to 0,\ x \to 0$

Using (20c) and 2nd condition of (18), we have

$$f'(\infty) = C_1 \int_0^\infty e^{-x^2}\, 2\sqrt{2}\ dx$$

$$= 2\sqrt{2}C_1 \int_0^\infty e^{-x^2}\, dx$$

$$\therefore \quad 1 = 2\sqrt{2}C_1 \frac{\sqrt{\pi}}{2} \quad = \quad C_1\sqrt{2\pi}$$

$$\therefore \quad C_1 = \frac{1}{\sqrt{2\pi}}$$

Putting C_1 in (20b), we get

$$\therefore f'(\eta) = \frac{1}{\sqrt{2\pi}} \int_0^\eta e^{-\frac{\eta^2}{8}}\, d\eta$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^\eta e^{-\frac{t^2}{8}}\, dt$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^{\eta/\sqrt{8}} e^{-y^2}\sqrt{8}\, dy \qquad \left(\text{taking } \frac{t}{\sqrt{8}} = y\right)$$

$$= \frac{\sqrt{8}}{\sqrt{2\pi}} \int_0^{\eta/\sqrt{8}} e^{-y^2}\, dy$$

$$f'(\eta) = \frac{2}{\sqrt{\pi}} \int_0^{\eta/\sqrt{8}} e^{-y^2}\, dy$$

$$\therefore \quad f'(\eta) = \mathrm{erf}\left(\frac{\eta}{\sqrt{8}}\right) \tag{21}$$

Integrating,

$$f(\eta) = \int \mathrm{erf}\left(\frac{\eta}{\sqrt{8}}\right) d\eta$$

$$= \int \left[\frac{2}{\sqrt{\pi}} \int_0^{\eta/\sqrt{8}} e^{-x^2} dx\right] d\eta$$

$$= \frac{2}{\sqrt{\pi}} \int \left[\int_0^{\eta/\sqrt{8}} e^{-x^2} dx\right] d\eta$$

$$= \frac{2}{\sqrt{\pi}} \int \left[\int_0^{\eta/\sqrt{8}} \left(1 - \frac{x^2}{1!} + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \ldots\ldots\right) dx\right] d\eta$$

$$= \frac{2}{\sqrt{\pi}} \int \left(x - \frac{x^3}{3} + \frac{x^5}{5.2!} - \frac{x^7}{7.3!} + \frac{x^9}{9.4!} - \ldots\ldots\right)_0^{\eta/\sqrt{8}} d\eta$$

$$f(\eta) = \frac{2}{\sqrt{\pi}} \int \left(\frac{\eta}{\sqrt{8}} - \frac{\left(\eta/\sqrt{8}\right)^3}{3} + \frac{\left(\eta/\sqrt{8}\right)^5}{5.2!} - \frac{\left(\eta/\sqrt{8}\right)^7}{7.3!} + \ldots\ldots\right) d\eta \tag{22}$$

From (21),

$$f'(\eta) = \mathrm{erf}\left(\frac{\eta}{\sqrt{8}}\right)$$

On integration

$$f'(\eta) = \int e\,r\,f\left(\frac{\eta}{\sqrt{8}}\right) d\eta$$
$$= \eta\, e\,r\,f\left(\frac{\eta}{\sqrt{8}}\right) - \int \eta d\left[e\,r\,f\left(\frac{\eta}{\sqrt{8}}\right)\right] d\eta$$

Using (22), we have

$$f(\eta) = \eta\, e\,r\,f\left(\frac{\eta}{\sqrt{8}}\right) - \int \eta \frac{2}{\sqrt{\pi}}\left[\frac{1}{\sqrt{8}} - \frac{3\eta^2}{3(\sqrt{8})^3} + \frac{5\eta^4}{5\,2!\,(\sqrt{8})^5} - \frac{7\eta^6}{7\,3!\,(\sqrt{8})^7}\right] d\eta$$
$$= \eta\, e\,r\,f\left(\frac{\eta}{\sqrt{8}}\right) - \frac{2}{\sqrt{\pi}}\left[\frac{\eta}{\sqrt{8}} - \frac{\eta^3}{8\sqrt{8}} + \frac{\eta^5}{2!\,8^2\sqrt{8}} + \frac{\eta^7}{3!\,8^3\sqrt{8}}\right] d\eta$$
$$= \eta\, e\,r\,f\left(\frac{\eta}{\sqrt{8}}\right) - \frac{2}{\sqrt{\pi}}\left[\frac{\eta^2}{2\sqrt{8}} - \frac{\eta^4}{4.8\sqrt{8}} + \frac{\eta^6}{6.2.8^2\sqrt{8}} - \frac{\eta^8}{8.3!.8^3\sqrt{8}}\right] +.\ldots\ldots\ldots\ldots\ldots\ldots$$
$$= \eta\, e\,r\,f\left(\frac{\eta}{\sqrt{8}}\right) - \frac{1}{\sqrt{\pi}}\left[\frac{\eta^2}{\sqrt{8}} - \frac{\eta^4}{2.8.\sqrt{8}} + \frac{\eta^6}{6.8^2.\sqrt{8}} - \frac{\eta^8}{4.3.8^3\sqrt{8}}\right] +.\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\eta\, e\,r\,f\left(\frac{\eta}{\sqrt{8}}\right) - \frac{\sqrt{8}}{\sqrt{\pi}}\left[\frac{\left(\eta^2/8\right)}{1!} - \frac{\left(\eta^2/8\right)^2}{2!} + \frac{\left(\eta^2/8\right)^3}{3!} - \frac{\left(\eta^2/8\right)^4}{4!} +.\ldots\ldots\ldots\ldots\ldots\ldots\ldots\right]$$
$$f(\eta) = \eta\, e\,r\,f\left(\frac{\eta}{\sqrt{8}}\right) - \frac{2\sqrt{2}}{\sqrt{\pi}}\left[1 - e^{-\eta^2/8}\right] \qquad (23)$$

From (16)

$$s'' + \frac{\eta}{4}s' = 0$$
$$\text{or} \frac{s''}{s'} = -\frac{\eta}{4}$$

On integration,

$$\int \frac{s''}{s'} d\eta = -\frac{1}{4}\int \eta\, d\eta$$
$$\text{or,}\quad \log\frac{s'}{C} = -\frac{\eta^2}{8}$$
$$\text{or}\quad \frac{s'}{C} = e^{-\frac{\eta^2}{8}}$$
$$s'(\eta) = C\, e^{-\frac{\eta^2}{8}} \qquad (23a)$$

On integration

$$s(\eta) = C\int e^{-\frac{\eta^2}{8}} d\eta + C_1$$

Using first condition in (24) of (20)

C1 = 1

$$\therefore\ s(\eta) = C\int e^{-\frac{\eta^2}{8}} d\eta + 1$$
$$= 1 + C\int_0^{\eta} e^{-\frac{\eta^2}{8}} d\eta$$
$$= 1 + C\int_0^{\eta} e^{-\frac{y^2}{8}} dy$$
$$= 1 + C\int_0^{\eta} e^{-\left(\frac{y}{2\sqrt{2}}\right)^2} dy$$

Put $\quad t = y/2\sqrt{2}$

$$dt = {dy}/{2\sqrt{2}}$$

$$\therefore s(\eta) = 1 + C\int_0^{\frac{\eta}{2\sqrt{2}}} e^{-t^2}\, 2\sqrt{2}\, dt$$

$$s(\eta) = 1 + C\,2\sqrt{2}\int_0^{\frac{\eta}{2\sqrt{2}}} e^{-t^2}dt \qquad (23b)$$

Using 2nd conditions of (20)

$$s(\infty) = 01 + 2\sqrt{2}\,C\int_0^{\infty} e^{-t^2}dt$$

$$0 = 1 + 2\sqrt{2}\,C\,\frac{\sqrt{\pi}}{2}$$

$$-1 = C\sqrt{2\pi}$$

$$\therefore \; C = -\frac{1}{\sqrt{2\pi}}$$

Putting C, in (23a) we get

$$\therefore s'(\eta) = -\frac{1}{\sqrt{2\pi}}e^{-\frac{\eta^2}{8}}$$

Putting C = $\frac{1}{\sqrt{2\pi}}$ in (23b), we get

$$s(\eta) = 1 + 2\sqrt{2}\left(-\frac{1}{\sqrt{2\pi}}\right)\int_0^{\frac{\eta}{2\sqrt{2}}} e^{-t^2}dt$$

or, $$s(\eta) = 1 - \frac{2}{\sqrt{\pi}}\int_0^{\frac{\eta}{2\sqrt{2}}} e^{-t^2}dt$$

$$= 1 - \mathrm{erf}\left({}^{\eta}/_{\sqrt{8}}\right)$$

or, $$s(\eta) = \mathrm{erfc}\left({}^{\eta}/_{\sqrt{8}}\right)$$

Similarly equation (17) is solved for θ'(η) and θ(η).

Equation (17) is solved for θ'(η) and θ(η) in the same way as (16) is solved for s'(η) and s(η)

Thus, the closed from solutions of (15) – (17) under boundary conditions (18) are

$$f'(\eta) = \mathrm{erf}\left({}^{\eta}/_{\sqrt{8}}\right) \qquad (24)$$

$$f(\eta) = \eta\,\mathrm{erf}\left({}^{\eta}/_{\sqrt{8}}\right) - \frac{2\sqrt{2}}{\sqrt{\pi}}\left(1 - e^{-\eta^2/8}\right) \qquad (25)$$

$$s(\eta) = \mathrm{erfC}\left({}^{\eta}/_{\sqrt{8}}\right) \qquad (26)$$

$$s'(\eta) = -\frac{1}{\sqrt{2\pi}}e^{-\eta^2/8} \qquad (27)$$

$$-\theta'(\eta) = \sqrt{\frac{P_r}{2\pi}}e^{-\frac{P_r\,\eta^2}{8}} \qquad (28)$$

$$\theta(\eta) = \mathrm{erfc}\sqrt{\frac{P_r}{8}}\,\eta \qquad (29)$$

Surface heat transfer for ξ = 0 (i.e. at the start of the motion) is given by

$$-\theta'(0) = -\left[\sqrt{\frac{P_r}{2\pi}}e^{-\frac{P_r\,\eta^2}{8}}\right]_{\eta=0}$$

$$= \sqrt{\frac{P_r}{2\pi}} \qquad (30)$$

III. Results and Conclusion

Numeric calculations are made for temperature distribution, velocity field and heat transfer. Variation of velocity distribution f '(η) and s(η) in the directions of x-axis and y-axis against η are shown in table 1 and 2; and shown graphically by curves in figure.

The heat transfer expression (26) is calculated for Prandtl number Pr (.71, 3.02, 10 and 19.6) and the numerical values are listed in Tables .Temperature distribution (27) is computed for Pr (=.71, 3.02, 10 and 19.6) and results of calculations are entered in Tables and illustrated in figures

This shows that the thermal boundary layer thickness decreases as Prandtl number Princreases.The surface heat transfer (28) increases with the Prandtl number Pr. The surface heat transfer (28) at the starting of motion is found to be strongely dependent on the Prandtl number Pr. But it is dependent of magnetic field, buoyancy force Bpand Rotation Parameter Ro.

For non-conduction fluid (M=0) and without boundary force Bp for Steady state (ξ=0) equations (12) and (13) become

$$f'''(\eta) + f(\eta)f''(\eta) + \frac{1}{2}\left[1 - f'^2(\eta) + R_0 s^2(\eta)\right] = 0$$

And

$$s''(\eta) + f(\eta)\, s'(\eta) - f'(\eta)\, s(\eta) = 0$$

These equations are same as that of Lee et. al. This deduction confirms the correctness of our approach.

Table – 1

Boundary layer flow in the vicinity of the forward stagnation point of the spinning and translating sphere

Value of velocity field f '(η) in the direction of x-axis f'(η) = e r t f (η/$\sqrt{8}$)

H	f'(η)
0	0
0.03	0.011128
0.14	0.05637
0.28	0.11246
0.42	0.16800
0.57	0.22270
0.71	0.27633
0.85	0.32863
0.99	0.37938
1.13	0.42839
1.27	0.47548
1.41	0.52050
1.56	0.56332
1.70	0.60386
1.84	0.64203
1.98	0.67780
2.12	0.71116
2.26	0.74210
1.40	0.77667
2.55	0.79691
2.69	0.82089
2.83	0.84270
2.97	0.86244
3.11	0.88021
3.25	0.89612
3.39	0.91031
…..	…..
6.79	0.99931
…..	…..
∞	1

Table 2

η	**s(**η)
0	1.00000
0.03	0.98872
0.14	0.94363
0.28	0.88754
0.42	0.83200
0.57	0.77730
0.71	0.72367
0.85	0.67137
0.99	0.62062
1.13	0.57161

1.41	0.47950
1.70	0.39614
1.98	0.32220
2.12	0.28884
2.40	0.22933
2.97	0.13756
3.25	0.10388
3.54	0.07710
3.82	0.05624
4.10	0.04030
4.53	0.02365
4.81	0.01621
5.09	0.01091
5.23	0.00889
5.52	0.00582
5.80	0.00374
6.08	0.00236
6.51	0.00114
6.65	0.00089
…..	…….

Table 3

η	θ(η)
0	1.00000
0.50	0.83200
1.01	0.67137
1.51	0.52452
2.02	0.39614
2.52	0.28884
3.02	0.20309
3.53	0.13756
4.03	0.089669
4.54	0.05624
5.04	0.03389
5.54	0.01962
6.05	0.01091
6.55	0.00582
7.06	0.00298
7.56	0.00146
8.08	0.00069
……	……….

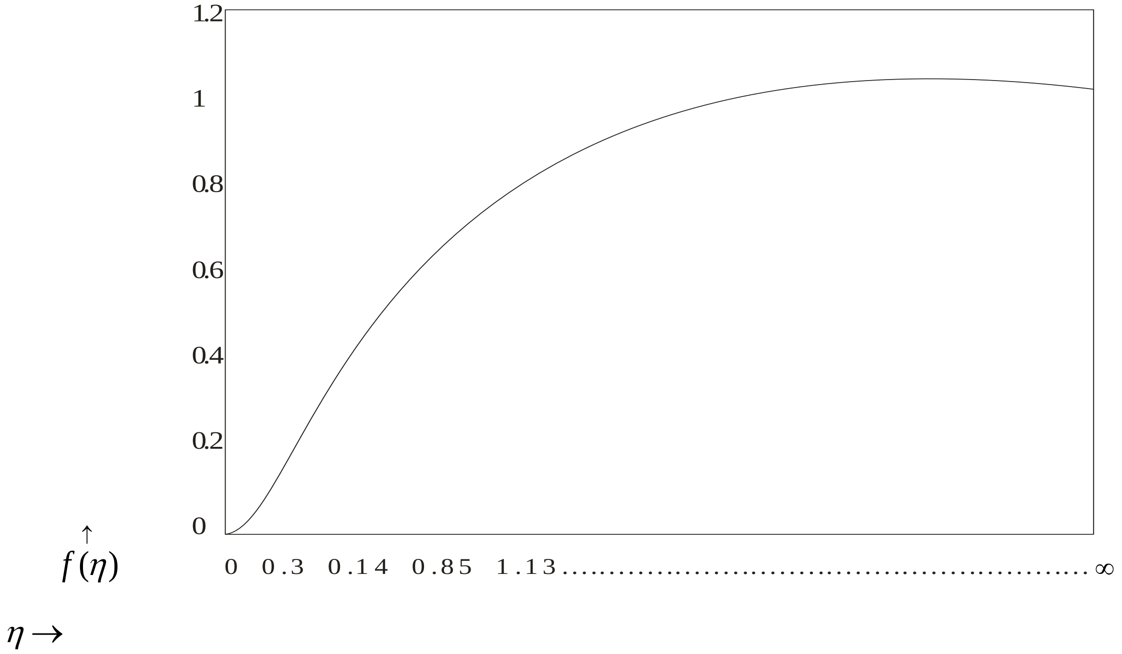


Figure 1 for Table 1

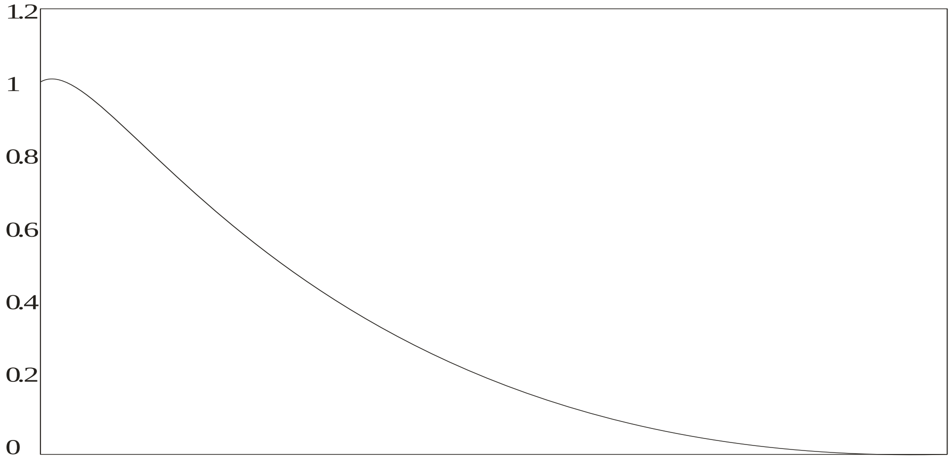


Figure 2 for Table 2

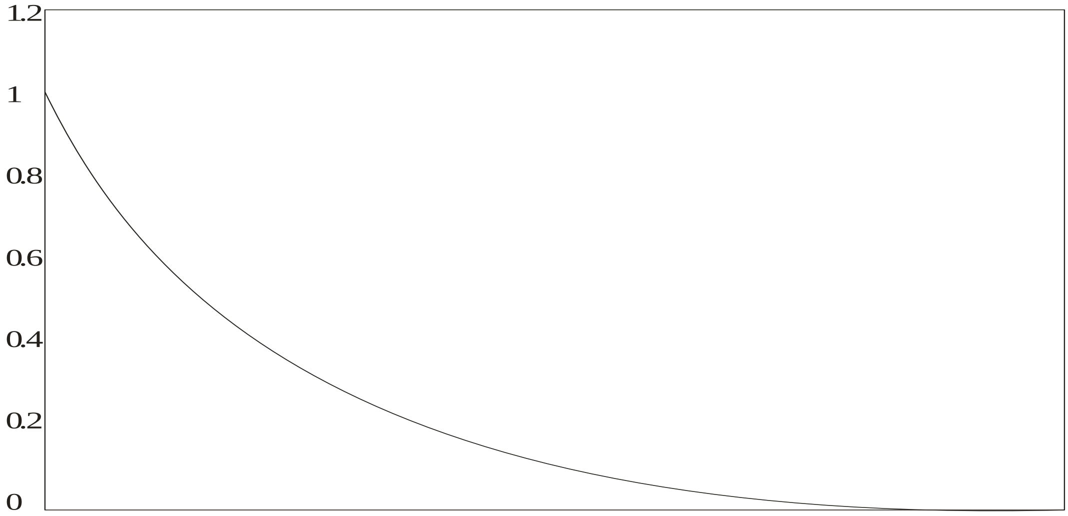


Figure 3 for Table 3

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