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Solving Age-old Transportation Problems by Nonlinear Programming methods

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Abstract: *In the present paper we propose to find solutions to transportation problems by non linear programming methods of Operations Research. No civilization can grow without development of better transportation facilities. Actually time and cost play an important role in its proper development. It may be realized that gradually the means of transportation improved and with the passage of time several methods were developed to minimize transportation cost and time. Here below, we have chosen to minimize the transportation cost.*

Keywords: *Innovations ,Transportation, Linear Programming, Non-linear Programming.*

I. Transportation in Ancient Times

Transportation means physical distribution of goods and services from one destination to another. The first form of transport was, of course, the human foot . Gradually animals such as donkeys, horses, camels etc. were domesticated for transportation. Meanwhile about 3500 BC the wheel was invented in Iraq and then animal-drawn wheeled vehicles were developed as the means of transportation. Till the end of 17th century transportation was slow and uncomfortable. Road transport was developed by the Romans but they were just dirt tracks. Goods were sometimes transported by horses with bags on their sides. Transportation was greatly improved during the 18th century. Transporting goods was also made much easier by digging canals. The contribution of Ancient India in water transportation cannot be ignored as the first canal irrigation system of the world was developed in North India around 2600 BC . People preferred to transport goods by water also. In the mid of 19th century transport was revolutionized by railways they made travel much faster. Although for the first time cars appeared at the end of the 19th century but after the 1st World War, they became cheaper and more common. The only purpose behind the development of all the means of transport since ancient times is to make transportation cheaper and faster. In the 20th century transportation was revolutionized when Operations Research came into existence as a discipline during the World War II. Several OR techniques such as linear programming, non-linear programming etc. were developed that enabled us to minimize transportation cost and time. Keeping these ideas in mind and with the help of these techniques everyday a new means of transport is being designed.

II. Linear Programming in Transportation Problem

Linear programming is a mathematical modelling technique useful for allocation of limited resources to several compelling activities. The transportation problem deals with the distribution of goods from m suppliers to n customers. The linear programming formulation of a typical transportation problem is as follows:
Minimize(Total cost)

$$Z=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

Subject to

$$\sum_{j=1}^{n} xij = ai\,;\, i = 1,2,\ldots\ldots\ldots.m \;\; \text{(Supply constraint)}$$

$$\sum_{i=1}^{m} xij = bi;\, j = 1,2,\ldots\ldots\ldots.n \;\; \text{(demand constraint)}$$

and $x_{ij} \geq 0$ for all i and j
Where c_{ij}= the cost of shipping one unit of the commodity from source I to destination j for each route.
x_{ij} = number of units shipped per route from source i to destination j.

Using simplex method of linear programming , any transportation problem formulated as above can be solved. But the most appropriate and simplest method to solve these transportation problems is well known Vogel Approximation method.

III. Nonlinear Programming in Transportation Problems

Generally it is assumed that in the transportation problems, the per unit transportation cost from a given source to a given distribution centre, is fixed, irrespective of the quantity transported. But in practical situation,volume or quantity discounts are often available. With the increase in volume, the unit transportation cost decreases. Hence the resulting cost C(x) of transporting x units becomes non-linear. If each combination of m sources and n destinations has a similar cost function, that is, the cost of transporting x_{ij} units from source i to destination j is given by a non-linear function $C_{ij}(x_{ij})$, then the final objective function is

$$\text{Minimize } f(x)=\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\, x_{ij}$$

where i= 1,2,3,……….,m
J=1,2,3,………,n
The constraints due to the availability of the sources and requirements at the destinations may remain linear functions or non-linear functions.

1.1. Karush-Kuhn-Tucker Conditions

Nonlinear programming problems with inequality constraints can be solved by KKT conditions. KKT conditions were first developed in 1939 by W.Karush at the University of Chicago. Later on the same conditions were developed independently by W.Kuhn and A.Tucker in 1951. So these conditions are called the KKT conditions, after the men who developed them. The KKT conditions for the non-linear minimization problem are given by

1) $\lambda_i \leq 0$; i=1,2,…..,m
2) $\frac{\partial L}{\partial xj} = 0$; j=1,2,3……..n
3) $\lambda i\, [G_i(x_1,x_2,\ldots\ldots,x_n)-b_i] = 0$; i = 1,2,3,………..m
4) $[G_i(x_1,x_2,\ldots\ldots,x_n)-b_i] \geq 0$; i = 1,2,3,………..m

where L is the lagrangian function and λ_i is the Lagrangian multiplier of the ith constrain**t.**

1.2. Illustration

60m^3 of a granular product are to be transported across a river. The transportation cost across the river is Rs. 100 per trip, irrespective of the amount transported, but there is restriction on the number of trips, which should not be more than 40. The cost of the container depends upon its dimensions as given below:
Cost of bottom = Rs. 10 per sq. metre
Cost of front and back sides = Rs. 10 per sq. metre.
And cost of ends = Rs 20 per sq metre
Find the dimensions of the container so that the sum of the container cost and transportation cost is minimized.
Sol: Let x,y and z be the length , breadth and depth of the container, then
Volume of container =xyz
Number of trips = 60/xyz
Transportation cost =6000/xyz
Cost of container = 10xy+20xz+40yz
The above problem can be formulated as
Minimize f(x,y,z)=6000/xyz +10xy+20xz +40yz
Subject to
g(x,y,z) =60/xyz ≤ 40
and x,y,z ≥ 0
Here both objective function and constraints are non-linear.
The KKT conditions for minima of the above problem are given by

1. $\lambda \leq 0$
2. (i) $\frac{\partial L}{\partial x}$ = 0, i.e. $-6000/x^2yz + 10y + 20z - 40\lambda yz = 0$

(ii) $\frac{\partial L}{\partial y}$ = 0, i.e. $-6000/xy^2z + 10x + 40z - 40\lambda xz = 0$
(iii) $\frac{\partial L}{\partial z}$ = 0, i.e. $-6000/xyz^2 + 20x + 40y - 40\lambda xy = 0$

3. λ (40xyz - 60) = 0
4. 40xyz-60 ≥ 0

Now from (3) either $\lambda = 0$ or $40xyz-60 = 0$. But $40xyz-60=0$ does not satisfy each of the above criteria. Hence $\lambda = 0$. Solving above equations, we get x=5.448140,y=2.72407,z=1.362035 and total cost = Rs. 742.056.

IV. Conclusion

It is all about the ideas that have helped mankind in solving transportation problems. It is true that India's contribution has not been much significant but its contribution cannot be denied. We have demonstrated how non-linear transportation problem can be solved efficiently by minimizing time and cost both.

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