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Temperature Distribution of an Inverse Steady State Thermo Elastic Problem of Thin Rectangular Plate by Numerical Method

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Abstract: *The main purpose of this paper to solve two- dimensional an inverse steady state thermo elastic problem of thin rectangular plate to determine temperature distribution at any point of rectangular plate with given boundary condition by applying numerical technique , finite difference method.*
Keywords: *Finite difference method, Rectangular plate, Thermo elasticity, Temperature distribution.*

I. Introduction

In recent few years, there have been considerable development in the area of the subject, thermo elasticity is motivated by various field of engineering science. the primary objective of the present paper is to gain effective solution and better understanding of temperature distribution in thin rectangular plate. Recently Mukhopadhaya Santwana and Roshan Kumar [5] have studied solution of generalized thermo elasticity of annular cylinder by finite difference method , Khobragade N.W.and Lamba N.K. [1] investigate an thermo elastic problem of thin rectangular plate due to partially distributed heat supply , Khobragade N.W and Wankhede [4] have studied an inverse unsteady state thermo elastic problem to determine the temperature distribution at the boundary of thin rectangular plate, they have used a finite fourier sine transform technique. In most of thermo elastic steady state problem of thin rectangular plate are investigated by integral transform technique.

In present paper an attempt has been made to determine the temperature distribution at any point of an inverse steady state thin rectangular plate is solved by using numerical techniques (finite difference method) and numerical calculation are done with the help of MATLAB.

II. Statement Of Problem

Consider thin rectangular plate occupying the space $0 \le x \le a$, $0 \le y \le b$. The displacement u_x and u_y in x and y direction represented in integral form as ,

$$u_x = \int [\frac{1}{E}\left(\frac{\partial^2 u}{\partial y^2} - v\frac{\partial^2 u}{\partial x^2}\right) + \alpha T\,]dx \qquad (1)$$

$$u_y = \int [\frac{1}{E}\left(\frac{\partial^2 u}{\partial x^2} - v\frac{\partial^2 u}{\partial y^2}\right) + \alpha T\,]dy \qquad (2)$$

Where v and α are poisons ratio and the linear coefficient of thermal expansion of the material of plate respy and $U(\,x, y\,)$ is Airy stress function which satisfy following relation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2}\right)^2 U = -\alpha E \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2}\right) T \qquad (3)$$

Where E is the young's modulus of elasticity and T is the temperature of plate satisfying differential equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial x^2} = 0 \qquad (4)$$

Subject to boundary condition,

$T(0, y) = 0$ (5)

$T(a, y) = g(y)$) unknown (6)

$T(x, 0) = 0$ (7)

$T(x, b) = 0$ (8)

$T(\zeta, y) = f(y), 0 < \zeta < a$ known (9)

The stress component in terms of U are given by

$$\sigma_{xx} = \frac{\partial^2 U}{\partial x^2} \qquad (10)$$

$$\sigma_{yy} = \frac{\partial^2 U}{\partial y^2} \qquad (11)$$

$$\sigma_{xy} = \frac{\partial^2 U}{\partial x \partial y} \qquad (12)$$

Equation (1) to (12) constitute mathematical formulation of problem.

III. Solution Of Problem

let (x, y) plane divided into network of rectangle of side $\Delta x = h, \Delta y = k$ by drawing the line

$x = ih, i = 0,1,2 \ldots \ldots$

$y = jk, j = 0,1,2 \ldots \ldots.$

The point of intersection of family of lines are mesh point. The forward finite difference approximation to the partial derivative with respect to independent variable x and y obtained as follows,

$$\frac{\partial T}{\partial x} = \frac{T_{i+1,j} - T_{i,j}}{h} + o(h)$$

$$= \frac{T_{i+1,j} - T_{i-1,j}}{2h} + o(h^2)$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i-1,j} - 2T_{i,j} + T_{i+1,j}}{h^2} + o(h^2) \qquad (13)$$

$$\frac{\partial T}{\partial y} = \frac{T_{i,j+1} - T_{i,j}}{k} + o(k)$$

$$= \frac{T_{i,j+1} - T_{i,j-1}}{2k} + o(k^2)$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{i,j-1} - 2T_{i,j} + T_{i,j+1}}{k^2} + o(k^2) \qquad (14)$$

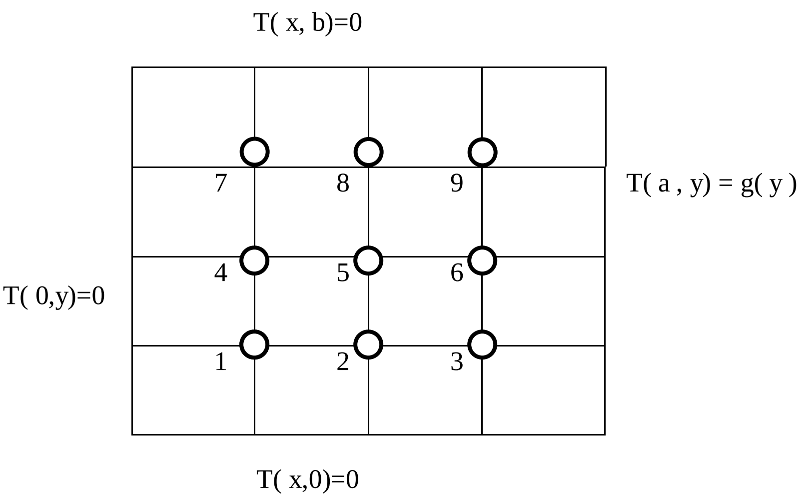
The equation are then replaced by explicit finite difference

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial x^2} = 0$$

$$\frac{1}{h^2}\left[T_{i+1,j} - 2T_{i,j} + T_{i-1,j}\right] + o(h^2) + \frac{1}{k^2}\left[T_{i,j-1} - 2T_{i,j} + T_{i,j+1}\right] + o(k^2) = 0$$

If $h = k$, then

$$T_{i+1,j} + T_{i-1,j} + T_{i,j-1} + T_{i,j+1} - 4T_{i,j} = 0 \qquad (15)$$



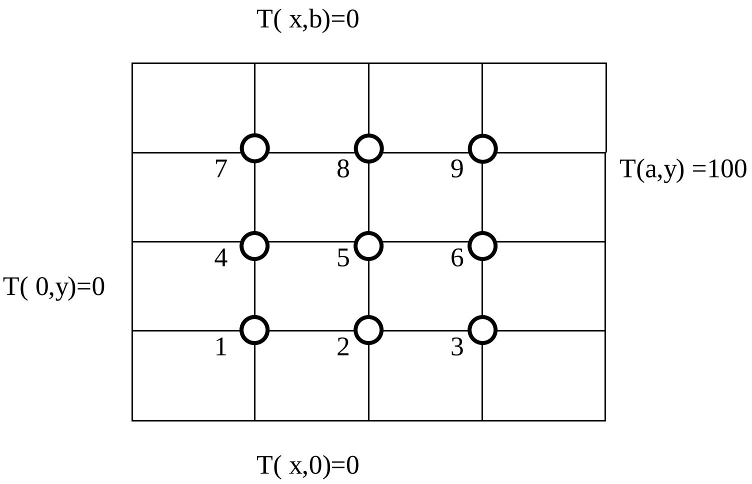
$$4T_1 - T_2 - T_4 = 0$$
$$4T_2 - T_3 - T_1 - T_5 = 0$$
$$4T_3 - T_2 - T_6 = g(y)$$
$$4T_4 - T_5 - T_1 - T_7 = 0$$
$$4T_5 - T_6 - T_4 - T_2 - T_8 = 0$$
$$4T_6 - T_5 - T_3 - T_9 = g(y)$$
$$4T_7 - T_8 - T_4 = 0$$
$$4T_8 - T_9 - T_7 - T_5 = 0$$
$$4T_9 - T_8 - T_6 = g(y)$$

$$\begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 4 & -1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 4 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ g(y) \\ 0 \\ 0 \\ g(y) \\ 0 \\ 0 \\ g(y) \end{bmatrix} \qquad (16)$$

Above system of equation (16) can be solve with the help of MATLAB, to get temperature at nodal point.

IV. Particular Case

Let g(y) =100.

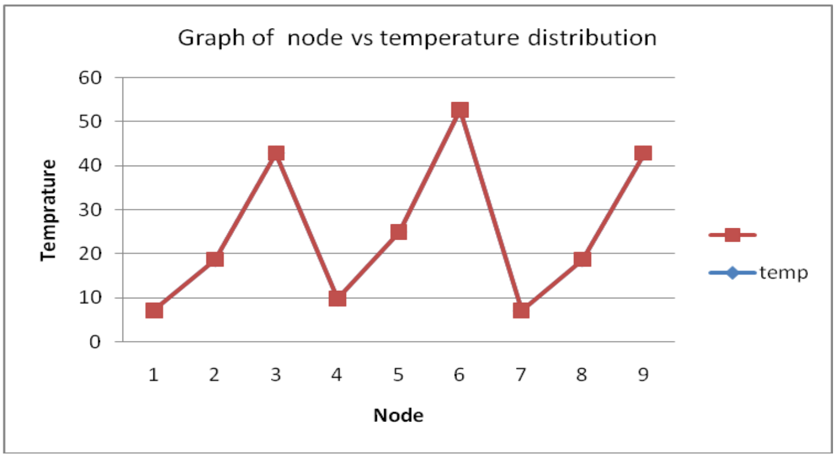


By using finite difference method, we write all 9 equation at nodal point in matrix form ,we get

$$\begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 4 & -1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 4 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 100 \\ 0 \\ 0 \\ 100 \\ 0 \\ 0 \\ 100 \end{bmatrix}$$

Above system of equation can be solve with the help of MATLAB, to get temp at nodal point
$T_1 = 7.14$, $T_2 = 18.75$, $T_3 = 42.85$, $T_4 = 9.82$, $T_5 = 25.00$, $T_6 = 52.67$, $T_7 = 7.14$, $T_8 = 18.75$, $T_9 = 42.85$.

Node	1	2	3	4	5	6	7	8	9
Temp	7.1429	18.75	42.8571	9.8214	25	52.6786	7.1429	18.75	42.8571



From numerical analysis and graph it is observed that,

1. Temperature at nodal point near the hot edge (convective boundary) warmer than the farther away.
2. The accuracy of a solution would be improved if node are closer together
3. The temperature at nodal point near the boundary T(x,0) = 0 and T(x,b) = 0 are similar.

V. Conclusion

In this paper ,we have discussed an inverse steady state thermo elastic problem of a thin rectangular plate $0 < x < a$, $0 < y < b$. The finite difference method is used to obtain a numerical result and may be applied in various industry application.

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