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Existence Theory for Second Order Nonlinear Functional Random Differential Equation in Banach Algebra

Mrs. M. K. Bhosale[1], Dr. R. N. Ingle[2]
[1]*Dept. of MathematicsShri. ChhatrapatiShivajiMaharaj College of Engineering Dist. Ahmednagar/Pune University,Maharashtra. India.*
[2]*Dept. of Mathematics BahirjiSmarak Mahavidyalaya, Basmathnagar /SRTMUN University Nanded, Maharashtra. India.*

Abstract: *In this paper we prove the existence of the solution for the second order nonlinear functional random differential equation in Banach Algebra under suitable condition.*
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I. Introduction

Consider the second order nonlinear functional random differential equation (in short NFRDE)

$$\left(\frac{x(t,\omega)}{f(t,x(t,\omega),\omega)}\right)'' = g(t, x_t(\omega), \omega) \quad \text{a.e. } t\in I$$
$$x_0(\omega) = \varphi_0(\omega)$$
$$x_0'(\omega) = \varphi_1(\omega)$$

for all $\omega \in \Omega$ where

$f: I \times \mathbb{R} \times \Omega \to \mathbb{R}$; $g: I \times C \times \Omega \to \mathbb{R}$; $\varphi_{1,}\varphi_2 : \Omega \to \mathbb{R}$

We shall obtain the existence of the random solution of the above NFRDE in the space $x = C(J,\mathbb{R}) \cap C(I_0,\mathbb{R}) \cap AC(J,\mathbb{R})$ under some suitable condition.

II. Statement Of Problem

Let $\mathbb{R}$ denote the real line and Let $I_0 = [-r, 0]$ and $I = [0, a]$ be two closed and bounded interval in $\mathbb{R}$ for some r> 0 and a > 0. Let $J = I_0$ UI. Let $C(I_0, \mathbb{R})$ denote the space of continuous $\mathbb{R}$ valued function I_0 . We equip the space C = $C(I_0, \mathbb{R})$ with a supremum norm $\|.\|_c$ defined by

$\|x\|_c = \sup_{t\epsilon I_0} |x(t)|$

Clearly C is a Banach Space which is also a Banach Algebra with respect to this norm.

For a given t ϵI define a continuous R-valued function.

$x_t: I_0 \to \mathbb{R}$ by

$x_t(\theta) = (t + \theta)$, $\theta \epsilon I_0$

Let (Ω, A) be a measurable space. Given a random variable $\varphi : \Omega \to C$

We consider a Nonlinear Functional Random Differential Equation (in short NFRDE)

$$\left.\begin{aligned}&\left(\frac{x(t,\omega)}{f(t,x(t,\omega),\omega)}\right)'' = g(t, x_t(\omega), \omega) \quad \text{a.e. } t\in I\\ &x_0(\omega) = \varphi_0(\omega)\\ &x_0'(\omega) = \varphi_1(\omega)\end{aligned}\right\} \quad \text{---(1.1)}$$

for all $\omega \epsilon \Omega$ where $f: I \times \mathbb{R} \times \Omega \to \mathbb{R}$; $g: I \times C \times \Omega \to \mathbb{R}\backslash\{0\}$; $\varphi_{1,}\varphi_2 : \Omega \to \mathbb{R}$.

Theorem 2.1 (Dhage 1) let X be a Banach algebra and let A,B,C: X →X be three operator such that

a) A and C are D- Lipschitzicians with D-functions φ and ψ respectively.
b) B is compact and continuous.
c) $M\,\varphi(r) + \psi(r) < r$, $r > 0$ where M= B(X) = sup {Bx: x ∈ X}.

Then

(i) The operator equation λ A(x/ λ) B x + λ C(x/ λ) = x has a solution for λ = 1 or
(ii) The solution set ϵ={ u ∈ X / λ A(x/ λ) B x + λ C(x/ λ) = x ; 0< λ <1 } is unbounded.

Before going to the main result of this paper, we state the following two useful lemmas.

Lemma 2.1: (Dhage 5): Assume that all the conditions of theorem 2.1 hold then map T: X → X define by T x = Ax B x + C x is continuous on X.

Lemma 2.2: (Dhage 6): Assume that all the conditions of theorem 2.1 hold then the set Fix(T) = { x∈ X / Ax B x + C x = x} is compact.

Theorem 2.2 : let X be a separable Banach Algebra and let A,B,C: $\Omega \times X \rightarrow X$ be three random operator satisfying for each $\omega \in \Omega$

a) $A(\omega)$ and $C(\omega)$ are D- Lipschitzicians with D-functions $\varphi_A(\omega)$and $\varphi_{Ac}(\omega)$ respectively .
b) $B(\omega)$ is compact and continuous.
c) $M(\omega)\varphi_A(\omega)(r) + \varphi_C(\omega)(r) < r, \quad r > 0$ for all $\omega \in \Omega$ where $M(\omega)=\|B(\omega)(x)\|$
d) The set ε ={ u ∈ X / λ(ω)A(ω) (u/ λ) B(ω)u + λ(ω)C(ω) (u/ λ) = u } is bounded for all measurable λ: $\Omega \rightarrow \mathbb{R}$ with $0 < \lambda(\omega) < 1$ Then the random equation

A (ω) x B (ω) x + C (ω) x = x ---(2.1)

has a random solution

Corollary 2.1 : Let X be a separable Banach Algebra and let A,B,C: $\Omega \times X \rightarrow X$ be three random operator satisfying for each $\omega \in \Omega$

a) $A(\omega)$ and $C(\omega)$ are D- Lipschitzicians with Lipschitz costant $\alpha(\omega)$and $\beta(\omega)$ respectively
b) $B(\omega)$ is compact and continuous.
c) $\alpha(\omega)M(\omega) + \beta(\omega) < 1$ for all $\omega \in \Omega$where $M(\omega)=\|B(\omega)(x)\|$
d) The set ε ={ u ∈ X / λA(ω) (u/ λ) B(ω)u + λ(ω)C(ω) (u/ λ) = u } is bounded for all 0< λ<1

Then the random equation (2.1) has a random solution and the set of such random solution is compact.

On taking C(ω) = 0 in theorem (2.2) we obtain

Theorem 2.3: Let X be a separable Banach Algebra and A,B: $\Omega \times X \rightarrow X$ be two random operator satisfying for each $\omega \in \Omega$

a) $A(\omega)$ is D- Lipschitzicians with D-functions $\varphi_A(\omega)$.
b) $B(\omega)$ is compact and continuous.
c) $M(\omega)\varphi_A(\omega) < r, \quad r > 0$ for all $\omega \in \Omega$ where $M(\omega)=\|B(\omega)(x)\|$
d) The set ϵ ={ u ∈ X / λ(ω)A(ω) (u/ λ) B(ω)u = u } is bounded for all measurable λ: $\Omega \rightarrow \mathbb{R}$ with $0 < \lambda(\omega) < 1$

Then the random equation

A(ω)x B(ω)x = x ---(2.2)

has a random solution.

Corollary 2.2 : Let X be a separable Banach Algebra and let A,B: $\Omega \times X \rightarrow X$ be two random operator satisfying for each $\omega \in \Omega$

a) $A(\omega)$ is D- Lipschitzicians with Lipschitz costant $\alpha(\omega)$.
b) $B(\omega)$ is compact and continuous.
c) $\alpha(\omega)M(\omega) < 1$ for all $\omega \in \Omega$where $M(\omega)=\|B(\omega)(x)\|$
d) The set ϵ ={ u ∈ X / λ A(ω) (u/ λ) B(ω)u = u } is bounded for all 0< λ(ω)< 1

Then the random equation (2.2) has a random solution and the set of such random solution is compact.

In the following section we shall prove an existence of the random solution of a nonlinear functional random differential equation (1.1) in Banach Algebra.

III. Existence Theory For Random Solution

Let M(J,ℝ), B(J,ℝ), BM(J,ℝ)and C(J,ℝ) denote respectively the space of all measurable, bounded, bounded and measurable and continuous real-valued function on J. Notice that C(J,ℝ)⊂ BM(J,ℝ)⊂ M(J,ℝ)

we shall obtain the existence of the random solution of the NFRDE (1.1) is the space X= C(J,ℝ)∩ C(I_0 ,ℝ)∩ AC(J,ℝ)under some suitable condition .

Define a norm ‖.‖ in X by

$\|x\|= \max_{t \in J} |x(t)|$ -- (3.1)

Clearly X is a separable Banach Algebra with this maximum norm. By $L'(J, \mathbb{R})$ we denote the space of all Lebesgue integral real valued function on J equipped with a norm $\|.\|_{L'}$ given by

$\| x \|_{L'} = \int_{t_0}^{t_1} |x(t)| ds$. --- (3.2)

Now the NFRDE(1.1) is equivalent to the functional Random Integral equation (in short FRIE)

$$x(t,\omega) = \begin{cases} f(t, X(t,\omega),\omega)[\, \varphi_0(0,\omega) + \varphi_1(0,\omega)t + \int_0^t g(s, x_s(\omega), \omega)\, ds \,], t \epsilon I \\ \varphi(t,\omega) \text{ if } t \epsilon I_0 \end{cases} \quad \text{---(3.3)}$$

i.e

$$x(t,\omega) = \begin{cases} \varphi_1(0,\omega) t f(t, x(t,\omega),\omega) + \\ \quad f(t, x(t,\omega),\omega)[\, \varphi_0(0,\omega) + \int_0^t g(s, x_s(\omega), \omega)\, ds \,], t \epsilon I \\ \varphi(t,\omega) \text{ if } t \epsilon I_0 \end{cases}$$

We need the following definition

Definition 3.1.:- A mapping β: $J \times C \times \Omega \to R$ is said to satisfy a condition of ω-Caratheodory or simply called ω-Caratheodory if for each $\omega \epsilon \Omega$

(i) $t \to \beta(t, x, \omega)$ is measurable for each $x \epsilon C$.

(ii) $x \to \beta(t, x, \omega)$ is continuous almost everywhere $t \epsilon I$

Further a ω-Caratheodory function β is called $L_\omega{}'$ –Caratheodory if

(iii) there exist a function h: $\Omega \to L'(J, \mathbb{R})$ such that

$|\beta(t, x, \omega)| \leq h(t,\omega)$ a. e. $t \epsilon I$ for all $x \epsilon \mathbb{R}$ and $\omega \epsilon \Omega$

We consider the following hypothesis in the sequel.

(H_1) The function $q:\Omega \to C(J,\mathbb{R})$ is measurable.

(H_2) The function $f:\Omega \to C(J \times \mathbb{R},\mathbb{R})$ is measurable and there exist a function$\alpha_1:\Omega \to B(I,\mathbb{R})$ with bound $\|\alpha_1(\omega)\|$ satisfying for each $\omega \epsilon \Omega$.

$|f(t, x, \omega) - f(t, y, \omega)| \leq \alpha_1(t, \omega)|x - y|$ a. e. $t \epsilon I$ for all $x, y \epsilon C$

(H_3) The function $\omega \to g(t, x, \omega)$ is measurable for all $t \epsilon I$ and

(H_4) The function $g(t, x, \omega)$ is $L_\omega{}'$ –Caratheodory.

(H_5) There exist function γ: $\Omega \to L'(I,\mathbb{R})$ with $\gamma(t,\omega) > 0$ a.e. $t \epsilon I$, for all $\omega \epsilon \Omega$ and conditions non decreasing function $\psi: [0,\infty) \to (0,\infty)$ satisfying for each $\omega \epsilon \Omega$.

$|g(t, x, \omega)| \leq \gamma(t,\omega)\psi(|x|)$ a.e. $t \epsilon J$ --- (3.4)

for all $x \epsilon$ C.

Theorem 3.1: Assume that the hypothesis (H_1) – (H_5) holds. Suppose further that

$\int_{C_1(\omega)}^{\infty} \frac{ds}{\psi(s)} > C_2(\omega) \| r \|_{L'}$ ---(3.5)

where

$$C_1(\omega) = \frac{[1+F(\omega)]\|\varphi(\omega)\|_C}{1-\|\alpha_1(\omega)\|[\|\varphi(\omega)\|_C+\|h(\omega)\|_{L'}]}$$

$$C_2(\omega) = \frac{F(\omega)}{1-\|\alpha_1(\omega)\|[\|\varphi(\omega)\|_C+\|h(\omega)\|_{L'}]}$$

Then the NFRDE (1.1) has a random solution on J.

Proof :- Let X = $C(J,\mathbb{R})$ and define three mapping A, B,C: $\Omega \times X \to X$ by

$$\begin{cases} f(t,x(t,\omega),\omega) \quad \text{if } t \epsilon I \end{cases} \quad \text{---- (3.6)}$$

A(ω)x(t) =

1 if t ϵ I_0

and

$$B(\omega)x(t) = \begin{cases} \varphi_0(0,\omega) + \int_0^t g(s, x_s(\omega), \omega)\, ds & \text{if } t \in I \\ \varphi(t,\omega) & \text{if } t \in I_0 \end{cases} \qquad \text{----(3.7)}$$

and

$$C(\omega)x(t) = \begin{cases} \varphi_1(0,\omega)t\, f(t, x(t,\omega), \omega) & \text{if } t \in I \\ \varphi_1(t,\omega) & \text{if } t \in I_0 \end{cases} \qquad \text{----(3.8)}$$

Then the FRIE (3.3) is transformed into the random operator equation

A(ω)x(t) B(ω)x(t) + C(ω)x(t) = x(t, ω) ---(3.9)

for t ϵ J and $\omega \epsilon \Omega$.

We shall show that the operator A(ω), B(ω) and C(ω) satisfy all the conditions of corollary 2.1 on X. This will be done in the following steps.

Step I :- First we show that A(ω) and B(ω) are random operator on X. Since the function f(t,x,ω) is measurable in ω for all t ϵ I and x ϵ ℝ and since constant function is measurable on Ω the function $\omega \to$A(ω)x is measurable for all x ϵ X. Hence A(ω) is a random operator on X . Now by (H_3) the function $\omega \to$g(t,x,ω) is measurable for all t ϵ I and x ϵ C. We know that the Riemann integral in a limit of a finite sum of measurable function, which is again measurable.

Therefore the function $\omega \to \int_0^t g(s, x_s(\omega), \omega)\, ds$ is measurable . Hence B(ω) is random operator on X.

Similarly it is shown that C(ω) is a randome operator on X.

Again since the function

$$t \to A(\omega)x(t) = \begin{cases} f(t, x(t,\omega), \omega) & \text{if } t \in I \\ 1 & \text{if } t \in I_0 \end{cases}$$

$$t \to B(\omega)x(t) = \begin{cases} \varphi_0(0,\omega) + \int_0^t g(s, x_s(\omega), \omega)\, ds & \text{if } t \in I \\ \varphi(t,\omega) & \text{if } t \in I_0 \end{cases}$$

$$t \to C(\omega)x(t) = \begin{cases} \varphi_1(0,\omega)t\, f(t, x(t,\omega), \omega) & \text{if } t \in I \\ \varphi_1(t,\omega) & \text{if } t \in I_0 \end{cases}$$

are continuous. The function A(ω)x(t) ,B(ω)x(t) and C(ω)x(t) are continuous and hence bounded and measurable on J for each $\omega \epsilon \Omega$. Hence A(ω), B(ω), and C(ω) define the random operator A, B,C: Ω×X→X.

Step II: Next we show that A(ω) is Lipschitzian random operator on X. Let x, yϵ X. Then by (H_1)

$|A(\omega)x(t) - A(\omega)y(t)| = |f(t, x(t,\omega), \omega) - f(t, y(t,\omega), \omega)| \le \| \alpha_1 \| \| x(\omega) - y(\omega) \|$

for all t ϵ I.

Similarly

| A(ω)x(t) - A(ω)y(t) | =0 $\leq\| \alpha_1 \| \| x(\omega)-y(\omega) \|$
for all t ϵ I_0.
Thus

|A(ω)x(t) - A(ω)y(t) | $\leq\| \alpha_1(\omega) \| \| x(\omega)-y(\omega) \|$ for all tϵJ and $\omega\epsilon\Omega$.

Taking the maximum over t in the above inequality. We obtain
|A(ω)x(t) - A(ω)y(t) | $\leq\| \alpha_1 \| \| x(\omega)-y(\omega) \|$

This shows that A (ω) is a Lipschitzian random operator on X with Lipschitz constant $\|\alpha_1(\omega)\|$.
Similarly it is shown that C(ω) is a Lipschitzian random operator on X with Lipschitz constant $\|\beta_1(\omega)\|$.

Step III.: Next we show that B(ω) is a continuous and compact random operator on X. Using the standard argument as in Granas et.al[9] it is shown that B(ω) is a continuous random operator on X. To show that B(ω) is compact. It is sufficient to show that B(ω)(x) is uniformly bounded and equi-contiuous set in X for each $\omega\epsilon\Omega$. First we show that B(ω)(x) is uniformly bounded for each $\omega\epsilon\Omega$. Let xϵ X be arbitrary . Thus

$$B(\omega)x(t)=\begin{cases}\varphi_0(0,\omega)+\int_0^t g(s,x_s(\omega),\omega)\,ds & \text{if } t\,\epsilon\, I\\ \varphi(t,\omega) & \text{if } t\,\epsilon\, I_0\end{cases}$$

for all $\omega\epsilon\Omega$ Since g is $L_x^{'}(\omega)$- Caratheodeory
We have

$$|B(\omega)x(t)| \leq \| \varphi_0(\omega) \|_C+\int_0^t g(s,x_s(\omega),\omega)\,ds$$
$$= \| \varphi_0(\omega) \|_C + \| h(\omega) \|_{L'}$$

Taking the maximum over t, one obtains $\| B(\omega)x \| \leq$ K for all xϵX where
K= $\| \varphi_0(\omega) \|_C + \| h(\omega) \|_{L'}$

This shows that $B(\omega)(x)$ is a uniformly bounded subset of X for each. Secondly we show that $B(\omega)(x)$ is an equicontinuous set in X for each $\omega\epsilon\Omega$.
Now there are three cases

Case I :- Let t,$\tau \in I$ Then for any x$\in X$ we have by (3.7)

$$|B(\omega)x(t)\text{-} B(\omega)x(\tau)| \leq \left| \int_0^t g(s,x_s(\omega),\ \omega)\,ds-\int_0^\tau g(s,x_s(\omega),\ \omega)\,ds \right|$$
$$\leq \left| \int_\tau^t g(s,x_s(\omega),\omega)\,ds \right|$$
$$\leq \left| \int_\tau^t h(s,\omega)\,ds \right|$$
$$=| p(t,\omega)-p(\tau,\omega) |$$

Where p(t) = $\int_0^t h(s,\omega)\,ds$
Now p is a continuous function on a compact interval I. So it is uniformly continuous there and hence
$|B(\omega)x(t)\text{-} B(\omega)x(\tau)| \rightarrow 0$ as t$\rightarrow \tau$ for each $\omega\epsilon\Omega$

Case II :- Again let $\tau\epsilon I_0$ and t ϵ I then we have

$$|B(\omega)x(t)\text{-} B(\omega)x(\tau)| \leq |\varphi_0(0,\omega)\text{-}\varphi_0(\tau,\omega)| +\int_\tau^t g(s,x_s(\omega),\ \omega)\,ds$$
$$\leq |\varphi_0(0,\omega)\text{-}\varphi_0(\tau,\omega)| + |p(t,\omega)\text{-}p(\tau,\omega)|$$

Where the function p defined above. Again φ_0 is a continuous on compact interval I_0
And the function p is continuous on compact interval I, so they are uniformly continuous and hence $|B(\omega)x(t)\text{-} B(\omega)x(\tau)| \rightarrow 0$ as t$\rightarrow \tau$

Case III :- similarly t, $\tau \in I_0$ Thus we have
$|B(\omega)x(t)\text{-} B(\omega)x(\tau)| = |\varphi(t,\omega)\text{-}\ \varphi(\tau,\omega)| \rightarrow 0$ as t$\rightarrow \tau$

Thus in all three case we have
$|B(\omega)x(t) - B(\omega)x(\tau)| \to 0$ as $t \to \tau$ for $t, \tau \in I_0$ and $\omega \epsilon \Omega$.
Hence $B(\omega)(x)$ is equicontinuous set in X for each $\omega\epsilon\Omega$. This further in view of Arzela Ascolli Theorem implies that $B(\omega)(x)$ is compact for each $\omega\epsilon\Omega$. Hence $B(\omega)$ is a continuous and compact random operator on X

Step IV :- Here

$$\begin{aligned}M(\omega) &= \| B(\omega)(x) \| \\ &= \sup \{ \| B(\omega)(x) \| : x\epsilon X \} \\ &= sup_{x\in X}\{max_{t\in J}|B(\omega)x(t)|\} \\ &\leq \| \varphi(\omega) \|_C + sup_{x\in X}\left\{max_{t\in J}\left|\int_0^t g(s,x_s(\omega),\ \omega)\, ds\right|\right\} \\ &= \| \varphi(\omega) \|_C + \| h(\omega) \|_{L'}\end{aligned}$$

Therefore
$\| \alpha_1(\omega) \| M(\omega) + \| \beta_1(\omega) \| = \| \alpha_1(\omega) \| [\| \varphi(\omega) \|_C + \| h(\omega) \|_{L'}] + \| \beta_1(\omega) \|_C$ for all $\omega\epsilon\Omega$

Step V :- Finally we show that condition (d) of corollary (2.1) is satisfied . Let $u\epsilon$ E be arbitrary. Then we have for all $\omega\epsilon\Omega$.

$$\lambda u(t,\omega) = A(\omega)\, u(t)B(\omega)u(t) + C(\omega)u(t)$$

$$= \begin{cases} \varphi_1(0,\omega)tf(t,u(t,\omega),\omega) + f(t,u(t,\omega),\omega)[\ \varphi_0(0,\omega) + \int_0^t g(s,u_s(\omega),\ \omega)\, ds\ ], t\epsilon I \\ \varphi(t,\omega) \text{ if } t\epsilon I_0 \end{cases}$$

For some real number $\lambda > 1$
Therefore

$$|u(t,\omega)| < \lambda^{-1}\varphi(t,\omega) + \lambda^{-1}[\varphi_1(0,\omega)tf(t,u(t,\omega),\omega) + f(t,u(t,\omega),\omega)[\ \varphi_0(0,\omega) + \int_0^t g(s,u_s(\omega),\omega)\, ds\ ]$$

$$\leq \| \varphi(\omega) \|_C + \lambda^{-1}|f(t,u(t,\omega),\omega)| + |\ f(t,u(t,\omega),\omega)|[\ |\varphi_0(0,\omega)| + \int_0^t h(s,\omega)ds\ ]$$

$$\leq C_1(\omega) + C_2(\omega) + \int_0^t \gamma(s,\omega)\,\Psi(\| u_s(\omega) \|_C)\, ds \qquad \text{---(3.10)}$$

Where

$$C_1(\omega) = \frac{[1 + F(\omega)\|\varphi(\omega)\|_C + F(\omega)]}{1 - \|\alpha_1(\omega)\|\left[\|\varphi(\omega)\|_C + \|h(\omega)\|_{L'}\right] - \|\beta_1(\omega)\|_C}$$

And

$$C_2(\omega) = \frac{F(\omega)}{1 - \|\alpha_1(\omega)\|\left[\|\varphi(\omega)\|_C + \|h(\omega)\|_{L'}\right]}$$

Let m $(t,\omega) = sup_{t\in[-r,t]}|u(t,\omega)|$ Then one has $|u(t,\omega)| \leq m(t)$ and $\| u_t(\omega) \|_C \leq m(t,\omega)$ for all $t\epsilon I$ and $\omega\epsilon\Omega$. Then there is a $t^* \in[-r,t]$ such that Let $m(t,\omega) = |u(t^*,\omega)|$ for all $\omega\epsilon\Omega$
Hence from inequality (3.10) it follows that

$$\begin{aligned}m(t,\omega) &= |u(t^*,\omega)| \\ &\leq C_1(\omega) + C_2(\omega)\int_0^t \gamma(s,\omega)\,\Psi(\| u_s(\omega) \|_C)\, ds \\ &\leq C_1(\omega) + C_2(\omega)\int_0^t \gamma(s,\omega)\,\Psi(m(s,\omega))\, ds\end{aligned}$$

Put $w(t,\omega) = C_1(\omega) + C_2(\omega)\int_0^t \gamma(s,\omega)\,\Psi(m(s,\omega))\, ds$

$$\left.\begin{aligned}&w'(t,\omega) = C_2(\omega)\gamma(t,\omega)\,\Psi(m(t,\omega)) \\ &w(0,\omega) = C_1(\omega) \\ &\text{This further implies that} \\ &w'(t,\omega) \leq C_2(\omega)\gamma(t,\omega)\,\Psi(m(t,\omega))\end{aligned}\right\}$$

w(0,ω)= $C_1(\omega)$
OR , ----(3.11)

$\frac{w'(t,\omega)}{\Psi(m(t,\omega))} \leq C_2(\omega)\gamma(t,\omega)$
w(0,ω)= $C_1(\omega)$

Integrating from 0 to t yield that
$\int_0^t \frac{w'(t,\omega)}{\Psi(m(t,\omega)} ds \leq C_2(\omega) \int_0^t \gamma(s,\omega)\, ds$

By changing the variable formula we get
$$\int_{C1(\omega)}^{w(t,\omega)} \frac{ds}{\Psi(s)} \leq C_2(\omega) \int_0^t \gamma(s,\omega)\, ds$$
$$\leq C_2(\omega) \int_0^a \gamma(s,\omega)\, ds$$
$$= C_2(\omega) \parallel \gamma(\omega) \parallel_{L'}$$
$$< \int_{C1(\omega)}^{\infty} \frac{ds}{\Psi(s)}$$
Now by an application of mean value theorem yield that there is a constant M >0 such that $w(t,\omega) \leq m$ for all t ϵ I and ωϵ Ω.

This further implies that |u(t,ω) | ≤ M for all t ϵ I and ωϵΩ.
Hence the set ε is bounded and condition (d) of corollary (2.1) yield

Hence the random operator equation (3.9) and consequently by the FRDE (1.1) has a random solution . This completes the proof.

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