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General Manpower and Machine System with Markovian Production and General Sales

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Abstract: *In this paper, Manpower and Machine System breaks down when both of them are in failed state and if one is alone in failed state, the failed one is hired till the other one also fails.Man power system breaks down due to attrition and machine breaks down due to shocks.The entire system has life time which is the maximum of the individuals. During the operation time, the system produces products for sale. When the system fails, the recruitments, the repairs and sales are attended. We study two models In Model-I, the vacancies caused by departure of employees are filled up one by one and in Model-II,when the operation time is more than a threshold time, the recruitment are done all together and when the operation time is less than the threshold time, the recruitments are done one by one. Joint Laplace transform of the pdf of the operation time, the repair time of the machine, therecruitment time and sales time has been found. Their expectations and covariance are presented with numerical illustration .*

Keywords: *Manpower Machine system, attrition, shocks, Joint Laplace transform*

I. Introduction

In an organization, the total flow out of the Manpower System (MPS) is termed as shortage, The flow out of the MPS of an organization happens due to resignation, dismissal and death. The shortages that have occurred due to the outflow of manpower are compensated by recruitment. But recruitment cannot be made frequently since it involves cost. Therefore, the MPS is allowed to undergo Cumulative Shortage Process (CSP). The basic idea is that accumulating random amount of shortages due to successive attritions leads to the breakdown of the system when the total shortage crosses a random threshold level. The breakdown point or the threshold can also be interpreted as that point at which immediate recruitment is necessitated.

The shortage of MPS depends on individual propensity to leave the organization, which in turn depends on various factors as discussed before. Manpower Planning models by Grinold and Marshall [2], For statistical approach one may refer to Bartholomew [1]. Lesson [6] has given methods to compute shortages (Resignations, Dismissals, Deaths). Markovian models are designed for shortage and promotion in MPS by Vassiliou [11]. Subramanian. V [10] has made an attempt to provide optimal policy for recruitment, training, promotion and shortages in manpower planning models with special provisions such as time bound promotions, cost of training and voluntary retirement schemes. For other manpower models one may refer Setlhare K [9]. For three characteristics system in manpower models one may refer to Mohan C and Ramanarayanan R [8].

Esary et al. [3] have discussed that any component or decice, when exposed to shocks which cause damage to the device or system, is likely to fail when the total accumulated damage exceeds a level called threshold. Stochastic Analysis of Manpower levels affecting business with varying Recruitment rate, K.Hari Kumar, P.Sekar and R.Ramanarayanan[4].

Manpower System with Erlang departure and one by one Recruitment, Hari Kumar.K [5]. For the study of Semi Markov Models for Manpower planning one may refer to the paper by Meclean [7].

In this paper, the manpower and the machine system fails when both are in failed state assuming the failed one is hired till the other also fails so that both are reequipped together. Two models are treated. In Model-I,the vacancies caused by departure of employees are filled up one by one and in Model-II,when the operation time is more than a threshold time the recruitments are done all together and when the operation time is less than the threshold time, the recruitments are done one by one.Joint Laplace transform of the pdf of the operation time, the repair time of the machine, the recruitment time and sales time has been found. Their expectations and covariance are presented with numerical illustration.

Model 1

Assumptions

1. Inter-departure time of employees are independent and identically(i.i.d) distributed random variables ‘F’ with Cdf F(x) and pdf f(x). The Manpower collapses with probability p when an employee leaves and with probability q manpower system continues operation, with p+q=1.

2. The machine attended by manpower gets shocks with inter-occurrences time distribution as exponential with parameter λ.The damage caused by shocks causes failure with probability α and with probability β the machine survives it and continues itsoperation, with $\alpha + \beta = 1$.
3. The manpower-machine system fails when both manpower and machine are in failed state. When either manpower or machine alone is in failed state, the failed one is hired till both became unavailable. The hiring stops when the other system also fails.
4. During the operation time the manpower-machine system produces one at a time with exponential inter-production time distribution with parameter μ.
5. When the manpower-machine system fails, the damages caused to the machine are repaired one by one with repair time distribution function R with Cdf R(y) and pdf r (y).
6. The sale time of products are i.i.d random variables G with Cdf G(w) and g (w).The sale time starts when production is stopped and sales are done one by one.
7. The vacancies caused by the departure of employees are filled up one by one with recruitment time Cdf H(z) and pdf h(z).

Analysis

To study the model 1, the joint probability density function of four variables namely $\left(X, \hat{R}, \hat{R}_1, \hat{S}\right)$, where X is the operation time of the manpower-machine system $\hat{\mathrm{R}}$ is the total repair time of the machine , $\hat{\mathrm{R}}_1$ is the total recruitment times of employees , and $\hat{\mathrm{S}}$ is the total sales time of products produced, may be written as follows.

We may note that the operation time X is the maximum of the life time of the machine and manpower service time. When K_1 damages have occurred to the machine, K_2 employees have left and K_3 products are produced during the operation time of manpower machine system, then the variables $\hat{R}, \hat{R}_1$ and $\hat{S}$ are given by

$\hat{R} = R_1 + R_2 + R_3 + ... + \mathrm{R}_{K_1}$, $\hat{\mathrm{R}}_1 = H_1 + H_2 + H_3 + \ldots + \mathrm{H}_{k_2}$ *and* $\hat{\mathrm{S}} = G_1 + G_2 + G_3 + \ldots + \mathrm{G}_{k_3}$ Here R_i , i = 1,2,3,. . . , H_j , j =1,2,3, . . . and G_n , n =1,2,3, . . . are i.i.d random variables with Cdfs R(y), H(z) and G(w) respectively.

We find the pdf of $\left(X, \hat{R}, \hat{R}_1, \hat{\mathrm{S}}\right)$ as

$$f(x,y,z,w) = \sum_{i=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=0}^{\infty}\left[f_n(x) q^{n-1} p h_n(z) \int_0^x e^{-\lambda u} \frac{(\lambda u)^{i-1}}{\underline{|i-1}} \beta^{i-1} \lambda\alpha du r_i(y) e^{-\mu x} \frac{(\mu x)^k}{\underline{|k}} g_k(w) \right] +$$

$$\sum_{i=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=0}^{\infty}\left[e^{-\lambda x} \frac{(\lambda x)^{i-1}}{\underline{|i-1}} \beta^{i-1} \lambda\alpha r_i(y) F_n(x) p q^{n-1} h_n(z) e^{-\mu x} \frac{(\mu x)^k}{\underline{|k}} g_k(w) \right] \quad ....(1)$$

The term of the first triple sum of equation (1) is the part of the pdf that the manpower system fails on the nth departure of an employees, when the machine is in failed state with i-damages and k products are produced during operation time. The n vacancies that occur during the operation time are filled up. The n recruitments, i repairs and k sales completions areindicated by corresponding pdfs.The suffix letter indicates the corresponding convolution of pdf or Cdf as the case may be.

The term under the second triple sum is the part of the pdf that the machinefails on the i-th damage when manpower system has collapsed already on the n-th departure of employees and k productsare produced during the operationtime. Therecruitments, repairs and sales completions are represented by corresponding pdfs.

The Laplace transform of the pdf f(x,y,z,w) of four variables is given by

$$f^*(\xi,\eta,\varepsilon,\delta) = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-\xi x - \eta y - \varepsilon z - \delta w} f(x,y,z,w) dxdydzdw \quad ... \quad (2)$$

Using the structure in the equation (1), equation (2) reducestoasingle integral

$$f^*(\xi,\eta,\varepsilon,\delta)=\int_0^\infty e^{-\xi x}\left[\sum_{n=1}^{\infty} f_n(x)\,pq^{n-1}h^{*n}(\varepsilon)\int_0^x e^{-\lambda u\left(1-\beta r^*(\eta)\right)}du\,\lambda\alpha r^*(\eta)e^{-\mu x\left(1-g^*(\delta)\right)}\right]dx\ +$$

$$\int_0^\infty e^{-\xi x}\left[\sum_{n=1}^{\infty} F_n(x)\,pq^{n-1}h^{*n}(\varepsilon)\lambda\alpha r^*(\eta)e^{-\lambda x\left(1-\beta r^*(\eta)\right)}e^{-\mu x\left(1-g^*(\delta)\right)}\right]dx \quad ...(3)$$

$$Let\ \ \psi=\xi+\mu\left(1-g^*(\delta)\right) \quad ...(4)$$

and $\chi=\xi+\lambda\left(1-\beta r^*(\eta)\right)+\mu\left(1-g^*(\delta)\right)$ ....(5)

Then,

$$f^*(\xi,\eta,\varepsilon,\delta)=\sum_{n=1}^{\infty}\int_0^\infty \frac{\left(e^{-\psi x}-e^{-\chi x}\right)}{\left(1-\beta r^*(\eta)\right)}\alpha r^*(\eta)f_n(x)\,pq^{n-1}h^{*n}(\varepsilon)\,dx+$$

$$\sum_{n-1}^{\infty}\int_0^\infty e^{-\chi x}F_n(x)pq^{n-1}h^{*n}(\varepsilon)\lambda\alpha r^*(\eta)\,dx \quad ....(6)$$

Now, we get

$$f^*(\xi,\eta,\varepsilon,\delta)=\frac{\alpha r^*(\eta)\,ph^*(\varepsilon)}{\left(1-\beta r^*(\eta)\right)}\left[\frac{f^*(\psi)}{\left(1-qh^*(\varepsilon)\,f^*(\psi)\right)}-\frac{f^*(\chi)}{\left(1-qh^*(\varepsilon)\,f^*(\chi)\right)}\right]+$$

$$\frac{\lambda\alpha r^*(\eta)\,ph^*(\varepsilon)\,f^*(\chi)}{\chi\left(1-qh^*(\varepsilon)\,f^*(\chi)\right)} \quad ....(7)$$

Now we can find expected operation time as

$E(X)=-\frac{\partial}{\partial\xi}f^*(\xi,0,0,0)\Big|_{\xi=0}$ It is seen as after simplification using equation (7) as

$$E(X)=\frac{E(F)}{p}+\frac{p}{\lambda\alpha}\frac{f^*(\lambda\alpha)}{\left(1-qf^*(\lambda\alpha)\right)} \quad ....(8)$$

Expected machine repair time is given by

$$E\left(\hat{R}\right)=-\frac{\partial}{\partial\eta}f^*(\xi,\eta,\varepsilon,\delta)\Big|_{\xi=\eta=\varepsilon=\delta=0}$$

This after simplification using equation (7) gives, E $(\hat{R}) = \frac{E(R)}{\alpha}$ (9)

Expected recruitment time can be seen as

$$E\left(\hat{R}_1\right) = -\frac{\partial}{\partial \varepsilon} f^*\left(\xi,\eta,\varepsilon,\delta\right)\Big|_{\xi=\eta=\varepsilon=\delta=0}$$

$$E\left(\hat{R}_1\right) = \frac{E(H)}{p} \qquad ....(10)$$

Expected Sales Time can be seen as

$$E\left(\hat{S}\right) = -\frac{\partial}{\partial \delta} f^*\left(\xi,\eta,\varepsilon,\delta\right)\Big|_{\xi=\eta=\varepsilon=\delta=0}$$

$$E\left(\hat{S}\right) = \mu E(G)\left[\frac{E(F)}{p} + \frac{pf^*(\lambda\alpha)}{\lambda\alpha\left(1-qf^*(\lambda\alpha)\right)}\right] \qquad ....(11)$$

We may obtain the Laplace transform of the pdf of operation time X and the recruitment time $\hat{R}_1$ of manpower as follows.

$$f^*\left(\xi,0,\varepsilon,0\right) = \frac{ph^*(\varepsilon) f^*(\xi)}{\left(1-qh^*(\varepsilon) f^*(\xi)\right)} - \left(\frac{\xi}{\xi+\lambda\alpha}\right)\frac{ph^*(\varepsilon) f^*(\xi+\lambda\alpha)}{\left(1-qh^*(\varepsilon) f^*(\xi+\lambda\alpha)\right)} \qquad .....(12)$$

The product moment of X and $\hat{R}_1$ namely, $E\left(X\hat{R}_1\right)$ is given by

$$E\left(X\hat{R}_1\right) = \frac{\partial^2}{\partial\xi\partial\varepsilon} f\left(\xi,0,\varepsilon,0\right)\Big|_{\xi=0=\varepsilon} \qquad ....(13)$$

On simplification we obtain from equation(12) and (13)

$$E\left(X\hat{R}_1\right) = \frac{(1+q)}{p^2}\mathrm{E}(F)\mathrm{E}(H) + \frac{pf^*(\lambda\alpha)E(H)}{\lambda\alpha\left(1-qf^*(\lambda\alpha)\right)^2} \qquad ....(14)$$

The covariance between X and $\hat{R}_1$ can be seen as

$$Cov\left(X,\hat{R}_1\right) = E\left(X\hat{R}_1\right) - E(X)E\left(\hat{R}_1\right) \qquad ...(15)$$

This becomes using equations (14), (10) and (8) as

$$Cov\left(X,\hat{R}_1\right) = qE(H)\left[\frac{E(F)}{p^2} - \frac{f^*(\lambda\alpha)\left(1-f^*(\lambda\alpha)\right)}{\lambda\alpha\left(1-qf^*(\lambda\alpha)\right)^2}\right] \qquad ....(16)$$

Model-II

In this section we treat the previous model with all assumptions (1), (2), (3), (4), (5) and (6) except assumption (7) for recruitment pattern.

Assumptions For Manpower Recruitment

(7.1) When the operation time X is more than a threshold time U, the recruitments are all done together. It is assigned to an agent to fill up all vacancies. His service time is H_1 to fill up all vacancies with Cdf $H_1(z)$ and pdf $h_1(z)$.
(7.2) When the operation time X is less than the threshold time U, the recruitments are done one by one and the recruitment time for each is H_2 with Cdf $H_2(z)$ and pdf $h_2(z)$
(7.3) The threshold U has exponential distribution with parameter θ.

Analysis

Using the arguments given for Model-1, the joint pdf of $\left(X,\hat{R},\hat{R}_1,\hat{S}\right)$(Operation Time, Repair Time of the machine, Recruitment time of the employees, Sales Time) may be obtained as follows.

We note the pdf f(x,y,z,w) as

$$f(x,y,z,w)=$$
$$\sum_{i=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=0}^{\infty}\left[f_n(x)pq^{n-1}\int_0^x e^{-\lambda u}\frac{(\lambda u)^{i-1}}{\lfloor i-1}\beta^{i-1}\lambda\alpha du r_i(y)e^{-\mu x}\frac{(\mu x)^k}{\lfloor k}g_k(w)\right]\left[\left(1-e^{-\theta x}\right)h_1(z)+e^{-\theta x}h_{2n}(z)\right]+$$
$$\sum_{i=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=0}^{\infty}\left[F_n(x)pq^{n-1}e^{-\lambda x}\frac{(\lambda x)^{i-1}}{\lfloor i-1}\beta^{i-1}\lambda\alpha r_i(y)e^{-\mu x}\frac{(\mu x)^k}{\lfloor k}g_k(w)\right]\left[\left(1-e^{-\theta x}\right)h_1(z)+e^{-\theta x}h_{2n}(z)\right]\ ...(17)$$

We use the same arguments given for Model 1 for all terms except the second square brackets appearing in the first and second sums. The first term in the bracket indicates the recruitment time pdf is $h_1(z)$ when the threshold is less than the Operation Time (U<X) and the recruitment time pdf is $h_2(z)$ when the threshold is greater than X, (U>X).
The suffix letter indicates the convolution of pdf or Cdf as the case may be. The function $h_{2,k}(z)$ indicates the k-fold convolution of $h_2(z)$ with itself.
The Laplace transform of the pdf of four variables is given by

$$f^*(\xi,\eta,\varepsilon,\delta)=\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty e^{-\xi x-\eta y-\varepsilon z-\delta w}f(x,y,z,w)dxdydzdw$$

This reduces to single integral

$$f^*(\xi,\eta,\varepsilon,\delta)=$$
$$\frac{p\alpha r^*(\eta)}{(1-\beta r^*(\eta))}h_1^*(\varepsilon)\left\{\frac{f^*(\psi)}{(1-qf^*(\psi))}-\frac{f^*(\psi+\theta)}{(1-qf^*(\psi+\theta))}-\frac{f^*(\chi)}{(1-qf^*(\chi))}+\frac{f^*(\chi+\theta)}{(1-qf^*(\chi+\theta))}\right\}+$$
$$\frac{p\alpha r^*(\eta)}{(1-\beta r^*(\eta))}h_2^*(\varepsilon)\left\{\frac{f^*(\psi+\theta)}{(1-qf^*(\psi+\theta)h_2^*(\varepsilon))}-\frac{f^*(\chi+\theta)}{(1-qf^*(\chi+\theta)h_2^*(\varepsilon))}\right\}+$$
$$\lambda\alpha\mathrm{pr}^*(\eta)h_1^*(\varepsilon)\left\{\frac{f^*(\chi)}{\chi(1-qf^*(\chi))}-\frac{f^*(\chi+\theta)}{(\chi+\theta)(1-qf^*(\chi+\theta))}\right\}+$$
$$\lambda\alpha\mathrm{pr}^*(\eta)h_2^*(\varepsilon)\frac{f^*(\chi+\theta)}{(\chi+\theta)(1-qh_2^*(\varepsilon)f^*(\chi+\theta))}\qquad .....(19)$$

Here ψ and χ are as defined in equation (4) and (5).
Since there is only change in the recruitment pattern of employees to fill up the manpower loss, $E(X)$, $E(\hat{R})$ and $E(\hat{S})$ remain the same as those of Model 1.

The Laplace transform of the pdf of $\left(X,\hat{R}_1\right)$ is

$$f^*\left(\xi,0,\varepsilon,0\right)=$$

$$ph_1^*(\varepsilon)\left\{\frac{f^*(\xi)}{\left(1-qf^*(\xi)\right)}-\frac{f^*(\xi+\theta)}{\left(1-qf^*(\xi+\theta)\right)}-\frac{f^*(\chi_1)}{\left(1-qf^*(\chi_1)\right)}\left(\frac{\xi}{\chi_1}\right)+\frac{f^*(\chi_1+\theta)}{\left(1-qf^*(\chi_1+\theta)\right)}\left(\frac{\xi+\theta}{\chi_1+\theta}\right)\right\}+$$

$$ph_2^*(\varepsilon)\left\{\frac{f^*(\xi+\theta)}{\left(1-qf^*(\xi+\theta)h_2^*(\varepsilon)\right)}-\frac{f^*(\chi_1+\theta)}{\left(1-qf^*(\chi_1+\theta)h_2^*(\varepsilon)\right)}\left(\frac{\xi+\theta}{\chi_1+\theta}\right)\right\} \quad ...(20)$$

Here $\chi_1=\xi+\lambda\alpha$ ....(21)

Now,

$$E\left(\hat{R}_1\right)=-\frac{\partial}{\partial\varepsilon}f\left(0,0,\varepsilon,0\right)\Big|_{\varepsilon=0}$$

$$E\left(\hat{R}_1\right)=E\left(H_1\right)\left[1-\frac{pf^*(\theta)}{\left(1-qf^*(\theta)\right)}+\frac{pf^*(\theta+\lambda\alpha)}{\left(1-qf^*(\theta+\lambda\alpha)\right)}\left(\frac{\theta}{\lambda\alpha+\theta}\right)\right]+$$

$$\mathrm{pE}\left(H_2\right)\left[\frac{f^*(\theta)}{\left(1-qf^*(\theta)\right)^2}-\frac{f^*(\lambda\alpha+\theta)}{\left(1-qf^*(\lambda\alpha+\theta)\right)^2}\left(\frac{\theta}{\lambda\alpha+\theta}\right)\right] \quad ....(22)$$

The product moment $E(X\hat{R}_1)$ can be seen as $E\left(X\hat{R}_1\right)=\frac{\partial^2}{\partial\xi\partial\varepsilon}f\left(\xi,0,\varepsilon,0\right)\Big|_{\xi=0=\varepsilon}$

After simplification, we get

$$E\left(X\hat{R}_1\right)=pE\left(H_1\right)\left[\begin{array}{l}\frac{E(F)}{p^2}+\frac{f^{*'}(\theta)}{\left(1-qf^*(\theta)\right)^2}+\left(\frac{1}{\lambda\alpha}\right)\frac{f^*(\lambda\alpha)}{\left(1-qf^*(\lambda\alpha)\right)}-\\ \left(\frac{\theta}{\lambda\alpha+\theta}\right)\frac{f^{*'}(\lambda\alpha+\theta)}{\left(1-qf^*(\lambda\alpha+\theta)\right)^2}-\frac{\lambda\alpha f^*(\lambda\alpha+\theta)}{(\lambda\alpha+\theta)^2\left(1-qf^*(\lambda\alpha+\theta)\right)}\end{array}\right]+$$

$$\mathrm{p}\ \ \mathrm{E}\left(H_2\right)\left[\begin{array}{l}-\frac{f^{*'}(\theta)\left(1+qf^*(\theta)\right)}{\left(1-qf^*(\theta)\right)^3}+\frac{f^{*'}(\lambda\alpha+\theta)\left(1+qf^*(\lambda\alpha+\theta)\right)}{\left(1-qf^*(\lambda\alpha+\theta)\right)^3}\left(\frac{\theta}{\lambda\alpha+\theta}\right)+\\ \qquad\frac{f^*(\lambda\alpha+\theta)}{\left(1-qf^*(\lambda\alpha+\theta)\right)}\left(\frac{\lambda\alpha}{(\lambda\alpha+\theta)2}\right)\end{array}\right]$$

.....(23)

The $\mathrm{Cov}\left(X,\hat{R}_1\right)$ may be written using the formula $Cov\left(X,\hat{R}_1\right)=E\left(X\hat{R}_1\right)-E(X)E\left(\hat{R}_1\right)$ *and* equation (23), (22) and (8).

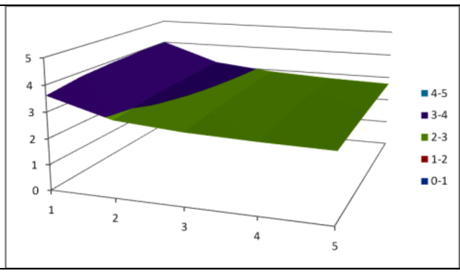
Numerical Illustration

Model –I

We assume the fixed values for E(R) = 5, E(H) = 3, E(S) = 8, p = 0.4, q = 0.6, α = 0.3, β = 0.7 and μ = 10We provide the different values for the parameter of exponential distribution of inter occurrence time of departure of employees and the parameter of inter occurrence time exponential distribution of shocks. (i.e λ = 2,4,6,8,10 and δ = 3,6,9,12,15)

Table and Graph of $E\left[X\right]$

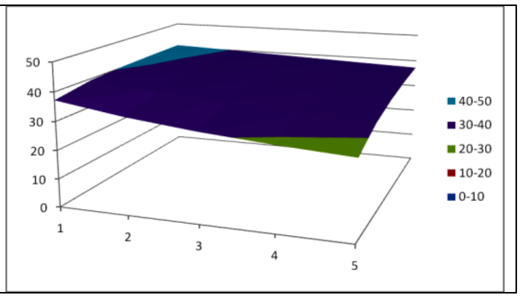
$\delta/\lambda \rightarrow$	2	4	6	8	10
3	3.611111	2.916667	2.722222	2.638889	2.595238
6	3.833333	3.055556	2.81746	2.708333	2.648148
9	3.928571	3.125	2.87037	2.75	2.681818
12	3.981481	3.166667	2.90404	2.777778	2.705128
15	4.015152	3.194444	2.92735	2.797619	2.722222



In the above table, when the value of λ increases, $E\left[X\right]$ decreases and when the value of δ increases , $E\left[X\right]$ increases.

Table and Graph of $E\left[\hat{R}\right]$

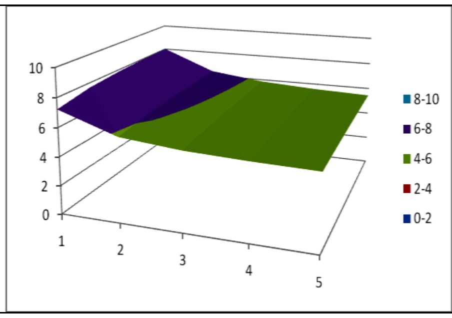
$\delta/\lambda \rightarrow$	2	4	6	8	10
3	37.22222	33.54167	30.86667	28.88889	27.38095
6	39.46667	37.22222	35.2381	33.54167	32.09877
9	40.2381	38.69792	37.22222	35.86667	34.64187
12	40.61728	39.46667	38.31956	37.22222	36.19329
15	40.84022	39.93056	39.00394	38.09524	37.22222



In the above table, when the value of λ increases, $E\left[\hat{R}\right]$ decreases and when the value of δ increases , $E\left[\hat{R}\right]$ increases.

Table and Graph of $E\left[\hat{S}\right]$

$\delta/\lambda \rightarrow$	2	4	6	8	10
3	7.222222	5.833333	5.444444	5.277778	5.190476
6	7.666667	6.111111	5.634921	5.416667	5.296296
9	7.857143	6.25	5.740741	5.5	5.363636
12	7.962963	6.333333	5.808081	5.555556	5.410256
15	8.030303	6.388889	5.854701	5.595238	5.444444



In the above table, when the value of λ increases, $E\left[\hat{S}\right]$ decreases and when the value of δ increases , $E\left[\hat{S}\right]$ increases

$\delta/\lambda \rightarrow$	2	4	6	8	10
3	7.5	7.5	7.5	7.5	7.5
6	7.5	7.5	7.5	7.5	7.5
9	7.5	7.5	7.5	7.5	7.5
12	7.5	7.5	7.5	7.5	7.5
15	7.5	7.5	7.5	7.5	7.5

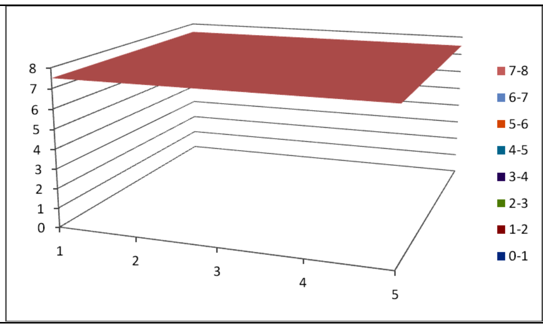
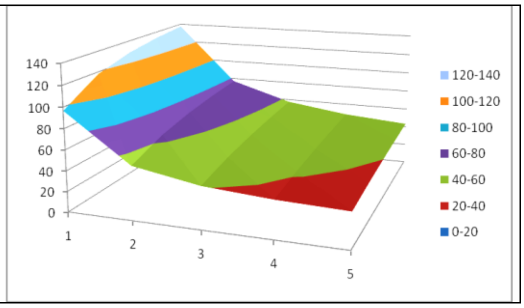


Table and Graph of $E\left[\hat{R}_1\right]$

As the value of λ and δ increases, the value of $E\left[\hat{R}_1\right]$ remains the same.

Table and Graph of $E\left[X\hat{R}_1\right]$

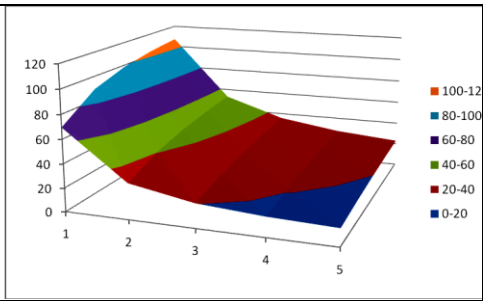
$\delta/\lambda \rightarrow$	2	4	6	8	10
3	96.66667	51.875	40.66667	36.25	34.08163
6	118	63.33333	47.68707	40.9375	37.40741
9	127.9592	69.84375	52.22222	44.25	39.91736
12	133.7037	74	55.34435	46.66667	41.83432
15	137.438	76.875	57.61341	48.4949	43.33333



In the above table, when the value of λ increases, $E\left[X\hat{R}_1\right]$ decreases and when the value of δ increases, $E\left[X\hat{R}_1\right]$ increases.

Table and Graph of $Cov\left(X,\hat{R}_1\right)$

$\delta/\lambda \rightarrow$	2	4	6	8	10
3	69.58333	30	20.25	16.45833	14.61735
6	89.25	40.41667	26.55612	20.625	17.5463
9	98.4949	46.40625	30.69444	23.625	19.80372
12	103.8426	50.25	33.56405	25.83333	21.54586
15	107.3244	52.91667	35.65828	27.51276	22.91667



In the above table, when the value of λ increases, $Cov\left(X,\hat{R}_1\right)$ decreases and when the value of δ increases, $Cov\left(X,\hat{R}_1\right)$ increases.

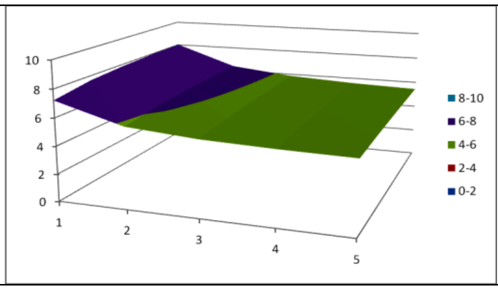
Model-II

We assume the fixed values for E(R) = 5, E(H) = 3, E(S) = 8, p = 0.4, q = 0.6, α = 0.3, θ=25 β = 0.7 and μ = 10

We provide the different values for the parameter of exponential distribution of inter occurrence time of departure of employees and the parameter of inter occurrence time exponential distribution of shocks. (i.e λ = 2,4,6,8,10 and δ = 3,6,9,12,15)

Table and Graph of $E\left[\hat{S}\right]$

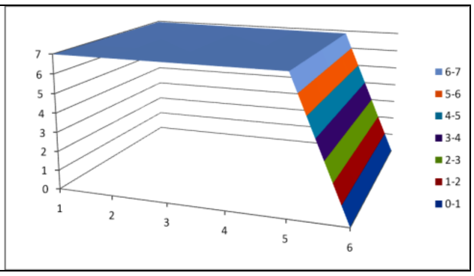
$\delta/\lambda \rightarrow$	2	4	6	8	10
3	7.222222	5.833333	5.444444	5.277778	5.190476
6	7.666667	6.111111	5.634921	5.416667	5.296296
9	7.857143	6.25	5.740741	5.5	5.363636
12	7.962963	6.333333	5.808081	5.555556	5.410256
15	8.030303	6.388889	5.854701	5.595238	5.444444



In the above table, when the value of λ increases, $E\left[\hat{S}\right]$ decreases and when the value of δ increases, $E\left[\hat{S}\right]$ increases.

Table and Graph of $E\left[\hat{R}_1\right]$

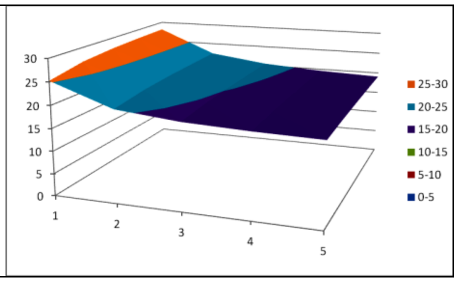
$\delta/\lambda \rightarrow$	2	4	6	8	10
3	25.27707	20.41531	19.05362	18.46975	18.16372
6	26.83217	21.38666	19.71902	18.95425	18.53214
9	27.49855	21.87222	20.08859	19.24488	18.76659
12	27.86875	22.16356	20.32381	19.43871	18.929
15	28.10436	22.35784	20.48673	19.57727	19.04825



In the above table, when the value of λ increases, $E\left[\hat{R}_1\right]$ decreases and when the value of δ increases, $E\left[\hat{R}_1\right]$ increases.

Table and Graph of $E\left[X\hat{R}_1\right]$

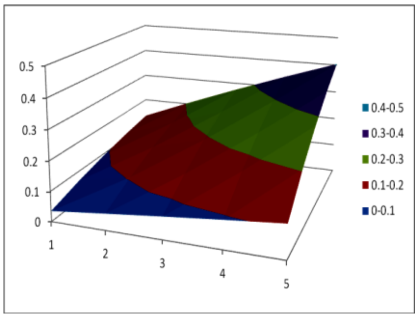
$\delta/\lambda \rightarrow$	2	4	6	8	10
3	6.990045	6.980744	6.972039	6.963881	6.956226
6	6.982063	6.965265	6.949512	6.934719	6.920809
9	6.975622	6.952747	6.931254	6.911034	6.891987
12	6.970396	6.942569	6.916378	6.891697	6.868411
15	6.966136	6.934254	6.9042	6.875837	6.84904



As the value of λ and δ increases, the value of $E\left[X\hat{R}_1\right]$ decreases.

Table and Graph of $Cov\left(X,\hat{R}_1\right)$

$\delta/\lambda \rightarrow$	2	4	6	8	10
3	0.035237	0.05481	0.074179	0.092846	0.110655
6	0.06759	0.103902	0.139046	0.172716	0.204814
9	0.094318	0.144883	0.193326	0.239541	0.283528
12	0.116248	0.17876	0.238368	0.295108	0.349066
15	0.134265	0.206749	0.275716	0.341294	0.403638



As the value of λ and δ increases, the value of $Cov\left(X,\hat{R}_1\right)$ increases.

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