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Forecasting the Price of Call Option Using Support Vector Regression

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Abstract: *Pricing option is a challenging task in finance. Numbers of parametric and nonparametric methods have been developed for pricing an option. Both methods have their own pros and cons. In this paper fair price of an option is predicted using ε-insensitive support vector regression and smooth ε-insensitive support vector regression. The methods are applied on five different money market conditions i.e. deep-in-the-money, in-the-money, at-the-money, out-of-money and deep-out-of-money. The experiments are performed on S&P CNX Nifty index option data set. The experimental study reflects that both methods performed fairly well in comparison to Black and Scholes model.*

Keywords: *Black Scholes model, Moneyness, Option pricing, ε- Insensitive Support Vector Regression, Smooth ε- Insensitive Support Vector Regression.*

I. Introduction

Option is a contract which gives its owner the right to buy or to sell a fixed quantity of a specified stock at a fixed price on or before a given date. In options trading, stocks are not traded but rights to own or sell stocks are traded. Option can be traded in two different ways i.e. European and American ways. In former option can be exercised on the specified date were as in latter it can be exercised before the specified date. Several models have been developed for pricing option using parametric and nonparametric methods [1, 2]. Black and Scholes (1973) derived a second-order parabolic partial differential equation for pricing European option. In this paper fair price of an option is predicted using ε-insensitive support vector regression and smooth ε-insensitive support vector regression. The methods are applied on five different money market conditions i.e. deep-in-the-money, in-the-money, at-the-money, out-of-money and deep-out-of-money instead of applying it directly on market data. The experiments are performed on S&P CNX Nifty index option data set.

II. Related Work

In recent year, nonparametric methods i.e. parameter-less methods are used for prediction. These methods are suitable to adjust changing behavior of the derivative securities as they can be trained from time to time [3].In this class neural network, genetic algorithm and various types of data mining techniques are used.

Hutchinson et al. [3] in 1994 used neural network for option pricing and compared its performance with Black Scholes model. The proposed model performed fairly well. M. Liu [4] in 1996, Yao et al. [5] in 2000 and Andreou [6] in 2008 successfully applied neural network in option pricing. Saxena [7] in 2008 and Mitra [8] in 2012 studied Nifty Options in India and forecasted it using neural network.

Support Vector Machine (SVM), part of statistical learning theory proposed by Vapnik [9] is a powerful tool. It has been widely used for classification, pattern recognition and nonlinear function estimation [10]. Tay Francis et al. [11, 12], K. J. Kim [13] and Lijuan Cao [14] proposed a modified version of support vector machines in financial time series forecasting. SVM provides a promising alternative to neural network for forecasting financial time series.

Trafalis et al. [15] and Michael M. Pires et al. [16] used Support Vector Regression and found that this approach provided much better results than neural network. B.V. Phani et al. [17] explored general regression neural network for prediction of option price. Andreou et al. [18] applied ε- insensitive support vector regression and least square support vector regression model for pricing European options and compared it with parametric option pricing models.

Conventional support vector machine problem is formulated as a constrained minimization problem [9, 19] and is solved using sequential minimal optimization algorithm [20]. Lee et al. [21, 22] proposed ε-insensitive smooth support vector machine and solved it using fast Newton-Armijo algorithm. In this paper Lee et al.'s ε-insensitive support vector regression is used to experimentally study forecasting of option on S&P CNX Nifty index option data set.

The paper is organized as follows: Section 2 provides basic concept about parametric and nonparametric methods. Section 3 describes data preparation and different performance measurement techniques. In Section 4 experimental results are discussed and finally conclusion is given in last section.

(A) Parametric Method

Black Scholes [1] in 1973 proposed a model for pricing option that fetched him noble prize in 1997. The model is normally used for pricing European options. The value of call option for without dividend paying stocks is

$$c = xN(d_1) - ke^{-rt}N(d_2)$$

where parameters d_1 and d_2 are

$$d_1 = \frac{\log(x/k) + (r + \sigma^2/2)t}{\sigma\sqrt{t}}, \qquad d_2 = d_1 - \sigma\sqrt{t}$$

and x is current price of the security, k is exercise price of option, r is risk free rate of interest, t is time to expiry of the option and σ is volatility of the underlying asset.

(B) Nonparametric Method

The methods that are not based on particular assumptions and parameters are known as nonparametric methods Artificial Neural Networks (ANN) methods, Genetic Programming, Kernel regression methods, Support Vector Regressions (SVR) etc. falls under this category.

ε-Insensitive Support Vector Regression

Support vector regression (SVR) proposed by Vapnik [9] used linear space to depict linear separability. Linearly separable problem can be easily tackled using standard support vector regression. In this case regression function f(x) is

$$y = \langle w \cdot x_i \rangle + b \qquad w \in X, b \in R \qquad (1)$$

and optimization problem is

$$\min \frac{1}{2}\|w\|^2$$

Such that

$$\begin{aligned} & y_i - \langle w \cdot x_i \rangle - b_i \le \in \\ & \langle w \cdot x_i \rangle + b_i - y_i \le \in \end{aligned} \qquad (2)$$

It is difficult to handle noisy data using eq^n (2) as there is no error coefficient. Therefore the optimization problem has been modified by introducing slack variables ξi, ξ_i^* as error function [9] and the formulation is known as soft margin.

$$\min \frac{1}{2}\|w\|^2 + c\sum_{i=1}^{N}(\xi_i - \xi_i^*)$$

Such that

$$\begin{aligned} & y_i - \langle w \cdot x_i \rangle - b_i \le \in + \xi \\ & \langle w \cdot x_i \rangle + b_i - y_i \le \in + \xi_i^* \\ & \xi_i, \xi_i^* \ge 0 \end{aligned} \qquad (3)$$

where c > 0 is a fixed constant. It is a trade-off parameter between the margin size and the amount up to which deviations larger than ε can be tolerated. The solution for the option

$$\max L(\lambda) = -\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\left(\lambda_i - \lambda_i^*\right)\left(\lambda_j - \lambda_j^*\right)x_i . x_j - \varepsilon\left(\lambda_i + \lambda_i^*\right) - \sum_{i=1}^{m} y_i\left(\lambda_i - \lambda_i^*\right)$$

$$s.t$$

$$\begin{aligned} & 0 \le \lambda_i, \lambda_i^* \le C \\ & \sum_{i=1}^{N}\left(\lambda_i - \lambda_i^*\right) = 0 \end{aligned} \qquad (4)$$

The solution of the above problem give the values of Lagrange's multipliers λ_i, λ_i^* and optimal regression function becomes

$$w = \sum_{i=1}^{m} \left(\lambda_i - \lambda_i^*\right) x_i$$
$$f(x) = \sum_{i=1}^{m} (\lambda_i - \lambda_i^*) x_i . x + b \qquad (5)$$

The above function is formulated for linear ε-insensitive SVR. In nonlinear ε-insensitive SVR data is mapped to higher dimensional feature space using kernel function to make it a linearly separable. The optimization problem is

$$\max L(\lambda) = -\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\left(\lambda_i - \lambda_i^*\right)\left(\lambda_j - \lambda_j^*\right)k(x_i . x_j) - \in \left(\lambda_i + \lambda_i^*\right) - \sum_{i=1}^{m} y_i\left(\lambda_i - \lambda_i^*\right)$$

such that $0 \le \lambda_i, \lambda_i^* \le C$

$$\sum_{i=1}^{N}\left(\lambda_i - \lambda_i^*\right) = 0 \qquad (6)$$

and optimal regression function is

$$w = \sum_{i=1}^{m} \left(\lambda_i - \lambda_i^*\right) \phi(x_i)$$
$$f(x) = \sum_{i=1}^{m} (\lambda_i - \lambda_i^*) k(x_i . x) + b \qquad (7)$$

Smooth ε- Insensitive Support Vector Regressions

Support vector regression is constrained quadratic minimization problem in which large training samples increase computation complexity [9] therefore Lee et al. [21, 22] introduced smooth ε-insensitive support vector regression in terms of unconstrained quadratic minimization.

$$\underset{w,b}{Min} \quad L(w,b) = \frac{1}{2}(\| w \|^2 + b^2) + \frac{C}{2}\left\| \left\langle w \cdot x_i \right\rangle + b_i - y_i \right\|_{\varepsilon}^2 \qquad (8)$$

It is a convex quadratic optimization problem without constrained. Lee et.al [21, 22] replaced the unsmooth part $\left\| \left\langle w \cdot x_i \right\rangle + b_i - y_i \right\|_{\varepsilon}^2$ of objective function in (8) with smooth function (9).

$$p_{\varepsilon}^2(x,\alpha) = (p(x-\varepsilon,\alpha))^2 + (p(-x-\varepsilon,\alpha))^2$$

where $p(x,\alpha) = x + \frac{1}{\alpha}\log(1+e^{-\alpha x}), \alpha > 0$ (9)

Smooth support vector regression function for nonlinear data is

$$\underset{w,b}{Min} \quad L(w,b) = \frac{1}{2}(\| w \|^2 + b^2) + \frac{C}{2}\sum_{i=1}^{m} p_{\varepsilon}^2(w.\phi(x_i) + b - y_i, \alpha)$$

where $w = \phi(x).u \ , \ \ for \ \ some \ \ u \in R^m$

after applying kernel function new formulation is

$$\underset{u,b}{Min} \quad L(u,b) = \frac{1}{2}(\| u \|^2 + b^2) + \frac{C}{2}\sum_{i=1}^{m} p_{\varepsilon}^2(k(.x_i.x_j)u + b - y_i, \alpha) \qquad (11)$$

The solution of this unconstrained minimization problem for u and b leads to the nonlinear regression function

$$f(x) = \sum_{i=1}^{m} k(x_i.x)u_i + b \tag{12}$$

(A) Data Preparation

In this study S&P CNX Nifty index option pricing data (23 January 2012 to 8 January 2014) is collected from NSE website [23]. There are 22,840 data points in data set. The daily closing values is considered as option value and to overcome the problems of over fitting data value having volume less than 100 is discarded [8]. Snapshot of sample data is shown in Fig. 1 and Fig. 2.

Figure 1: Snapshot of Input Data for Black Scholes Model

Underlying Value	Strike Price	Rate	Time in year	Volatility	Closing Price	Ratio
6174.6	5000	0.1	0.134247	0.666207	1206.95	1.23492
6168.35	5000	0.1	0.131507	0.666207	1195.7	1.23367
6272.75	5000	0.1	0.120548	0.666207	1296.4	1.25455
6261.65	5000	0.1	0.109589	0.666207	1282	1.25233
6303.95	5000	0.1	0.10137	0.666207	1332.5	1.26079
6313.8	5000	0.1	0.09863	0.666207	1339.35	1.26276
6338.95	5000	0.1	0.09589	0.666207	1375.65	1.26779
6345.65	5000	0.1	0.093151	0.666207	1370.6	1.26913
6266.75	5000	0.1	0.090411	0.666207	1289.8	1.25335

Figure 2: Snapshot of Input Data for SVR Model

Closing Price	Time	Moneyness Ratio
984.5	0.082192	1.179971
937.15	0.076712	1.176971
893.25	0.073973	1.168019
829.2	0.060274	1.154019
842	0.054795	1.160827
867	0.052055	1.166
890.9	0.038356	1.17
799.9	0.035616	1.154058
847.2	0.032877	1.163144

(B) Cross Validation

In this experimental study 5-fold cross-validation is used to select the best model parameters. The training set is divided into five subsets of equal size and one subset is used as testing and remaining ones are used as training in 5-fold cross-validation. The cross-validation accuracy is measured in terms of mean-squared error. Number of researches have used combination of different parameters for their model like time to maturity, volatility, risk free interest rate, spot price and strike price. Hutchinson[3] observed that volatility and risk free interest rate may discarded. In this experiment time to maturity moneyness is used as input parameters. The data set is divided into five categories based on different money market conditions i.e. deep-in-the-money, in-the-money, at-the-money, out-of-money and deep-out-of-money to narrow the range of the data for better training. The market condition can be measured using moneyness. It is defined as ratio of spot price and strike price. The empirical data is clustered using k mean clustering based on moneyness (S/K). Each cluster is divided into two parts training data set (70%) and testing data set (30%). The five clusters are

1. Deep in-the-money (DITM) ($S/K < 0.91$)
2. In-the-money (ITM) ($0.91 \leq S/K < 0.98$)
3. At-the-money (ATM) ($0.98 \leq S/K < 1.06$)
4. Out-of-money (OTM) ($1.06 \leq S/K < 1.18$)
5. And deep out-of-money (DOTM) ($1.18 \leq S/K$)

The option price is predicted using following algorithm

Step 1. Filtering of data: Remove the data that have trading volume less than 100
Step 2. Clustering the data: Cluster the data using k mean clustering into five set based on moneyness.
Step 3. Normalization: Normalized the data using minmax normalization.
Step 4.Tune model parameters: The model parameters are tuned using 5-fold cross validation by dividing the data set into training data 70% and testing data 30%.
Step 5. Forecasting of option prices: Option value is forecasted using ε- Insensitive Support Vector Regression and Smooth ε- Insensitive Support Vector Regression.

(C) Performance Measurement

The performance of the model is evaluated by calculating the difference between actual and theoretical option values. Common measure for measuring the performance of the model are total error, mean error, mean square error, root mean square error and normalized root mean square error. Where N is total number of option pricing data, y_i^{mo} is empirically evaluated option prices and y_i^{ma} is actual option prices from the market.

Total Error (TE)

$$TE = \sum_{i=1}^{N} (y_i^{mo} - y_i^{ma})$$

Mean Error (ME)

$$ME = \frac{1}{N} \sum_{i=1}^{N} (y_i^{mo} - y_i^{ma})$$

Mean Square Error (MSE)

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (y_i^{mo} - y_i^{ma})^2$$

Root Mean Square Error (RMSE)

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i^{mo} - y_i^{ma})^2}$$

Normalized Root Mean Square Error (NRMSE)

$$NRMSE = \frac{RMSE}{y_{max}^{ma} - y_{min}^{ma}}$$

III. Experimental Results

The option is priced using ε-insensitive SVR, ε-insensitive SSVR and Black Scholes Model. Indian stock market data is collected to reflect different market conditions. Experiments are performed on MATLAB along with LIBSVM [24] and SSVM [22] toolboxes. The performance on five different market conditions is shown in 4.1, 4.2, 4.3, 4.4 and 4.5 respectively.

Table 1: Performance of various model when market condition is deep in the money market

Models	Data-Points	TE	ME	MSE	RMSE	NRMSE
BS	4724	1.3136×10^7	2.7807×10^3	9.1375×10^6	3022.8	275.08
ε-SVR	4724	-2.7157×10^3	.5749	53.9760	7.3468	.6354
ε-SSVR	4724	-2.5986×10^3	-.5501	42.7703	6.5399	.5727

Table 2: Performance of various model when market condition is in the money market

Models	Data-Points	TE	ME	MSE	RMSE	NRMSE
BS	7025	2.1830×10^7	3.1075×10^3	1.0826×10^7	3290.2	71.6486
ε-SVR	7025	-3.1512×10^4	-4.4857	640.0180	25.2986	.5356
ε-SSVR	7025	-1.4764×10^4	-2.1016	484.4335	22.0099	.4717

Table 3: Performance of various model when market condition is at the money market

Models	Data-Points	TE	ME	MSE	RMSE	NRMSE
BS	6110	1.3592×10^7	2.2246×10^3	6.2100×10^6	2492.0	275.08
ε-SVR	6110	-2.2600×10^4	-3.6988	1087.2	32.9734	.3318
ε-SSVR	6110	3.3897×10^3	.5548	960.8654	30.9978	.3116

Table 4: Performance of various model when market condition is out of the money market

Models	Data-Points	TE	ME	MSE	RMSE	NRMSE
BS	3589	2.9702×10^6	8.2759×10^2	1.1528×10^6	1073.7	275.08
ε-SVR	3589	7.3363×10^4	10.0006	3.6523×10^3	60.4344	.4125
ε-SSVR	3589	3.7030×10^4	10.3176	3.6367×10^3	60.3050	.4117

Table 5: Performance of various model when market condition is deep out of the money market

Models	Data-Points	TE	ME	MSE	RMSE	NRMSE
BS	1392	3.0960×10^5	2.2241×10^2	1.7264×10^5	415.5362	2.0384
ε-SVR	1392	3.0888×10^4	22.1894	1.2396×10^4	111.3358	.5215
ε-SSVR	1392	2.8134×10^4	20.2109	1.1728×10^4	108.2967	.5075

IV. Conclusion

In this study fair price of option is predicted using ε-insensitive support vector regression and smooth ε-insensitive support vector regression. The empirical results are compared with the Black and Scholes model. The performance of the ε-insensitive smooth support vector regression is better than ε-insensitive support vector regression and Black and Scholes.

References

[1]. J. C. Hull, Options, futures, and other derivatives (Prentice-Hall, 1997).
[2]. Heston, A closed form solution for options with stochastic volatility with applications to bond and currency options, Review of Financial Studies, 6(2), 1993, 327-343.
[3]. Hutchinson, A nonparametric approach to pricing and hedging derivative securities via learning networks, Journal of Finance, 49(3), 1998,851-889 .
[4]. M. Liu, Option pricing with neural networks, in Progress in Neural Information Processing, S. I. Amari, L. Xu, L. W. Chan, I. King, and K. S. Leung, Eds. New York: Springer-Verlag, 2, 1996, 760–765.
[5]. Yao. J., Li. Y., and Tan. C. L., Option price forecasting using neural networks, Omega, 28(4), 2000, 455-466.
[6]. Andreou, Panayiotis C., Chris Charalambous, and Spiros H. Martzoukos, Pricing and trading European options by combining artificial neural networks and parametric models with implied parameters, European Journal of Operational Research, Elsevier, vol. 185(3), 2008, 1415-1433.
[7]. Anant Saxena, Valuation of S&P CNX Nifty Options: Comparison of Black-Scholes and Hybrid ANN Mode, SAS Global Forum, 162, 2008.
[8]. Dr. S. K. Mitra ,An Option Pricing Model That Combines Neural Network Approach and Black Scholes Formula, Global Journals Inc. (USA), 12(4), 2012,6-16.
[9]. Vapnik, V. N. The Nature of Statistical Learning Theory, (Springer, New York. John Wiley & Sons, 1995).
[10]. Vapnik, V., Golowich, S., & Smola, Support vector method for function approximation, regression estimation and signal processing, Advances In Neural Information Processing Systems , 9,1997, 281-287.
[11]. Tay, F. E. H. and Cao, L., Application of support vector machines in financial time-series forecasting, Omega, 29(4), 2001, 309–317.
[12]. Tay, F. E. H. and Cao, L., Modified support vector machines in financial time series forecasting, Neurocomputing, 48(1-4), 2002, 847-861.
[13]. Kyoung- jae Kim,Financial time series forecasting using support vector machines Neurocomputing, 55, 2003,307 – 319.
[14]. Lijuan Cao, Support vector machines experts for time series Forecasting, Neurocomputing , 51(1-2),2003, 321 – 339.
[15]. Trafalis, T.B., Ince, H., Mishina, T., Support vector regression in option pricing, .In Conference on Computational Intelligence and Financial Engineering, CIFer, 2003.
[16]. Michael M. Pires Tshilidzi Marwala, American Option Pricing Using Multi-Layer Perceptron and Support Vector Machine, IEEE International Conference on Systems, Man and Cybernetics, 2, 2004, 1279-1285.
[17]. B.V. Phani, B. Chandra, Vijay Raghav, Quest for Efficient Option Pricing Prediction model using Machine Learning Techniques, In Neural Networks (IJCNN) , 2011, 654-657.
[18]. Panayiotis C. Andreou, Chris Charalambous, and Spiros H. Martzoukos, European Option Pricing By Using The Support Vector Regression Approach, C. Alippi , Artificial Neural Networks–ICANN, 2009, 874-883.
[19]. Drucker, H., Burges, C. J., Kaufman, L., Smola, A., & Vapnik, V., Support vector regression machines. Advances in neural information processing systems, 9, 1997, 155-161.
[20]. Platt J., Fast training of support vector machines using sequential minimal optimization [A], Advances in Kernel Method: Support Vector Learning [C]. Cambridge: MIT Press, 1999, 185-208.
[21]. Lee, Y. J., & Mangasarian, O. L. SSVM: A smooth support vector machine for classification. Computational optimization and Applications, 20(1), 2001, 5-22.
[22]. Lee, Y. J., Hsieh, W. F., Huang, C. M.: “epsilon-SSVR: A Smooth Support Vector Machine for ε-Insensitive Regression” IEEE
[23]. Trans on Knowledge and Data Engineering, software available at http://dmlab8.csie.ntust.edu.tw/ssmtoolbox.html, 17(5), 2005,678-685
[24]. National stock exchange of India ltd. http://www.nseindia.com/. Accessed: January 2014.
[25]. C-C. Chang and C-J. Lin. LIBSVM: a library for support vector machines, 2001. Software available at http://www.csie.ntu.edu.tw/~cjlin/libsvm.