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On αrω–Continuous and αrω–Irresolute Maps in Topological Spaces

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Abstract: *The aim of this paper is to introduce a new type of functions called the α REGULAR ω continuous maps , αrω–irresolute maps, strongly αrω–continuous maps , perfectly αrω–continuous maps and study some of these properties.*
Keywords: *αrω–open sets, αrω–closed sets, αrω–continuous maps, αrω-irresolute maps, strongly αrω-continuous maps, perfectly αrω–continuous.*

I. Introduction

The concept of regular continuous and Completely–continuous functions was first introduced by Arya. S. P. and Gupta.R [1]. Later Y. Gnanambal [2] studied the concept of generalized pre regular continuous functions. Also, the concept of ωα-continuous functions was introduced by S S Benchalli et al [3]. Recently R S Wali et al[4] introduced and studied the properties of αrω-closed sets. The purpose of this paper is to introduce a new class of functions, namely, αrω-continuous functions and αrω-irresolute functions strongly αrω-continuous maps , perfectly αrω-continuous maps. Also, we study some of the characterization and basic properties of αrω-continuous functions.

II. Preliminaries

Throughout this paper, (X, τ) and (Y, σ) (or simply X and Y) represent a topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. X\A or A^c denotes the complement of A in X.

We recall the following definitions and results.

Definition 2.1: A subset A of a topological space (X, τ) is called.
(1) semi-open set [5] if $A \subseteq cl(int(A))$ and semi-closed set if $int(cl(A)) \subseteq A$.
(2) pre-open set [6] if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$.
(3) α-open set [7] if $A \subseteq int(cl(int(A)))$ and α -closed set if $cl(int(cl(A)))\subseteq A$.
(4) semi-pre open set [8] (=β-open9] if $A\subseteq cl(int(cl(A))))$ and a semi-pre closed set (=β-closed) if $int(cl(int(A)))\subseteq A$.
(5) regular open set [10] if A = int(clA)) and a regular closed set if A = cl(int(A)).
(6) Regular semi open set [11] if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$.
(7) Regular α-open set[12] (briefly,rα-open) if there is a regular open set U s.t $U \subseteq A \subseteq \alpha cl(U)$.

Definition 2.2 : A subset A of a topological space (X, τ) is called
1) generalized pre regular closed set(briefly gpr-closed)[2] if $pcl(A)\subseteq U$ whenever $A\subseteq U$ and U is regular open in X.
2) ωα- closed set [3] if $\alpha cl(A) \subseteq U$ whenever $A\subseteq U$ and U is ω-open in X.
3) α regular ω- closed (briefly αrω -closed) set [4] if $\alpha cl(A) \subseteq U$ whenever $A\subseteq U$ and U is rω-open in X.
4) regular generalized α-closed set (briefly, rgα-closed)[12] if $\alpha cl (A) \subseteq U$ whenever $A\subseteq U$ and U is regular α-open in X.
5) generalized closed set(briefly g-closed) [13] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
6) generalized semi-closed set(briefly gs-closed)[14] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
7) generalized semi pre regular closed (briefly gspr-closed) set [15] if $spcl(A) \subseteq U$ whenever $A\subseteq U$ and U is regular open in X.
8) strongly generalized closed set [15](briefly,g*-closed) if $Cl(A)\subseteq U$ whenever $A\subseteq U$ and U is g-open in X.

9) α-generalized closed set(briefly αg -closed)[16] if αcl(A)⊆U whenever A⊆U and U is open in X.
10) ω-closed set[17] if cl(A) ⊆ U whenever A⊆ U and U is semi-open in X.
11) weakely generalized closed set(briefly, wg-closed)[18] if cl(int(A))⊆U whenever A⊆U and U is open in X.
12) regular weakly generalized closed set (briefly, rwg-closed)[18] if cl(int(A)) ⊆ U whenever A⊆U and U is regular open in X.
13) semi weakly generalized closed set (briefly, swg-closed)[18] if cl(int(A))⊆U whenever A⊆U and U is semi open in X.
14) generalized pre closed (briefly gp-closed) set [19] if pcl(A) ⊆U whenever A⊆U and U is open in X.
15) regular ω-closed (briefly rω -closed) set [20] if cl(A) ⊆ U whenever A⊆U and U is regular semi-open in X.
16) g*-pre closed (briefly g*p-closed) [21] if pcl(A) ⊆ U whenever A⊆ U and U is g-open in X
17) generalized regular closed (briefly gr–closed)set[22] if rcl(A)⊆ U whenever A⊆ U and U is open in X.
18) regular generalized weak (briefly rgw-closed) set[23] if cl(int(A)) ⊆ U whenever A⊆ U and U is regular semi open in X.
19) weak generalized regular–α closed (briefly wgrα-closed) set[24] if cl(int(A) ⊆ U whenever A⊆ U and U is regular α-open in X.
20) regular pre semi–closed (briefly rps-closed) set [25] if spcl(A)⊆U whenever A⊆U and U is rg- open in X.
21) generalized pre regular weakly closed (briefly gprw-closed) set [26] if pcl(A) ⊆U whenever A⊆ U and U is regular semi- open in X.
22) α-generalized regular closed (briefly αgr-closed) set [27] if αcl(A)⊆U whenever A⊆U and U is regular open in X.
23) R*-closed set [28] if rcl(A) ⊆U whenever A⊆ U and U is regular semi- open in X.

The compliment of the above mentioned closed sets are their open sets respectively.

Definition 2.3: A map f: (X,τ)→(Y,σ) is said to be
(i) regular-continuous(r-continuous) [1] if $f^{-1}(V)$ is r-closed in X for every closed subset V of Y.
(ii) Completely–continuous[1] if $f^{-1}(V)$ is regular closed in X for every closed subset V of Y.
(iii) Strongly–continuous[10] if $f^{-1}(V)$ is Clopen (both open and closed) in X for every subset V of Y.
(iv) α–continuous[7] if $f^{-1}(V)$ is α–closed in X for every closed subset V of Y.
(v) strongly α–continuous [29] if $f^{-1}(V)$ is α-closed in X for every semi-closed subset V of Y.
(vi) αg–continuous[16] if $f^{-1}(V)$ is αg–closed in X for every closed subset V of Y.
(vii) wg–continuous[18] if $f^{-1}(V)$ is wg–closed in X for every closed subset V of Y.
(viii) rwg–continuous18[ ] if $f^{-1}(V)$ is rwg–closed in X for every closed subset V of Y.
(ix) gs–continuous[14] if $f^{-1}(V)$ is gs–closed in X for every closed subset V of Y.
(x) gp–continuous[19] if $f^{-1}(V)$ is gp–closed in X for every closed subset V of Y.
(xi) gpr–continuous[2] if $f^{-1}(V)$ is gpr–closed in X for every closed subset V of Y.
(xii) αgr–continuous[27] if $f^{-1}(V)$ is αgr–closed in X for every closed subset V of Y.
(xiii) ωα–continuous[3] if $f^{-1}(V)$ is ωα–closed in X for every closed subset V of Y.
(xiv) gspr–continuous[15] if $f^{-1}(V)$ is gspr–closed in X for every closed subset V of Y.
(xv) g–continuous[3] if $f^{-1}(V)$ is g–closed in X for every closed subset V of Y
(xvi) ω–continuous[17] if $f^{-1}(V)$ is ω–closed in X for every closed subset V of Y
(xvii) rgα–continuous[12] if $f^{-1}(V)$ is rgα–closed in X for every closed subset V of Y.
(xviii) gr–continuous[22] if $f^{-1}(V)$ is gr–closed in X for every closed subset V of Y.
(xix) g*p–continuous[21] if $f^{-1}(V)$ is g*p–closed in X for every closed subset V of Y.
(xx) rps–continuous[25] if $f^{-1}(V)$ is rps–closed in X for every closed subset V of Y.
(xxi) R*–continuous[28] if $f^{-1}(V)$ is R*–closed in X for every closed subset V of Y.
(xxii) gprw–continuous[26] if $f^{-1}(V)$ is gprw–closed in X for every closed subset V of Y.
(xxiii) wgrα–continuous[24] if $f^{-1}(V)$ is wgrα–closed in X for every closed subset V of Y.
(xxiv) swg–continuous[18] if $f^{-1}(V)$ is swg–closed in X for every closed subset V of Y.
(xxv) rω–continuous[20] if $f^{-1}(V)$ is rw–closed in X for every closed subset V of Y.
(xxvi) rgw–continuous23] if $f^{-1}(V)$ is rgw–closed in X for every closed subset V of Y.

Definition 2.4: A map f: (X,τ)→(Y,σ) is said to be
(i) α–irresolute [7] if $f^{-1}(V)$ is α-closed in X for every α-closed subset V of Y.
(ii) irresolute [3] if $f^{-1}(V)$ is semi- closed in X for every semi-closed subset V of Y.

(iii) contra ω–irresolute [17] if $f^{-1}(V)$ is ω-open in X for every ω-closed subset V of Y.
(iv) contra irresolute [7] if $f^{-1}(V)$ is semi-open in X for every semi-closed subset V of Y.
(v) contra r–irresolute [1] if $f^{-1}(V)$ is regular-open in X for every regular-closed subset V of Y
(vi) contra continuous [30] if $f^{-1}(V)$ is open in X for every closed subset V of Y .
(vii) rω*-open(resp rω*-closed) [20] map if f(U) is rω-open (resp rω-closed) in Y for every rω-open (resp rω-closed) subset U of X.

Lemma 2.5 see[4] :
1) Every closed (resp regular-closed, α-closed) set is αrω-closed set in X.
2) Every αrω-closed set is αg-closed set
3) Every αrω-closed set is αgr-closed (resp ωα-closed, gs-closed, gspr-closed, wg-closed, rwg-closed , gp-closed, gpr-closed) set in X

Lemma 2.6: see [4] If a subset A of a topological space X, and
1) If A is regular open and αrω-closed then A is α-closed set in X
2) If A is open and αg-closed then A is αrω-closed set in X
3) If A is open and gp-closed then A is αrω-closed set in X
4) If A is regular open and gpr-closed then A is αrω-closed set in X
5) If A is open and wg-closed then A is αrω-closed set in X
6) If A is regular open and rwg-closed then A is αrω-closed set in X
7) If A is regular open and αgr-closed then A is αrω-closed set in X
8) If A is ω-open and ωα-closed then A is αrω-closed set in X

Lemma 2.7: see [4] If a subset A of a topological space X, and
1) If A is semi-open and sg-closed then it is αrω-closed.
2) If A is semi-open and ω-closed then it is αrω-closed.
3) A is αrω-open iff $U \subseteq \alpha int(A)$, whenever U is rω-closed and $U \subseteq A$.

Definition 2.8 : A topological space (X, τ) is called
(1) an α-space if every α-closed subset of X is closed in X.

III. 3. arω– Continuous Functions:

Definition 3.1: A function f from a topological space X into a topological space Y is called α regular ω continuous (αrω–Continuous) if $f^{-1}(V)$ is αrω–Closed set in X for every closed set V in Y.

Theorem 3.2: If a map f is continuous, then it is αrω–continuous but not converserly.
Proof: Let f: X→Y be continuous. Let F be any closed set in Y. Then the inverse image $f^{-1}(F)$ is closed set in X. Since every closed set is αrω–closed Lemma 2.5, $f^{-1}(F)$ is αrω–closed in X. Therefore f is αrω–continuous.

Theorem 3.3: If a map f: X→Y is α–continuous, then it is αrω–continuous but not converserly.
Proof: Let f: X→Y be α–continuous. Let F be any closed set in Y. Then the inverse image $f^{-1}(F)$ is α–closed set in X. Since every α–closed set is αrω– closed by Lemma 2.5, $f^{-1}(F)$ is αrω– closed in X. Therefore f is αrω–continuous.

The converse need not be true as seen from the following example.
Example 3.5: Let X=Y={a,b,c,d}, τ ={X,ϕ,{a},{b},{a,b},{a,b,c}} and σ ={Y, ϕ,{a},{b},{a,b},{a,b,c}}Let map f: X→Y defined by f(a)=c , f(b)= a , f(c)=b, f(d)=d , then f is αrω–continuous but not continuous and not α–continuous , as closed set F= {c,d} in Y, then $f^{-1}(F)$={a,d} in X which is not α–closed , not closed set in X.

Theorem 3.6: If a map f: X→Y is continuous, Then the following holds.
(i) If f is αrω–continuous, then f is αg–continuous.
(ii) If f is αrω–continuous, then f is wg–continuous (resp gs–continuous, rwg–continuous, gp–continuous, gspr–continuous, gpr–continuous, ωα–continuous, αgr–continuous).

Proof: (i) Let F be a closed set in Y. Since F is αrω–continuous, then $f^{-1}(F)$ is αrω–closed in X. Since every αrω–closed set is αg–closed by Lemma 2.5, then $f^{-1}(F)$ is αg–closed in X. Hence f is αg–continuous.
Similarly we can prove (ii).

The converse need not be true as seen from the following example.
Example 3.7: Let X=Y={a,b,c},τ = {X, ϕ,{a},{b,c}} σ ={Y, ϕ,{a}}, Let map f: X→Y defined by , f(a)=b , f(b)=a , f(c)=c then f is αg–continuous, wg–continuous, gs–continuous, gp–continuous, gspr–continuous, gpr–continuous , rwg–continuous , αgr–continuous but not αrω–continuous as closed set F= {b,c} in Y, then $f^{-1}(F)$={a,c} in X which is not αrω–closed set in X.

Remark 3.8: The following examples shows that αrω-continuous maps are independent of pre–continuous, β–continuous, g–continuous, ω–continuous, rω–continuous, swg–continuous , rgw–continuous , wgrα–continuous , rgα–continuous, gprw–continuous, g*p–continuous, gr–continuous, R*–continuous , rps–continuous , semi-continuous .

Example 3.9: Let X=Y={a,b,c}, τ = {X, ϕ,{a},{b,c}} σ ={Y, ϕ,{a}}, Let map f: X→Y defined by , f(a)=b , f(b)=a , f(c)=c then f is pre–continuous, β–continuous, g–continuous, ω–continuous, rω–continuous, swg–continuous, rgw–continuous, wgrα–continuous, rgα–continuous, gprw–continuous, g*p–continuous, gr–continuous, R*–continuous , rps–continuous but f is not αrω–continuous , as closed set F= {b,c} in Y, then f^{-1}(F)={a,c} in X , which is not αrω–closed set in X.

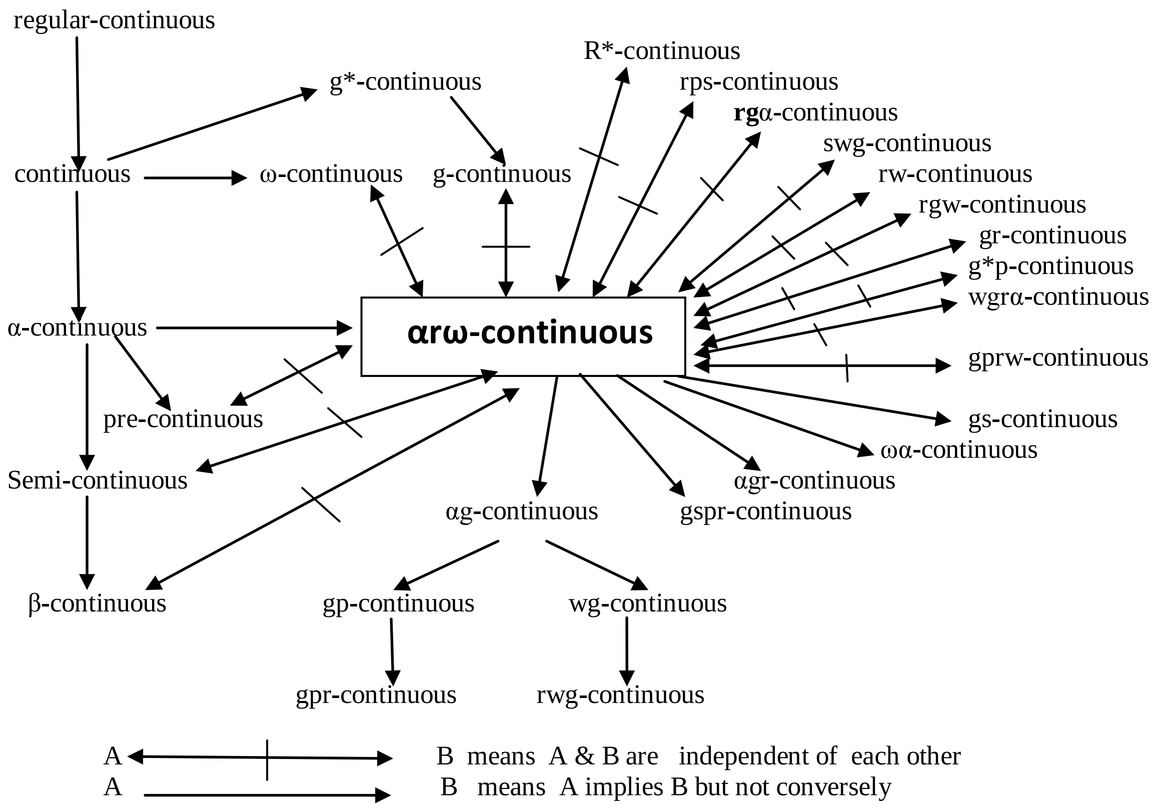
Example 3.10: X={a,b,c,d}, Y={a,b,c} τ = {X, ϕ,{a},{b},{a,b},{a,b,c}} σ ={Y, ϕ,{a}}, Let map f: X→Y defined by , f(a)=b , f(b)=a , f(c)=a , f(d)=c then f is αrω–continuous but f is not gprw–continuous, rps–continuous, wgrα–continuous , rgw–continuous, rgα–continuous, swg–continuous, pre–continuous, R*–continuous , rω–continuous , ω-continuous , as closed set F= {b,c} in Y, then f^{-1}(F)={a,d} in X , which is not gprw -closed (resp rps–closed, wgrα–closed, rgw–closed , rgα–closed, swg–closed, pre–closed, R*–closed, rω–closed , ω-closed) set in X.

Example 3.11: X=Y={a,b,c,d}, τ = {X, ϕ {a},{b},{a,b},{a,b,c}} σ ={Y, ϕ,{b,c},{b,c,d},{a,b,c}}, Let map f: X→Y defined by , f(a)=c , f(b)=b , f(c)=a , f(d)=d then f is αrω–continuous but f is not R*–continuous , rω–continuous, ω-continuous, gr-continuous ,g–continuous ,g*p-continuous , as closed set F={a} in Y, then f^{-1}(F)={c} in X, which is not R*-closed (resp rω–closed, ω–closed, gr–closed, g-closed, g*p-closed) set in X.

Example 3.12: X={a,b,c,d}, Y={a,b,c} τ = {X, ϕ,{a},{b},{a,b},{a,b,c}} σ ={Y, ϕ,{a}}, Let map f: X→Y defined by , f(a)=b , f(b)=a , f(c)=c , f(d)=b then f is αrω–continuous but f is not semi–continuous, β–continuous , as closed set F= {b,c} in Y, then f^{-1}(F)={a,c,d} in X which is not semi-closed (resp β–closed) set in X.

Example 3.13: Let X=Y={a,b,c}, τ = {X, ϕ,{a},{b},{a,b}} σ ={Y, ϕ,{a},{b,c}}, Let map f: X→Y defined by , f(a)=b , f(b)=a , f(c)=c then f is semi–continuous, β–continuous but f is not αrω–continuous , as closed set F= {a} in Y, then f^{-1}(F)={b} in X , which is not αrω–closed set in X.

Remark 3.14: From the above discussion and know results we have the following implications. **(Fig**)



Theorem 3.15: Let f: X→Y be a map. Then the following statements are equivalent :
(i) f is αrω–continuous.
(ii) the inverse image of each open set in Y is αrω–open in X
Proof: Assume that f: X→Y is αrω–continuous. Let G be open in Y. The G^c is closed in Y. Since f is αrω–continuous, $f^{-1}(G^c)$ is αrω–closed in X. But $f^{-1}(G^c) = X–f^{-1}(G)$. Thus $f^{-1}(G)$ is αrω–open in X.
Converserly , Assume that the inverse image of each open set in Y is αrω–open in X. Let F be any closed set in Y. By assumption $f^{-1}(F^c)$ is αrω–open in X. But $f^{-1}(F^c) = X–f^{-1}(F)$. Thus X– $f^{-1}(F)$ is αrω–open in X and so $f^{-1}(F)$ is αrω–closed in X. Therefore f is αrω–continuous. Hence (i) and (ii) are equivalent.

Theorem 3.16: If f: (X,τ)→(Y,σ) is map . Then the following holds.
1) f is αrω–continuous and contra r–irresolute map then f is α–continuous
2) f is αg–continuous and contra continuous map then f is αrω –continuous.
3) f is gp–continuous and contra continuous map then f is αrω –continuous
4) f is gpr–continuous and contra r–irresolute map then f is αrω –continuous.
5) f is wg–continuous and contra continuous e map then f is αrω –continuous
6) f is rwg–continuous and contra r–irresolute map then f is αrω –continuous
7) f is αgr–continuous and contra r–irresolute map then f is αrω –continuous
8) f is ωα–continuous and contra ω–irresolute map then f is αrω –continuous
Proof:
1) Let V be regular closed set of Y, As every regular set is closed, V is closed set in Y. Since f is αrω–continuous and contra r–irresolute map, $f^{-1}(V)$ is αrω–closed and regular open in X , Now by Lemma 2.6 , $f^{-1}(V)$ is α-closed in X. Thus f is α–continuous.
2) Let V be closed set of Y. Since f is αg–continuous and contra continuous map, $f^{-1}(V)$ is αg–closed and open in X , Now by Lemma 2.6, $f^{-1}(V)$ is αrω–closed in X. Thus f is αrω–continuous.

Similarly , we can prove 3), 4) , 5) , 6), 7), 8) .

Theorem 3.17: If f: (X,τ)→(Y,σ) is map . Then the following holds.
1) f is sg–continuous and contra irresolute map then f is αrω–continuous.
2) f is ω–continuous and contra irresolute map then f is αrω –continuous
Proof:
1) Let V be closed set of Y. As every closed set is semi-closed, V is semi-closed set in Y. Since f is sg–continuous and contra irresolute map, $f^{-1}(V)$ is sg–closed and semi-open in X , Now by Lemma 2.7, $f^{-1}(V)$ is αrω–closed in X. Thus f is αrω–continuous.
2) The proof is in the similar manner.

Theorem 3.18: Let A be a subset of a topological space X. Then x ∈ αrωcl(A) if and only if for any αrω–open set U containing x, A∩U≠ ϕ.
Proof: Let x ∈ αrωcl(A) and suppose that, there is a αrω–open set U in X such that x ∈ U and A∩U= ϕ implies that A ⊂ U^c which is αrω–closed in X implies αrωcl(A) ⊆ αrωcl(U^c) = U^c. since x ∈ U implies that x ∉ U^c implies that x ∉ αrωcl(A), this is a contradiction.
Converserly, Suppose that, for any αrω–open set U containing x, A∩U≠ ϕ. To prove that x ∈ αrωcl(A). Suppose that x ∉ αrωcl(A),then there is a αrω–closed set F in X such that x ∉ F and A ⊆ F. Since x ∉ F implies that x ∈ F^c which is αrω–open in X. Since A ⊆ F implies that A ∩ F^c = ϕ, this is a contradiction. Thus x ∈ αrωcl(A).

Theorem 3.19: Let f: X→Y be a function from a topological space X into a topological space Y. If f: X→Y is αrω–continuous, then f(αrωcl(A)) ⊆ cl(f(A)) for every subset A of X.
Proof: Since f(A) ⊆ cl(f(A)) implies that A ⊆ f^{-1}(cl(f(A))). Since cl(f(A)) is a closed set in Y and f is αrω–continuous, then by definition f^{-1}(cl(f(A))) is a αrω–closed set in X containing A. Hence αrωcl(A)⊆f^{-1}(cl(f(A))). Therefore f(αrωcl(A)) ⊆ cl(f(A)).

The converse of the above theorem need not be true as seen from the following example
Example 3.20 : Let X=Y={a,b,c,d}, τ = {X, ϕ, {a},{c,d},{a,c,d}} σ ={Y, ϕ,{b,c},{b,c,d},{a,b,c}}, Let map f: X→Y defined by , f(a)=b , f(b)=d , f(c)=c , f(d)=d . For every subset of X, f(αrωcl(A)) ⊆ cl(f(A)) holds . But f is not αrω–continuous since closed set V={d} in Y , f^{-1}(V)={b,d} which is not αrω–closed set in X.

Theorem 3.21 : Let f: X→Y be a function from a topological space X into a topological space Y. Then the following statements are equivalent:
(i) For each point x in X and each open set V in Y with f(x) ∈ V, there is a αrω–open set U in X such that x ∈ U and f(U) ⊆ V.

(ii) For each subset A of X, $f(\alpha r\omega cl(A)) \subseteq cl(f(A))$.
(iii) For each subset B of Y, $\alpha r\omega cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$.
(iv) For each subset B of Y, $f^{-1}(int(B)) \subseteq \alpha r\omega int(f^{-1}(B))$.
Proof:

(i) → (ii) Suppose that (i) holds and let $y \in f(\alpha r\omega cl(A))$ and let V be any open set of Y. Since $y \in f(\alpha r\omega cl(A))$ implies that there exists $x \in \alpha r\omega cl(A)$ such that f(x) = y. Since $f(x) \in V$, then by (i) there exists a αrω–open set U in X such that $x \in U$ and $f(U) \subseteq V$. Since $x \in f(\alpha r\omega cl(A))$, then by theorem 3.18 $U \cap A \neq \phi$. $\phi \neq f(U \cap A) \subseteq f(U) \cap f(A) \subseteq V \cap f(A)$, then $V \cap f(A) \neq \phi$ .Therefore we have $y = f(x) \in cl(f((A))$. Hence $f(\alpha r\omega cl(A)) \subseteq cl(f(A))$.
(ii) → (i) Let if (ii) holds and let $x \in X$ and V be any open set in Y containing f(x). Let $A = f^{-1}(V^c)$ this implies that $x \notin A$. Since $f(\alpha r\omega cl(A)) \subseteq cl(f(A)) \subseteq V^c$ this implies that $\alpha r\omega cl(A) \subseteq f^{-1}(V^c) = A$. Since $x \notin A$ implies that $x \notin \alpha r\omega cl(A)$ and by theorem 3.18 there exists a αrω–open set U containing x such that $U \cap A = \phi$,then $U \subseteq A^c$ and hence $f(U) \subseteq f(A^c) \subseteq V$.

(ii) → (iii) Suppose that (ii) holds and Let B be any subset of Y. Replacing A by $f^{-1}(B)$ we get from (ii) $f(\alpha r\omega cl(f^{-1}(B))) \subseteq cl(f(f^{-1}(B))) \subseteq cl(B)$. Hence $\alpha r\omega cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$.
(iii) → (ii) Suppose that (iii) holds, let B = f(A) where A is a subset of X. Then we get from (iii) , $\alpha r\omega cl(f^{-1}(f(A)) \subseteq f^{-1}(cl(f(A)))$ implies $\alpha r\omega cl(A) \subseteq f^{-1}(cl(f(A)))$. Therefore $f(\alpha r\omega cl(A)) \subseteq cl(f(A))$.

(iii) → (iv) Suppose that (iii) holds. Let $B \subseteq Y$, then $Y–B \subseteq Y$. By (iii) , $\alpha r\omega cl(f^{-1}(Y–B)) \subseteq f^{-1}(cl(Y–B))$ this implies $X– \alpha r\omega int(f^{-1}(B)) \subseteq X– f^{-1}(int(B))$. Therefore $f^{-1}(int(B)) \subseteq \alpha r\omega int(f^{-1}(B))$.
(iv) → (iii) Suppose that (iv) holds Let $B \subseteq Y$, then $Y–B \subseteq Y$. By (iv), $f^{-1}(int(Y–B)) \subseteq \alpha r\omega int(f^{-1}(Y–B))$ this implies that $X– f^{-1}(cl(B)) \subseteq X–\alpha r\omega cl(f^{-1}(B))$. Therefore $\alpha r\omega cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$.

Definition 3.22: Let (X, τ) be topological space and $\tau_{\alpha r\omega} = \{V \subseteq X : \alpha r\omega–cl(V^c) = V^c \}$, $\tau_{\alpha r\omega}$ is toplogy on X.

Definition 3.23: 1) A space (X, τ) is called $T_{\alpha r\omega}$–space if every αrω–closed is closed.
2) A space (X, τ) is called ${}_{\alpha r\omega}T_{\alpha}$ – space if every αrω–closed set is α-closed set.

Theorem 3.24: Let f: X→Y be a function. Let (X,τ) and (Y,σ) be any two spaces such that $\tau_{\alpha r\omega}$ is a topology on X. Then the following statements are equivalent:
(i) For every subset A of X, $f(\alpha r\omega cl(A)) \subseteq cl(f(A))$ holds,
(ii) f: $(X, \tau_{\alpha r\omega}) \to (Y,\sigma)$ is continuous.
Proof: Suppose (i) holds. Let A be closed in Y. By hypothesis $f(\alpha r\omega cl(f^{-1}(A))) \subseteq cl(f(f^{-1}(A))) \subseteq cl(A) = A$. i.e. $\alpha r\omega cl(f^{-1}(A)) \subseteq f^{-1}(A)$. Also $f^{-1}(A) \subseteq \alpha r\omega cl(f^{-1}(A))$. Hence $\alpha r\omega cl(f^{-1}(A)) = f^{-1}(A)$. This implies $f^{-1}(A) \in \tau_{\alpha r\omega}$. Thus $f^{-1}(A)$ is closed in $(X, \tau_{\alpha r\omega})$ and so f is continuous. This proves (ii).

Suppose (ii) holds. For every subset A of X, cl(f(A)) is closed in Y. Since f: $(X, \tau_{\alpha r\omega}) \to (Y,\sigma)$ is continuous, $f^{-1}(cl(A))$ is closed in $(X, \tau_{\alpha r\omega})$ that implies by Definition 3.22 $\alpha r\omega cl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$. Now we have, $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(cl(f(A)))$ and by αrω-closure, $\alpha r\omega cl(A) \subseteq \alpha r\omega cl(f^{-1}(cl(f(A))) = f^{-1}(cl(f(A))$. Therefore $f(\alpha r\omega cl(A)) \subseteq cl(f(A))$. This proves (i).

Remark 3.25 : The Composition of two αrω–continuous maps need not be αrω–continuous map and this can be shown by the following example.

Example 3.26 : Let X=Y=Z={a,b,c} , τ = {X, ϕ,{a},{b},{a,b}} , σ ={Y, ϕ,{a}} , η ={Z, ϕ,{a},{a,b},{a,c}} and a maps f : X→Y , g : Y→Z and gof : X→Z are identity maps. Both f and g are αrω–continuous maps. But g∘f not αrω–continuous map , since closed set V={b} in Z, $f^{-1}(V)$={b} which is not αrω–closed set in X.

Theorem 3.27: Let f: X→Y is αrω–continuous function and g:Y→Z is continuous function then g∘f: X→Z is αrω–continuous.
Proof: Let g be continuous function and V be any open set in Z then$g^{-1}(V)$ is open in Y. Since f is αrω–continuous, $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is αrω–open in X. Hence g∘f is αrω–continuous.

Theorem 3.28: Let f: X→Y is αrω–continuous function and g:Y→Z is αrω–continuous function and Y is $T_{\alpha r\omega}$–space , then g∘f: X→Z is αrω–continuous.
Proof: Let g be αrω–continuous function and V be any open set in Z then $g^{-1}(V)$ is αrω–open in Y and Y is $T_{\alpha r\omega}$–space, then $g^{-1}(V)$ is open in Y. Since f is αrω–continuous, $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is αrω–open in X. Hence g∘f is αrω–continuous.

Theorem 3.29: If a map f: X→Y is completely–continuous, then it is αrω– continuous.
Proof : Suppose that a map f: (X,τ)→(Y,σ) is completely–continuous. Let F closed set in Y. Then $f^{-1}(F)$ is regular closed in X and hence $f^{-1}(F)$ is is αrω–closed in X. Thus f is αrω–continuous.

Theorem 3.30: If a map f: X→Y is α–irresolute, then it is αrω– continuous.
Proof : Suppose that a map f: (X,τ)→(Y,σ) is α–irresolute. Let V be an open set in Y. Then V is α-open in Y. Since f is α–irresolute , $f^{-1}(V)$ is α–open and hence αrω–open in X. Thus f is αrω–continuous.

Definition 3.31: A function f from a topological space X into a topological space Y is called perfectly α regular ω continuous (briefly perfectly αrω–Continuous) if $f^{-1}(V)$ is clopen (closed and open) set in X for every αrω–open set V in Y.

Theorem 3.32: If a map f: X→Y is continuous, Then the following holds.
(i) If f is perfectly αrω–continuous, then f is αrω–continuous.
(ii) If f is perfectly αrω–continuous, then f is αg–continuous.
(iii) If f is perfectly αrω–continuous, then f is wg–continuous(resp gs–continuous, rwg–continuous, gp–continuous, gspr–continuous, gpr–continuous, ωα–continuous, αgr–continuous).
Proof:
(i) Let F be a open set in Y, as every open is αrω–open in Y , since F is perfectly αrω–continuous, then $f^{-1}(F)$ is both closed and open in X, as every open is αrω–open , $f^{-1}(F)$ is αrω–open in X. Hence f is αrω–continuous.
(ii) Let F be a open set in Y, as every open is αrω–open in Y, since F is perfectly αrω–continuous, then $f^{-1}(F)$ is both closed and open in X, as every open is αrω–open that implies is αg–open, then $f^{-1}(F)$ is αg–open in X. Hence f is αg–continuous.
Similarly , we can prove (iii).

Definition 3.33: A function f from a topological space X into a topological space Y is called α regular ω*- continuous (briefly αrω*–continuous) if $f^{-1}(V)$ is αrω–closed set in X for every α-closed set V in Y.

Theorem 3.34: If A map f: (X,τ)→(Y,σ) be function,
(i) f is αrω–irresolute then it is αrω*–continuous.
(ii) f is αrω*–continuous then it is αrω–continuous.
Proof:
(i) Let f: X→Y be αrω–irresolute. Let F be any α-closed set in Y. Then F is αrω–closed in Y. Since f is αrω–irresolute , the inverse image $f^{-1}(F)$ is αrω–closed set in X. Therefore f is αrω*–continuous.
(ii) Let f: X→Y be αrω*–continuous. Let F be any closed set in Y. Then F is α–closed in Y. Since f is αrω*–continuous., the inverse image $f^{-1}(F)$ is αrω–closed set in X. Therefore f is αrω–continuous.

Theorem 3.35: If a bijection f: (X,τ)→(Y,σ) is rω*-open , αrω*–continuous , then it is αrω–irresolute.
Proof: Let A be αrω–closed in Y. Let $f^{-1}(A) \subseteq U$ where U is rω-open set in X, Since f is rω*-open map, f(U) is rω-open set in Y. $A \subseteq f(U)$ implies $\alpha cl(A) \subseteq f(U)$. That is , $f^{-1}(\alpha cl(A)) \subseteq U$. Since f is αrω*–continuous , $\alpha cl(f^{-1}(\alpha cl(A))) \subseteq U$. and so $\alpha cl(f^{-1}(A)) \subseteq U$ This shows $f^{-1}(A)$ is αrω–closed set in X. Hence f is αrω–irresolute.

Theorem 3.36 If f: (X,τ)→(Y,σ) is αrω-continuous and rω*-closed and if A is αrω-open(or αrω-closed) subset of (Y,σ) and (Y,σ) is α-space, then $f^{-1}(A)$ is αrω-open (or αrω -closed) in (X,τ).
Proof : Let A be a αrω-open set in (Y,σ) and G be any rω-closed set in (X,τ) such that $G \subseteq f^{-1}(A)$. Then $f(G) \subseteq A$. By hypothesis f(G) is rω-closed and A is αrω-open in (Y,σ). Therefore $f(G) \subseteq \alpha Int(A)$ by Lemma 2.7 and so $G \subseteq f^{-1}(\alpha Int(A))$. Since f is αrω-continuous, αInt(A) is α-open in (Y,σ) and (Y,σ) is α-space, so αInt(A) is open in (Y,σ). Therefore $f^{-1}(\alpha Int(A))$ is αrω-open in ((X,τ). Thus $G \subseteq \alpha Int(f^{-1}(\alpha Int(A))) \subseteq \alpha Int(f^{-1}(A))$; that is, $G \subseteq \alpha Int(f^{-1}(A))$, $f^{-1(}A)$ is αrω-open in (X,τ).
By taking the complements we can show that if A is rω-closed in (Y,σ), $f^{-1}(A)$ is αrω-closed in (X,τ).

Theorem 3.37: Let (X,τ) be a discrete topological space and (Y,σ) be any topological space. Let f: (X,τ)→ (Y,σ) be a map. Then the following statements are equivalent:
(i) f is strongly αrω-continuous.
(ii) f is perfectly αrω-continuous.
Proof:

(i)=>(ii) Let U be any αrω -open set in (Y,σ).By hypothesis $f^{-1}(U)$ is open in (X,τ). Since(X,τ) is a discrete space,$f^{-1}(U)$ is also closed in (X,τ). $f^{-1}(U)$ is both open and closed in (X,τ). Hence f is perfectly αrω-continuous.
(ii)=>(i) Let U be any αrω-open set in (Y,σ). Then $f^{-1}(U)$ is both open and closed in (X,τ). Hence f is strongly αrω-continuous.

IV. αrω–IRRESOLUTE AND STRONGLY αrω–CONTINUOUS FUNCTIONS:
V.

Definition 4.1: A function f from a topological space X into a topological space Y is called α regular ω irresolute (αrω–irresolute) map if $f^{-1}(V)$ is αrω–Closed set in X for every αrω–closed set V in Y.

Definition 4.2: A function f from a topological space X into a topological space Y is called strongly α regular ω continuous (strongly αrω–continuous) map if $f^{-1}(V)$ is closed set in X for every αrω–closed set V in Y.

Theorem 4.3: If A map f: (X,τ)→(Y,σ) is αrω–irresolute, then it is αrω–continuous but not conversely.
Proof: Let f: X→Y be αrω–irresolute. Let F be any closed set in Y. Then F is αrω–closed in Y. Since f is αrω–irresolute , the inverse image $f^{-1}(F)$ is αrω–closed set in X. Therefore f is αrω–continuous.

The converse of the above theorem need not be true as seen from the following example.
Example 4.4 : X={a,b,c,d}, Y={a,b,c} τ = {X, ϕ,{a},{b},{a,b},{a,b,c}} σ ={Y, ϕ,{a}}, Let map f: X→Y defined by , f(a)=b , f(b)=a , f(c)=a , f(d)=c then f is αrω–continuous but f is not αrω–irresolute, as αrω-closed set F= {b} in Y, then $f^{-1}(F)$={a} in X , which is not αrω–closed set in X.

Theorem 4.5: If A map f: (X,τ)→(Y,σ) is αrω–irresolute, if and only if the inverse image $f^{-1}(V)$ is αrω–open set in X for every αrω–open set V in Y.
Proof: Assume that f: X→Y is αrω–irresolute. Let G be αrω–open in Y. The G^c is αrω–closed in Y. Since f is αrω– irresolute, $f^{-1}(G^c)$ is αrω–closed in X. But $f^{-1}(G^c) = X–f^{-1}(G)$. Thus $f^{-1}(G)$ is αrω–open in X.

Converserly , Assume that the inverse image of each open set in Y is αrω–open in X. Let F be any αrω–closed set in Y. By assumption $f^{-1}(F^c)$ is αrω–open in X. But $f^{-1}(F^c) = X–f^{-1}(F)$. Thus X– $f^{-1}(F)$ is αrω–open in X and so $f^{-1}(F)$ is αrω–closed in X. Therefore f is αrω– irresolute.

Theorem 4.6: If A map f: (X,τ)→(Y,σ) is αrω–irresolute, then for every subset A of X ,f(αrωcl(A)⊂ αcl(f(A)).
Proof : If A⊂ X then consider αcl(f(A)) which is αrω-closed in Y. since f is αrω–irresolute , f^{-1}(αcl(f(A))) is αrω-closed in X. Furthermore A⊆ f^{-1}(f(A))⊆ f^{-1}(αcl(f(A))). Therefore by αrω-closure, αrωcl(A)⊆f^{-1}(αcl(f(A))), consequently , f(αrωcl(A) ⊆ f(f^{-1}(αcl(f(A)))) ⊆αclf((A)).

Theorem 4.7: Let f: (X, τ)→(Y,σ) and g: (Y, σ) →(Z,η) be any two functions. Then
(i) g o f : (X, τ)→(Z,η) is αrω–continuous if g is r-continuous and f is αrω– irresolute.
(ii) g o f : (X, τ)→(Z,η) is αrω–irresolute if g is αrω–irresolute a nd f is αrω–irresolute.
(iii) g o f : (X, τ)→(Z,η) is αrω–continuous if g is αrω–continuous and f is αrω–irresolute.
Proof:
(i) Let U be a open set in (Z, η) . Since g is r-continuous, $g^{-1}(U)$ is r-open set in (Y, σ). Since every r-open is αrω–open then $g^{-1}(U)$ is αrω–open in Y , siince f is αrω–irresolute $f^{-1}(g^{-1}(U))$ is an αrω–open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an αrω–open set in (X, τ) and hence gof is αrω–continuous.
(ii) Let U be a αrω–open set in (Z, η) . Since g is αrω–irresolute, $g^{-1}(U)$ is αrω–open set in (Y, σ). Since f is αrω–irresolute, $f^{-1}(g^{-1}(U))$ is an αrω–open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an αrω–open set in (X, τ) and hence gof is αrω– irresolute.
(iii) Let U be a open set in (Z, η) . Since g is continuous, $g^{-1}(U)$ is open set in (Y, σ). As every open set is αrω–open, $g^{-1}(U)$ is αrω–open set in (Y, σ). Since f is αrω– irresolute $f^{-1}(g^{-1}(U))$ is an αrω–open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an αrω–open set in (X, τ) and hence gof is αrω–continuous.

Theorem 4.8: Let f: (X, τ)→(Y,σ) is strongly αrω–continuous then it is continuous.
Proof: Assume that f: (X, τ)→(Y,σ) is strongly αrω–continuous, Let F be closed set in Y. As every closed is αrω–closed, F is αrω–closed in Y. since f is strongly αrω–continuous then $f^{-1}(F)$ is closed set in X. Therefore f is continuous.

Theorem 4.9: Let f: (X, τ)→(Y,σ) is strongly αrω–continuous then it is strongly α–continuous but not conversely.

Proof: Assume that f: (X, τ)→(Y,σ) is strongly αrω–continuous, Let F be α-closed set in Y. As every α-closed is αrω–closed, F is αrω–closed in Y. since f is strongly αrω–continuous then $f^{-1}(F)$ is closed set in X. Therefore f is strongly α–continuous.

The converse of the above theorem 4.9 need not be true as seen from the following example

Example 4.10: Let X=Y={a,b,c,d}, τ ={X, ϕ,{a},{b},{a,b},{a,b,c}} and σ ={Y,ϕ,{a},{b},{a,b},{a,b,c}}Let map f: X→Y defined by f(a)=a , f(b)=f(c)=f(d)=b , then f is strongly α–continuous but not continuous and not strongly αrω–continuous , as closed set F={a,c,d} in Y, then $f^{-1}(F)$={a} in X which is not closed set in X.

Theorem 4.11: Let f: (X, τ)→(Y,σ) is strongly αrω–continuous if and only if $f^{-1}(G)$ is open set in X for every αrω–open set G in Y.
Proof : Assume that f: X→Y is strongly αrω–continuous. Let G be αrω–open in Y. The G^c is αrω–closed in Y. Since f is strongly αrω–continuous, $f^{-1}(G^c)$ is closed in X. But $f^{-1}(G^c) = X–f^{-1}(G)$. Thus $f^{-1}(G)$ is open in X.

Converserly , Assume that the inverse image of each open set in Y is αrω–open in X. Let F be any αrω–closed set in Y. By assumption F^c is αrω–open in X. But $f^{-1}(F^c) = X–f^{-1}(F)$. Thus $X– f^{-1}(F)$ is open in X and so $f^{-1}(F)$ is closed in X. Therefore f is strongly αrω–continuous.

Theorem 4.12: Let f: (X, τ)→(Y,σ) is strongly continuous then it is strongly αrω–continuous.
Proof: Assume that f: X→Y is strongly continuous. Let G be αrω–open in Y and also it is any subset of Y since f is strongly continuous, $f^{-1}(G)$ is open (and also closed) in X. $f^{-1}(G)$ is open in X Therefore f is strongly αrω–continuous.

Theorem 4.13: Let f: (X, τ)→(Y,σ) is strongly αrω–continuous then it is αrω–continuous.
Proof: Let G be open in Y , every open is αrω–open , G is αrω–open in Y, since f is strongly αrω–continuous, $f^{-1}(G)$ is open in X. and therefore $f^{-1}(G)$ is αrω–open in X . Hence f is αrω–continuous.

Theorem 4.14: In discrete space**,** a map f: (X, τ)→(Y,σ) is strongly αrω–continuous then it is strongly continuous.
Proof: F any subset of Y, in discrete space, Every subset F in Y is both open and closed , then subset F is both αrω–open or αrω–closed, i) let F is αrω–closed in Y, since f is strongly αrω–continuous, then $f^{-1}(F)$ is closed in X. ii) let F is αrω–open in Y, since f is strongly αrω–continuous, then $f^{-1}(F)$ is open in X. Therefore $f^{-1}(F)$ is closed and open in X. Hence f is strongly continuous.

Theorem 4.15 : Let f: (X, τ)→(Y,σ) and g: (Y, σ) →(Z,η) be any two functions. Then
(i) g o f : (X, τ)→(Z,η) is strongly αrω–continuous if g is strongly αrω–continuous and f is strongly αrω–continuous.
(ii) g o f : (X, τ)→(Z,η) is strongly αrω–continuous if g is strongly αrω–continuous and f is continuous.
(iii) g o f : (X, τ)→(Z,η) is αrω–irresolute if g is strongly αrω–continuous and f is αrω–continuous.
(iv) g o f : (X, τ)→(Z,η) is continuous if g is αrω–continuous and f is strongly αrω–continuous
Proof:
(i) Let U be a αrω–open set in (Z, η) . Since g is strongly αrω–continuous, $g^{-1}(U)$ is open set in (Y, σ). As every open set is αrω–open, $g^{-1}(U)$ is αrω–open set in (Y, σ). Since f is strongly αrω–continuous $f^{-1}(g^{-1}(U))$ is an open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is strongly αrω–continuous.
(ii) Let U be a αrω–open set in (Z, η) . Since g is strongly αrω–continuous, $g^{-1}(U)$ is open set in (Y, σ). Since f is continuous $f^{-1}(g^{-1}(U))$ is an open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is strongly αrω–continuous.
(iii) Let U be a αrω–open set in (Z, η) . Since g is strongly αrω–continuous, $g^{-1}(U)$ is open set in (Y, σ). Since f is αrω–continuous $f^{-1}(g^{-1}(U))$ is an αrω–open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an αrω–open set in (X, τ) and hence gof is αrω–irresolute
(iv) Let U be open set in (Z, η) . Since g is αrω–continuous, $g^{-1}(U)$ is αrω–open set in (Y, σ). Since f is strongly αrω–continuous $f^{-1}(g^{-1}(U))$ is an open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is continuous.

Theorem 4.16 : Let f: (X, τ)→(Y,σ) and g: (Y, σ) →(Z,η) be any two functions. Then
1. g o f : (X, τ)→(Z,η) is strongly αrω–continuous if g is perfectly αrω–continuous and f is continuous.

2. g o f : (X, τ)→(Z,η) is perfectly αrω–continuous if g is strongly αrω–continuous and f is perfectly αrω–continuous.

Proof:

1. Let U be a αrω–open set in (Z, η) . Since g is perfectly αrω–continuous, $g^{-1}(U)$ is clopen set in (Y, σ). $g^{-1}(U)$ is open set in (Y, σ). Since f is continuous $f^{-1}(g^{-1}(U))$ is an open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is strongly αrω–continuous .
2. Let U be a αrω–open set in (Z, η) . Since g is strongly αrω–continuous, $g^{-1}(U)$ is open set in (Y, σ). $g^{-1}(U)$ is open set in (Y, σ). Since f is perfectly αrω-continuous , $f^{-1}(g^{-1}(U))$ is an clopen set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an clopen set in (X, τ) and hence gof is perfectly αrω–continuous.

Theorem 4.17: If A map f: (X, τ)→(Y,σ) is strongly αrω–continuous and A is open subset of X then the restriction f/A: A→Y is strongly αrω–continuous.
Proof: Let V be any αrω-open set of Y, since f is strongly αrω–continuous , then $f^{-1}(V)$ is open in X. since A is open in X, $(f/A)^{-1}(V)=A\cap f^{-1}(V)$ is open in A. hence f/A is strongly αrω–continuous.

Theorem: 4.18 Let (X,τ) be any topological space and (Y,σ) be a $T_{\alpha r\omega}$-space and f: (X,τ) →(Y,σ) be a map. Then the following are equivalent:
(i) f is strongly αrω-continuous.
(ii) f is continuous.
Proof:
(i) =>(ii) Let U be any open set in (Y,σ). Since every open set is αrω-open, U is αrω-open in (Y,σ). Then $f^{-1}(U)$ is open in (X,τ). Hence f is continuous.
(ii) =>(i) Let U be any αrω-open set in (Y,σ). Since (Y,σ) is a $T_{\alpha r\omega}$-space, U is open in (Y,σ). Since f is continuous. Then $f^{-1}(U)$ is open in (X,τ). Hence f is strongly wgrα-continuous.

Theorem 4.19: Let (X,τ) be a discrete topological space and (Y,σ) be any topological space. Let f: (X,τ)→ (Y,σ) be a map. Then the following statements are equivalent:
(i) f is strongly αrω-continuous.
(ii) f is perfectly αrω-continuous.
Proof:
(i)=>(ii) Let U be any αrω -open set in (Y,σ).By hypothesis $f^{1}(U)$ is open in (X,τ). Since(X,τ) is a discrete space,$f^{1}(U)$ is also closed in (X,τ). $f^{1}(U)$ is both open and closed in (X,τ). Hence f is perfectly αrω-continuous.
(ii)=>(i) Let U be any αrω-open set in (Y,σ). Then $f^{1}(U)$ is both open and closed in (X,τ). Hence f is strongly αrω-continuous.

Theorem 4.20: Let f: (X,τ) →(Y,σ) be a map. Both (X,τ) and (Y,σ) are $T_{\alpha r\omega}$-space . Then the following are equivalent:
(i) f is αrω-irresolute.
(ii) f is strongly αrω-continuous
(iii) f is continuous.
(iv) f is αrω-continuous.
Proof : Straight forward.

Theorem 4.21: Let X and Y be ${}_{\alpha r\omega}T_{\alpha}$-spaces, then for a function f: (X,τ) → (Y,σ), the following are equivalent:
(i) f is α-irresolute.
(ii) f is αrω-irresolute.
Proof: (i)=> (ii): Let f: (X,τ) →(Y,σ) be a α-irresolute. Let V be a αrω -closed set in Y. As Y ${}_{\alpha r\omega}T_{\alpha}$-space, then V be a α-closed set in Y. Since f is α -irresolute, $f^{-1}(V)$ is α -closed in X. But every α-closed set is αrω -closed in X and hence $f^{-1}(V)$ is a αrω-closed in X. Therefore, f is αrω-irresolute.
(ii)=> (i): Let f: (X,τ) → (Y,σ) be a αrω -irresolute. Let V be a α-closed set in Y. But every α-closed set is αrω -closed set and hence V is αrω-closed set in Y and f is αrω-irresolute implies $f^{-1}(V)$ is αrω-closed in X. But X is ${}_{\alpha r\omega}T_{\alpha}$-space and hence $f^{-1}(V)$ is α-closed set in X. Thus, f is α-irresolute.

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