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On RAM Finite Hyperbolic Transforms

R.A.Muneshwar[1], K.L.Bondar[2], V.S.Thosare[3]
[1]*Department of Mathematics, NES Science College, SRTMU Nanded, India – 431602*
[2]*Department of Mathematics, NES Science College, SRTMU Nanded, India – 431602*
[3]*Department of Mathematics, NES Science College, SRTMU Nanded, India – 431602*

Abstract: *In this paper we have introduced the new concept of RAM finite hyperbolic transforms. Transform of some standard functions are obtained and some properties are proved.*
Keywords: *Generalized Transform, Finite transform, RAM Finite hyperbolic transform, Transform of some standard functions.*

I. Introduction:

The Laplace transform method is normally used to find the response of a linear system at any time t to the initial data at t = 0 and disturbance f(t) acting for t > 0. If the disturbance is f(t) = e^{at^2} , for a > 0, the usual Laplace transform cannot be used to find the solution of an initial value problem because Laplace transform of f(t) does not exist. It is often true that the solution at times later than t would not affect the state at time t. This leads to define Finite Laplace transform.
The finite Laplace transform of a continuous or an almost piecewise continuous function f(t) in (0,T) is denoted by L_T (f(t)) = F(p,T), and is defined by

$$L_T(f(t)) = F(p,T) = \int_0^T f(t)e^{-pt}dt \qquad (1.1)$$

Where p is a real or complex number and T be a finite number which may be positive or negative.
Note : Above definition is defined for any bounded interval ($-T_1$, T_2).
Finite Laplace transform motivate us to define RAM Finite Sine Hyperbolic transform and RAM Finite Cosine Hyperbolic transform in $0 \le t \le T$ in order to extend the power and usefulness of usual Laplace transform in $0 \le t < \infty$. In section 2, the concept of RAM Finite Hyperbolic Transforms is introduced. Section 3 is devoted to explain existence conditions for these transforms. Sections 4 and 5 are devoted to obtain these transforms of some standard functions. Some properties like Linearity, Scalar Multiplication, and Scaling are proved in sections 6 and 7. Section 8 is devoted to Discussion and Conclusion.

II. RAM Finite Hyperbolic Transforms:

Definition 2.1: Let $p \in C$ and T be a finite number which may be positive or negative and f(t) is a continuous or an almost piecewise continuous function defined over the interval (0,T). Then RAM Finite Sine Hyperbolic transform of f(t) is denoted by R_{sh} (f(t)) = F_S(p,T), and defined by

$$R_{sh}(f(t)) = F_S(p,T)= \int_0^T \sinh(pt)f(t)dt,$$

where sinh(pt) is a Kernel of R_{sh}.
Here R_{sh} is called RAM Finite Sine Hyperbolic transformation operator.
Definition 2.2: Let $p \in C$ and T be a finite number which may be positive or negative and f(t) is a continuous or an almost piecewise continuous function defined over the interval (0,T). Then RAM Finite Cosine Hyperbolic transform of f(t) is denoted by R_{ch}(f(t))= F_C(p,T), and defined by

$$R_{ch}(f(t)) = F_C(p,T)= \int_0^T \cosh(pt) f(t)dt,$$

where cosh(pt) is a Kernel of R_{ch}.
Here R_{ch} is called RAM Finite Cosine Hyperbolic transformation operator.
Note : sinht, cosht are bounded for any bounded interval ($-T_1$,T_2).

III. Existence of R_{sh} and R_{ch}.

Theorem 3.1 If f(t) is a piecewise continuous and absolutely integrable function on (0,T), then R_{sh}(f(t)) exists.
Proof: As sinht is bounded on (0,T), there exist $K \in [0, \infty)$ such that $|\sinh(pt)| \le K$ on (0,T). Since f(t) is absolutely integrable, there exist $M \in [0, \infty)$ such that $\int_0^T |f(t)|dt \le M$.

Consider

$$|R_{sh}\,(f\,(t))| \quad = \quad |\int_0^T \sinh(pt)\, f\,(t)dt|$$

$$\leq \int_0^T |\sinh(pt)|\,|\,f\,(t)||dt|$$

$$\leq \int_0^T K\,|\,f\,(t)\,dt|$$

$$\leq K\int_0^T |\,f\,(t)\,dt\,|$$

$$\Rightarrow |R_{sh}\,(f(t)| \leq K.M.$$

Thus R_{sh} (f(t)) exists. Hence proved.

Theorem 3.2 If f(t) is a piecewise continuous and absolutely integrable function on (0, T), then R_{ch} (f(t)) exists.

Proof: Consider

$$|R_{ch}\,(f(t))| \quad = \quad |\int_0^T \cosh(pt)\, f(t)dt\,|$$

$$\leq \int_0^T |\cosh(pt)|\,|f(t)||dt\,|$$

$$\leq \int_0^T K|\,f\,(t)dt\,| \qquad (\text{ since } |\cosh(pt)| \leq K \text{ on } (0, T),\ 0 \leq K < \infty)$$

$$\leq K\int_0^T |\,f\,(t)dt\,|$$

$$\Rightarrow |\,R_{ch}\,(f\,(t))| \leq M.K. \ (\text{since } \int_0^T |\,f\,(t)|\,dt \leq M,\ 0 \leq M < \infty)$$

Thus R_{ch} (f (t)) exists. Hence proved.

Theorem 3.3: If f (t) is a piecewise continuous and bounded function on (0, T), then R_{sh} (f (t)) exists.

Proof: Consider

$$|R_{sh}\,(f(t))\,| \quad = \quad |\int_0^T \sinh(pt)\, f\,(t)\,dt\,|$$

$$\leq \int_0^T |\sinh(pt)|\,|\,f\,(t)|\,|\,dt\,|$$

$$\leq \int_0^T M.K.\,|dt\,| \quad (\text{since } |f\,(t)| \leq M \text{ on } (0, T),\ 0 \leq K, M < \infty)$$

$$\Rightarrow \quad |R_{sh}\,(f\,(t))| \leq M.K.T.$$

Thus R_{sh} exists. Hence proved.

Theorem 3.4: If f(t) is a piecewise continuous and bounded function on (0, T), then R_{ch}(f (t)) exists.

Proof : consider

$$|R_{sh}\,(f(t))| \quad = \quad |\int_0^T \cosh(pt)\, f(t)\,dt|$$

$$\leq \int_0^T |\cosh(pt)|\,|\,f(t)\,dt|$$

$$\leq MK\int_0^T |dt|\ (\text{since } |\cosh\,(pt)| \leq K,\ |\,f\,(t)| \leq M \text{ on } (0,T),\ 0 \leq K,M < \infty)$$

$$\Rightarrow |R_{ch}\,(f(t))| \leq M.K.T.$$

Thus R_{ch} (f(t)) exists. Hence Proved.

IV. RAM Finite Sine Hyperbolic transform of some standard functions:

1. $R_{sh}(1) = \dfrac{\cosh(pT)-1}{p}$

Proof:

$R_{sh}(1) = \int_0^T \sinh(pt)dt$

$= \dfrac{\cosh(pT)-1}{p}$

2. $R_{sh}(t) = \dfrac{T\cosh(pT)}{p} - \dfrac{\sinh(pT)}{p^2}$

Proof:

$R_{sh}(t) = \int_0^T t\sinh(pt)dt$

$= \dfrac{T\cosh(pT)}{p} - \dfrac{\sinh(pT)}{p^2}$

3. $R_{sh}(t^2) = \dfrac{T^2\cosh(pT)}{p} - \dfrac{2T\sinh(pT)}{p^2} + \dfrac{(2\cosh(pT)-2)}{p^3}$.

Proof:

$R_{sh}(t^2) = \int_0^T t^2 \sinh(pt)dt$

$= \dfrac{T^2\cosh(pT)}{p} - \dfrac{2T\sinh(pT)}{p^2} + \dfrac{(2\cosh(pT)-2)}{p^3}$.

4. $$R_{sh}(t^k) = \begin{cases} \dfrac{T^k\cosh(pT)}{p} - \dfrac{kT^{k-1}\sinh(pT)}{P^2} + ... + \dfrac{k!(-1)^k[\cosh(pT)-1]}{p^k}, \text{if k is even} \\ \dfrac{T^k\cosh(pT)}{p} - \dfrac{kT^{k-1}\sinh(pT)}{P^2} + ... + \dfrac{k!(-1)^k\sinh(pT)}{p^k}, \text{if k is odd} \end{cases}$$

Proof:

$R_{sh}(t^k) = \int_0^T t^k \sinh(pt)dt$

$$= \begin{cases} \left[\dfrac{t^k\cosh(pt)}{p}\right]_0^T - \left[\dfrac{kt^{k-1}\sinh(pt)}{p^2}\right]_0^T + ... + \left[\dfrac{k!(-1)^k\cosh(pt)}{p^k}\right]_0^T, \text{if k is even,} \\ \left[\dfrac{t^k\cosh(pt)}{p}\right]_0^T - \left[\dfrac{kt^{k-1}\sinh(pt)}{p^2}\right]_0^T + ... + \left[\dfrac{k!(-1)^k\sinh(pt)}{p^k}\right]_0^T, \text{if k is odd.} \end{cases}$$

$$= \begin{cases} \dfrac{T^k\cosh(pT)}{p} - \dfrac{kT^{k-1}\sinh(pT)}{P^2} + ... + \dfrac{k!(-1)^k[\cosh(pT)-1]}{p^k}, \text{if k is even} \\ \dfrac{T^k\cosh(pT)}{p} - \dfrac{kT^{k-1}\sinh(pT)}{P^2} + ... + \dfrac{k!(-1)^k\sinh(pT)}{p^k}, \text{if k is odd} \end{cases}$$

5. $R_{sh}(\sin(at)) = \left(\dfrac{-a}{p^2+a^2}\right)\sinh(pT)\cos(aT) + \left(\dfrac{p}{p^2+a^2}\right)\cosh(pT)\sin(aT)$.

Proof:

R_{sh} (sin(at)) = $\int_0^T$ sin(at) sinh (pt) dt

$$= \frac{\sinh(pT)\cos(aT)}{-a} + \frac{p\cos(pT)\sin(aT)}{a^2} - \frac{p^2 R_{sh}(\sin(at))}{a^2}.$$

$$\Rightarrow \left(1+\frac{p^2}{a^2}\right) R_{sh}(\sin(at)) = \frac{\sinh(pT).\cos(aT)}{-a} + \frac{p.\cos(pT).\sin(aT)}{a^2}.$$

i.e. $R_{sh}(\sin(at)) = \left(\frac{-a}{p^2+a^2}\right)$ sinh(pT). cos(aT) + $\left(\frac{p}{p^2+a^2}\right)$ cosh(pT) sin(aT).

6. $R_{sh}(\cos(at)) = \left(\frac{a}{p^2+a^2}\right)$ sinh (pT). sin (aT) + $\left(\frac{p}{p^2+a^2}\right)$ [cosh(pT). cos(aT) – 1].

Proof:

R_{sh} (cos(at)) = $\int_0^T$ cos(at) sinh (pt) dt

$$= \frac{\sinh(pT).\sin(aT)}{a} + \frac{[p.\cosh(pT).\cos(aT)-p]}{a^2} - \frac{p^2.R_{sh}(\cos(at))}{a^2}$$

$$\Rightarrow \left(1+\frac{p^2}{a^2}\right) R_{sh}(\cos(at)) = \frac{\sinh(pT)\sin(aT)}{a} + \frac{[p.\cosh(pT).\cos(aT)-p]}{a^2}$$

i.e. $R_{sh}(\cos(at)) = \left(\frac{a}{p^2+a^2}\right)$ sinh(pT). sin (aT) + $\left(\frac{p}{p^2+a^2}\right)$ [cosh (pT). cos (aT) – 1].

7. $R_{sh}(e^{at}) = \left(\frac{-a}{p^2-a^2}\right)$ sinh(pT). e^{aT} + $\left(\frac{p}{p^2-a^2}\right)$ [cosh (pT). e^{aT} – 1], provided $p^2 \neq a^2$.

Proof:

$R_{sh}(e^{at})$ = $\int_0^T$ e^{at} sinh (pt) dt

$$= \frac{\sinh(pT).e^{aT}}{a} - \frac{[p.\cos(pT)e^{aT}-p]}{a^2} + \frac{p^2.R_{sh}(e^{at})}{a^2}$$

$\Rightarrow$ $R_{sh}(e^{at}) = \left(\frac{-a}{p^2-a^2}\right)$ -sinh(pT). e^{aT} + $\left(\frac{p}{p^2-a^2}\right)$ [cosh(pT). e^{aT} – 1], provided $p^2 \neq a^2$.

8. $R_{sh}(e^{-at}) = \left(\frac{a}{p^2-a^2}\right)$ sinh(pT). e^{-aT} + $\left(\frac{-p}{p^2-a^2}\right)$ [1-cosh (pT). e^{-aT}], Provided $P^2 \neq a^2$.

Proof:

$R_{sh}(e^{-at})$ = $\int_0^T$ e^{-at} sinh (pt) dt

$$= \frac{\sinh(pT).e^{-aT}}{-a} - \frac{[p.\cosh(pT).e^{-aT}-p]}{a^2} + \frac{p^2 R_{sh}(e^{-at})}{a^2}$$

$$\Rightarrow \left(1-\frac{p^2}{a^2}\right) R_{sh}(e^{-at}) = \frac{\sinh(pT).e^{-aT}}{-a} - \frac{[p.\cosh(pT).e^{-aT}-p]}{a^2}$$

i.e. $R_{sh}(e^{-at}) = \left(\frac{a}{p^2-a^2}\right)$ sinh(pT).e^{-aT} + $\left(\frac{-p}{p^2-a^2}\right)$ [1-cosh(pT). e^{-aT}], provided $p^2 \neq a^2$.

V. RAM Finite Cosine Hyperbolic Transform of some standard functions:

1. $R_{ch}(1) = \dfrac{\sinh(pT)}{p}$.

Proof:

$$R_{ch}(t) = \int_0^T \cosh(pt)\,dt$$

$$= \frac{\sinh(pT)}{p}$$

2. $R_{ch}(t) = \dfrac{T\sinh(pT)}{p} - \left(\dfrac{\cosh(pT)-1}{p^2}\right)$

Proof:

$$R_{ch}(t) = \int_0^T t\cosh(pt)\,dt$$

$$= \frac{T\sinh(pT)}{p} - \left(\frac{\cosh(pT)-1}{p^2}\right)$$

3. $R_{ch}(t^2) = \dfrac{T^2.\sinh(pT)}{p} - \dfrac{2.T.\cosh(pT)}{P^2} + \dfrac{2.\sinh(pT)}{p^3}$.

Proof:

$$R_{ch}(t) = \int_0^T t^2 . \cosh(pt)\,dt$$

$$= \frac{T^2.\sinh(pT)}{p} - \frac{2.T.\cosh(pT)}{P^2} + \frac{2.\sinh(pT)}{p^3}.$$

4. $R_{ch}(t^k) = \begin{cases} \dfrac{T^k\sinh(pT)}{p} - \dfrac{kT^{k-1}\cosh(pT)}{p^2} + \ldots\ldots + \dfrac{k!(-1)^k\sinh(pT)}{p^k}, \text{if k is even,} \\ \dfrac{T^k\sinh(pT)}{p} - \dfrac{kT^{k-1}\cosh(pT)}{p^2} + \ldots\ldots + \dfrac{k!(-1)^k[\cosh(pT)-1]}{p^k}, \text{if k is odd.} \end{cases}$.

Proof:

$$R_{ch}(t^k) = \int_0^T t^k \cosh(pt)\,dt$$

$$= \begin{cases} \left[\dfrac{t^k\sinh(pt)}{p}\right]_0^T - \left[\dfrac{kt^{k-1}\cosh(pT)}{P^2}\right]_0^T + \ldots + \left[\dfrac{k!(-1)^k\sinh(pt)}{p^k}\right]_0^T, if \text{ k is even} \\ \left[\dfrac{t^k.\sinh(pt)}{p}\right]_0^T - \left[\dfrac{kt^{k-1}\cosh(pt)}{P^2}\right]_0^T + \ldots + \left[\dfrac{k!(-1)^k\cosh(pt)}{p^k}\right]_0^T, if \text{ k is odd} \end{cases}$$

$$= \begin{cases} \dfrac{T^k\sinh(pT)}{p} - \dfrac{kT^{k-1}\cosh(pT)}{p^2} + \ldots\ldots + \dfrac{k!(-1)^k\sinh(pT)}{p^k}, \text{if k is even,} \\ \dfrac{T^k\sinh(pT)}{p} - \dfrac{kT^{k-1}\cosh(pT)}{p^2} + \ldots\ldots + \dfrac{k!(-1)^k[\cosh(pT)-1]}{p^k}, \text{if k is odd.} \end{cases}.$$

5. $R_{ch}(\text{Sin}(at)) = \left(\dfrac{a}{p^2+a^2}\right)[1 - \cosh(pT)\cos(aT)] + \left(\dfrac{p}{p^2+a^2}\right)\sinh(pT)\sin(aT)$.

Proof:

R_{ch} (sin(at)) $= \int_0^T \sin(at)\cosh(pt)\,dt$

$= \frac{[\cos(pT)\cos(aT)-1]}{-a} + \frac{p\sinh(pT)\sin(aT)}{a^2} - \frac{p^2 R_{ch}(\sin(at))}{a^2}$

$\Rightarrow \left(1+\frac{p^2}{a^2}\right) R_{ch}(\sin(at)) = \frac{[\cos(pT)\cos(aT)-1]}{-a} + \frac{p\sinh(pT)\sin(aT)}{a^2}$

i.e. R_{ch} (Sin(at)) $= \left(\frac{a}{p^2+a^2}\right)$ [1 – cosh(pT) cos(aT)] + $\left(\frac{p}{p^2+a^2}\right)$ sinh(pT) sin(aT).

6. R_{ch} (cos(at)) $= \left(\frac{a}{p^2+a^2}\right)$ cosh(pT) sin(aT) + $\left(\frac{p}{p^2+a^2}\right)$ sinh(pT) cos(aT).

Proof:

R_{ch} cos(at) $= \int_0^T \cos(at)\cosh(pt)\,dt$

$= \frac{\cosh(pT)\sin(aT)}{a} + \frac{p\sinh(pT)\cos(aT)}{a^2} - \frac{p^2 R_{ch}(\cos(at))}{a^2}$.

$\Rightarrow \left(1+\frac{p^2}{a^2}\right) R_{ch}(\cos(at)) = \frac{\cosh(pT)\sin(aT)}{a} + \frac{p\sinh(pT)\cos(aT)}{a^2}$

i.e. R_{ch} (cos(at)) $= \left(\frac{a}{p^2+a^2}\right)$ cosh(pT) sin(aT) + $\left(\frac{p}{p^2+a^2}\right)$ sinh(pT) cos(aT).

7. $R_{ch}(e^{at}) = \left(\frac{a}{p^2-a^2}\right)$ [cosh(pT) e^{aT} – 1] + $\left(\frac{p}{p^2-a^2}\right)$ sinh(pT) e^{aT}, Provided $p^2 \neq a^2$.

Proof:

$R_{ch}(e^{at}) = \int_0^T e^{at}\cosh(pt)\,dt$

$= \frac{[\cosh(pT)e^{aT}-1]}{a} - \frac{p.\sinh(pT)e^{aT}}{a^2} + \frac{p^2 R_{ch}(e^{at})}{a^2}$.

$\Rightarrow \left(1-\frac{p^2}{a^2}\right) R_{ch}(e^{at}) = \frac{[\cosh(pT)e^{aT}-1]}{a} - \frac{p.\sinh(pT)e^{aT}}{a^2}$

i.e. $R_{ch}(e^{at}) = \left(\frac{a}{p^2-a^2}\right)$ [cosh(pT) e^{aT} – 1] + $\left(\frac{p}{p^2-a^2}\right)$ sinh(pT) e^{aT}, Provided $p^2 \neq a^2$.

8. $R_{ch}(e^{-at}) = \left(\frac{a}{p^2-a^2}\right)$ cosh(pT) e^{-aT} + $\left(\frac{-p}{p^2-a^2}\right)$ [1-sinh(pT) e^{-aT}], provided $p^2 \neq a^2$.

Proof :

$R_{ch}(e^{-at}) = \int_0^T e^{-at}\cosh(pt)\,dt$

$= \frac{[\cosh(pT)e^{-aT}-1]}{-a} - \frac{p\sinh(pT)e^{-aT}}{a^2} + \frac{p^2 R_{ch}(e^{-at})}{a^2}$

$\Rightarrow \left(1-\frac{p^2}{a^2}\right) R_{ch}(e^{-at}) = \frac{[\cosh(pT)e^{-aT}-1]}{-a} - \frac{p\sinh(pT)e^{-aT}}{a^2}$

i.e. $R_{ch}(e^{-at}) = \left(\frac{a}{p^2-a^2}\right)(\cosh(pT)\,e^{-aT} - 1) + \left(\frac{-p}{p^2-a^2}\right)[\sinh(pT)\,e^{-aT}]$; provided $p^2 \neq a^2$.

VI. Some Properties of RAM Finite Sine Hyperbolic transform:

1. **Linearity:** $R_{sh}(f_1(t) + f_2(t)) = R_{sh}(f_1(t)) + R_{sh}(f_2(t))$.

Proof: Let $0 < t < T$, then by definition

$$R_{sh}(f_1(t) + f_2(t)) = \int_0^T (f_1(t) + f_2(t))\sinh(pt)\,dt$$

$$= \int_0^T f_1(t)\sinh(pt)\,dt + \int_0^T f_2(t)\sinh(pt)\,dt$$

$$= R_{sh}(f_1(t)) + R_{sh}(f_2(t)).$$

2. **Scalar Multiplication:** If c be any constant, then $R_{sh}(cf(t)) = cR_{sh}(f(t))$.

Proof: Let c be any constant, then by definition

$$R_{sh}(c\,f(t)) = \int_0^T c\,f(t)\sinh(pt)\,dt$$

$$= c\int_0^T f(t)\sinh(pt)\,dt$$

$$= cR_{sh}(f(t)).$$

3. **Scaling:** If $R_{sh}(f(t)) = F_S(p, T)$ then $R_{sh}(f(at)) = \frac{F_S\left(\frac{p}{a}, aT\right)}{a}$

Proof: Let $R_{sh}(f(t)) = F_S(p, T)$, then by definition

$$R_{sh}(f(at)) = \int_0^T f(at)\sinh(pt)\,dt$$

$$= \int_0^T \frac{f(x)\sinh\left(\frac{xp}{a}\right)}{a}\,dx$$

$$= \frac{F_S\left(\frac{p}{a}, aT\right)}{a}$$

VII. Some Properties of RAM Finite Cosine Hyperbolic transform:

1. **Linearity :** $R_{ch}(f_1(t) + f_2(t)) = R_{ch}(f_1(t)) + R_{ch}(f_2(t))$

Proof: Let $0 < t < T$, then by definition

$$R_{ch}(f_1(t) + f_2(t)) = \int_0^T (f_1(t) + f_2(t))\cosh(pt)\,dt$$

$$= \int_0^T f_1(t)\cosh(pt)\,dt + \int_0^T f_2(t)\cosh(pt)\,dt$$

$$= R_{ch}(f_1(t)) + R_{ch}(f_2(t)).$$

2. **Scalar Multiplication:** If c is any constant, then $R_{ch}(c.\,f(t)) = c.\,R_{ch}(f(t))$

Proof: Let c be any constant, then by definition

$$R_{ch}(c\,f(t)) = \int_0^T (c\,f(t))\cosh(pt)\,dt$$

$$= c\int_0^T f(t)\cosh(pt)\,dt$$

$$= c\,R_{ch}(f(t)).$$

3. **Scaling:** If R_{ch} (f(t)) = F_C(p, T), then R_{ch} (f (at)) = $\dfrac{F_C\left(\frac{p}{a}, aT\right)}{a}$

Proof: Let $R_{ch}(f(t)) = F_C$ (p, T), then

$$R_{ch}(f(at)) = \int_0^T f(at)\cosh(pt)\,dt$$

$$= \frac{\int_0^T (f(x))\cosh\left(\frac{xp}{a}\right)}{a}dx$$

$$= \frac{F_C\left(\frac{p}{a}, aT\right)}{a}$$

VIII. Discussion and Conclusion:

Unlike the usual Laplace transform of a function f(t), there is no restriction needed on the transform variable p for the existence of $R_{ch}(f(t))$ and $R_{sh}(f(t))$. Further, the existence of $R_{ch}(f(t))$ and $R_{sh}(f(t))$does not require exponential order property of a function f(t) . If a function f(t) has the usual Laplace transform, then it also has the RAM Finite Sine Hyperbolic transform and RAM Finite Cosine Hyperbolic transform. In other words, if L(f(t)) exists, then $R_{ch}(f(t))$ and $R_{sh}(f(t))$exists as shown below. We have

$$L(f(t)) = \int_0^\infty f(t)\,e^{-pt}\,dt$$

$$= \int_0^T f(t)\cosh(pt)\,dt - \int_0^T f(t)\sin(pt)\,dt + \int_T^\infty f(t)\,e^{-pt}\,dt$$

$$= R_{ch}(f(t)) - R_{sh}(f(t)) + \int_T^\infty f(t)\,e^{-pt}\,dt\,.$$

Since L(f(t)) exists, all the three integrals on R.H.S. exist. Hence, if L(f(t)) exists then R_{ch} (f(t)) and $R_{sh}(f(t))$exists but converse is not necessarily true. This can be shown by an example. It is well known that the usual Laplace transform of f(t) = e^{at} , for a > 0, does not exist but $R_{ch}(e^{at})$ and $R_{sh}(e^{at})$ both exists.

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