IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 10, Issue 6 Ver. IV (Nov - Dec. 2014), PP 01-06
www.iosrjournals.org

An inventory Model for Deteriorating and repairable Items with Linear demand

Dr. Ravish Kumar Yadav[1], Rajeev Kumar[2]
Associate Professor, Hindu College Moradabad, Devta College Morana,

Abstract: *In this paper an inventory model for deteriorating and repairable item developed with linear demand. It extends a lot size inventory model with inventory-level dependent demand. Shortages are partially backordered and defective product can be repaired. Optimal lot size with minimum total cost is derived using Taylor and calculus analysis. The business strategies, technological change and free market economy is forcing organizations to reconsider their planning for maintaining competitive advantage. The most effective way of sustaining competitive advantage in modern economy, besides producing quality and innovative products, is to make products that can be repaired at minimal cost. Making products that can be repaired will help environment as well as save cost.*

Keywords: *deteriorating items, repairable product, linear demand, Taylor analysis*

I. Introduction

This paper is concerned with the study of continually deteriorating items inventory when shortage is partially backordered and the defective products can be repaired. Deterioration is defined as decay, damage, spoilage, evaporation , obsolescence, pilferage, and loss of utility or loss of marginal value of a commodity that results in decreasing usefulness from the original one. Blood, fish, strawberries, alcohol, gasoline, radioactive chemicals and grain products are examples of deteriorating commodities. Several researchers have studied deteriorating inventory in the past. Ghare and Schrader (1963) were the first authors to develop an economic order quantity (EOQ) model for items with an exponentially decaying inventory. Covert and Philip (1973) devised an EOQ model for items with deterioration patterns explained by the Weibull distribution. Other researchers, such as Dave (1979) began to conduct a study on the discrete-in-time order-level inventory model for deteriorating items. Shah (1976) assume an instantaneous replenishment rate with different assumptions on the patterns of deterioration, while Misra (1975), Elsayed and Teresi (1983), Mak (1982), Raafat et al. (1991)) Heng et al. (1991) and Wee (1993) assumed a finite replenishment rate.

Demand pattern are different for different items. Research given attention in this regards in their models. Datta, T. K. and Pal, A. K. (1988) given order level inventory system with power demand pattern for items with variable rate of deterioration. Mandal, B. N. and Ghosh, A. K. (1991) write A note on an inventory model with different demand rates during stock in and stock and period.Mandal, B. N. and Pal A. K. (2000) given order level inventory system for perishable items with power demand pattern. Teng, J. T. (1996) work on model with linear trend in demand.

The process of returning and reusing of defective materials is not a new concept. In the last decades, it has been a common practice to reuse metal, paper and glass. This is due to the growth of environmental concerns as well as the economy of repairing. There are four synonyms of reuse according to Thierry et al. (1995) They are: direct reuse, repair, recycling and remanufacturing. Schrady (1967) was the first author to consider reuse in a deterministic model. He assumed constant demand, return rates and fixed lead times for external orders and recovery. In the model, he proposed a control policy with fixed lot sizes. Demand is satisfied as far as possible from the recovered products. Recently, Mabini et al. (1992) extended Schrady s model to consider stockout service level constraints and multi-item system sharing the same repair facility. In the policy, expressions for the optimal control parameter values were derived. Koh(2002) developed a joint EOQ and EPQ model in which the stationary demand can be satisfied by recycled products and newly purchased products. The model assumes a fixed proportion of the used products are collected from the customers.

In this study, we extend above concept by assuming a linear demand and shortage partial backorder. To meet demand, either new products are ordered externally or old products are recovered in a recovery process. The objective of this inventory management system is to minimize the fixed and the variable costs and find the optimal production lot-size policy for a recycling production-inventory system with partial backordering.

II. Mathematical modeling and analysis

Assumptions

The mathematical model in this paper is developed on the basis of the following assumptions:

1. A single item with a constant deteriorating rate of the on-hand inventory is considered.
2. Demand is linearly depend upon time which is continuous and deceasing bya factor on inventory.
3. The production rate is finite and constant, which is larger than the demand rate and is unaffected by the lot size.
4. Shortage is allowed.
5. Each cost parameter is constant and known.
6. To repair has a less cost than to buy.
7. The quality for reuse item is the same as new one.
8. All defective products can be repaired and reused.

Notations

The following notations are used:

1. K= the production rate (units/unit time)
2. D= $\alpha + \beta t$ the demand rate (units/unit time).α,β are demand constants.
3. a=the deterioration rate (units/unit time)
4. b=the decreasing rate of the demand (units/unit time)
5. c=fraction of defective product
6. r =a constant, between zero and one, which indicates the fraction of stock-out demand sales lost due to some stock-out demands
7. I_1 (t)= inventory level at time in i^{th} the period
8. I_{max} =the maximum inventory level (units/cycle)
9. I_{min} =stockout demand (units/cycle)
10. $C_{h=}$ the carrying cost (Rs /unit/unit time)
11. $C_{sh=}$ the shortage cost (Rs /unit/unit time)
12. C_d=the deterioration cost ($/unit/unit time)
13. C_p= the penalty cost of a lost sale including lost profit (/unit time)
14. C_s=the setup cost (Rs /cycle)
15. TRC= the total relevant cost
16. p= profit per recycle unit ($/unit)
17. n =fraction of defective products per recycle unit
18. Q(t) =the recycle quantity

III. The mathematical formulation

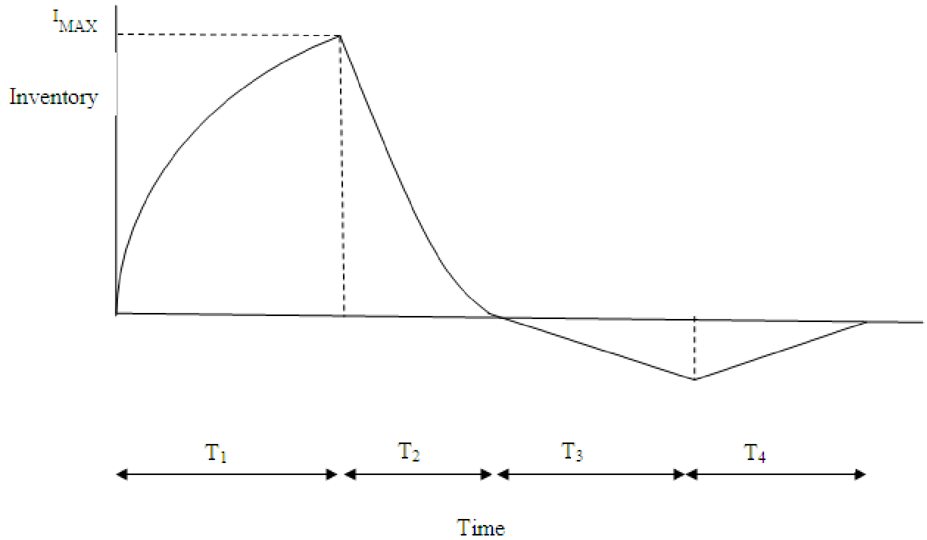

Fig. 1. The inventory system of a continuously decaying inventory with declining demand, partial back-ordering and defective recycling

$$\frac{dI_1(t)}{dt} = K - \{(\alpha+\beta t) - bI_1(t) + cI_1(t)\} - \text{a}\, I_1(t), 0 \le \text{t} \le \text{T}_1 \quad \text{…(1)}$$

$$\frac{dI_2(t)}{dt} = -\{(\alpha+\beta t) - bI_2(t)\} - \text{a}\, I_1(t), 0 \le \text{t} \le \text{T}_2 \quad \text{…(2)}$$

$$\frac{dI_3(t)}{dt} = -\{(1-r)(\alpha+\beta t)\}, 0 \le \text{t} \le \text{T}_3 \quad \text{…(3)}$$

$$\frac{dI_4(t)}{dt} = (1-r)\{K - (\alpha+\beta t) - cI_4(t)\}, 0 \le \text{t} \le \text{T}_4 \quad \text{…(4)}$$

Where solution of (1),(2),(3) and (4) is the position of the inventory level at time t. The solution of the above differential equations and various boundary conditions, $I_1(0)=0$, $I_2(T_2)=0$, $I_3(0)=0$ and $I_4(T_4)=0$, are

$$I_1(t) = \{\frac{K-\alpha}{a-b+c} + \frac{\beta}{(a-b+c)^2}\}\{1-e^{(a-b+c)t}\} - \frac{\beta t}{(a-b+c)}, 0 \le \text{t} \le \text{T}_1 \quad \text{…(5)}$$

$$I_2(t) = \{\frac{-\alpha}{a-b} + \frac{\beta}{(a-b)^2}\}\{1-e^{(a-b)(\text{T}_2-t)}\} - \frac{\beta}{(a-b)}(\text{T}_2 \text{e}^{(a-b)(\text{T}_2-t)} - t), 0 \le \text{t} \le \text{T}_2 \quad \text{…(6)}$$

$$I_3(t) = (1-r)\{\alpha\, \text{t} + \frac{\beta t^2}{2}\}, 0 \le \text{t} \le \text{T}_3 \quad \text{…(7)}$$

$$I_4(t) = (1-r)[\{\frac{K-\alpha}{c} + \frac{\beta}{c^2}\}(1-e^{c(T_4-t)}) + \frac{\beta}{c}(\text{T}_4 \text{e}^{\text{c}(\text{T}_4-t)} - t)], 0 \le \text{t} \le \text{T}_4 \quad \text{…(8)}$$

Carrying cost

The carrying cost per unit time is

$$C_h = \frac{c_h}{t_4}[\int_0^{T_1} I_1(\text{t})\,\text{dt} + \int_0^{T_2} I_2(\text{t})\,\text{dt}]$$

$$= \frac{c_h}{t_4}[\{(\frac{K-\alpha}{a-b+c} - \frac{\beta}{(a-b+c)^2})(\text{T}_1 - \frac{e^{(a-b+c)T_1}-1}{(a-b+c)}) - \frac{\beta T_1^2}{2(a-b+c)}\} + \{(\frac{\alpha}{(a-b)} - \frac{\beta}{(a-b)^2})$$
$$(\frac{1-e^{(a-b)T_2}}{(a-b)} - T_2) + \frac{\beta T_2}{(a-b)}(\frac{1-e^{(a-b)T_2}}{(a-b)} - 1)\})$$

…(9)

Back-order cost

From Fig. 1, we can see that back-order occurs during T_3 and T_4; the inventory backlogs linearly because there is no deterioration. Due to partial backlogging, the maximum inventory shortage which is backlogged is:

$$I_b = (1-r)\{-\alpha\, \text{T}_3 - \beta\frac{\text{T}_3^{\ 2}}{2}\} = (1-r)\{(\text{K}-\alpha)\,\text{T}_4 - \beta\frac{\text{T}_4^{\ 2}}{2}\}$$

The backlogged cost per unit time is then

$$C_s = \frac{c_s}{t_4}[\int_0^{T_3} I_3(\text{t})\,\text{dt} + \int_0^{T_4} I_4(\text{t})\,\text{dt}]$$

$$= \frac{c_s}{t_4}[(1-r)\{\frac{\alpha T_3^{\ 2}}{2} + \frac{\beta T_3^{\ 3}}{6} + (\frac{K-\alpha}{c} + \frac{\beta}{c^2})(\text{T}_4 - \frac{1-e^{cT_4}}{c}) + \frac{\beta T_4}{c}(\frac{1-e^{cT_4}}{(a-b)} - \frac{T_4}{2})\}) \quad \text{…..(10)}$$

Deteriorating cost

Deterioration occurs in periods T_1 and T_2. The deterioration cost per unit time is

$$C_d = \frac{c_d}{t_4}[\int_0^{T_1} I_1(\mathrm{t})\,\mathrm{dt} + \int_0^{T_2} I_2(\mathrm{t})\,\mathrm{dt}]$$

$$C_d = \frac{c_d \alpha}{t_4}[\{(\frac{K-\alpha}{a-b+c} - \frac{\beta}{(a-b+c)^2})(\mathrm{T}_1 - \frac{e^{(a-b+c)T_1}-1}{(a-b+c)}) - \frac{\beta T_1^2}{2(a-b+c)}\} + \{(\frac{\alpha}{(a-b)} - \frac{\beta}{(a-b)^2}) (\frac{1-e^{(a-b)T_2}}{(a-b)} - T_2) + \frac{\beta T_2}{(a-b)}(\frac{1-e^{(a-b)T_2}}{(a-b)} - 1)\}) \quad \ldots(11)$$

Lost sales cost

The shortage due to lost sale is

$$I_l = r\int_0^{T_3} (\alpha + \beta t)\,\mathrm{dt}$$

$$= r(\alpha T_3 + \frac{\beta T_3}{2})$$

The lost sales cost per unit time is

$$C_p = \frac{c_p r}{t_4}(\alpha T_3 + \frac{\beta T_3}{2}) \quad \ldots.(12)$$

Recycling revenue

The defective items occur during periods T_1 and T_4. The collected inventory is

Taylors series expansion of e^x and the assumption of x <<1, can be approximated as $1+x+\frac{x^2}{2}$. So by expanding the exponential functions and neglecting the second and higher powers of (a-b+c) for a small value of (a-b+c), we can obtain

$$Q(t) = (\frac{K-\alpha}{2})\{T_1^2 - T_4^2\}$$

The recycling profit per cycle is

C_{rp} =p{cQ(t)-nc Q(t)}

$$= pc(1-n)\,(\frac{K-\alpha}{2})\{T_1^2 - T_4^2\} \quad \ldots(13)$$

Total cost

Summing the carrying cost, back-order cost, lost sales cost, deteriorating cost and then deducting the recycling revenue, the total cost per unit time, TRC,total revenue cost can be obtained:

TRC = C_h+ C_s+ C_d+ C_p+ C_{sh}-C_{rp}

$$= \frac{c_h}{t_4}[\{(\frac{K-\alpha}{a-b+c} - \frac{\beta}{(a-b+c)^2})(\mathrm{T}_1 - \frac{e^{(a-b+c)T_1}-1}{(a-b+c)}) - \frac{\beta T_1^2}{2(a-b+c)}\} + \{(\frac{\alpha}{(a-b)} - \frac{\beta}{(a-b)^2}) (\frac{1-e^{(a-b)T_2}}{(a-b)} - T_2) + \frac{\beta T_2}{(a-b)}(\frac{1-e^{(a-b)T_2}}{(a-b)} - 1)\}) + \frac{c_s}{t_4}[(1-r)\{\frac{\alpha T_3^2}{2} + \frac{\beta T_3^3}{6} + (\frac{K-\alpha}{c} + \frac{\beta}{c^2})(\mathrm{T}_4 - \frac{1-e^{cT_4}}{c}) + \frac{\beta T_4}{c}(\frac{1-e^{cT_4}}{(a-b)} - \frac{T_4}{2})\}) + \frac{c_d \alpha}{t_4}[\{(\frac{K-\alpha}{a-b+c} - \frac{\beta}{(a-b+c)^2})(\mathrm{T}_1 - \frac{e^{(a-b+c)T_1}-1}{(a-b+c)}) - \frac{\beta T_1^2}{2(a-b+c)}\} + \{(\frac{\alpha}{(a-b)} - \frac{\beta}{(a-b)^2})(\frac{1-e^{(a-b)T_2}}{(a-b)} - T_2) + \frac{\beta T_2}{(a-b)}(\frac{1-e^{(a-b)T_2}}{(a-b)} - 1)\}) + \frac{c_p r}{t_4}(\alpha T_3 + \frac{\beta T_3}{2}) + C_s - pc(1-n\ )(\frac{K-\alpha}{2})\{T_1^2 - T_4^2\} \quad \ldots.(14)$$

From Fig. 1, $I_1(T_1)$= I_{max} = $I_2(0)$; hence from Equation (5) and Equation (6),

$I_{max} = I_1(T_1) = \{\frac{K-\alpha}{a-b+c} + \frac{\beta}{(a-b+c)^2}\}\{1 - e^{(a-b+c)T_1}\} - \frac{\beta T_1}{(a-b+c)}.$

$$I_2(0) = \{\frac{-\alpha}{a-b} + \frac{\beta}{(a-b)^2}\}\{1 - e^{T_2(a-b)}\} - \frac{\beta}{(a-b)} T_2 e^{T_2(a-b)}$$

$$= \frac{1}{(a-b)^2}[\{-\alpha(a-b)+\beta\}\{1-e^{T_2(a-b)}\} - (a-b)\beta T_2 e^{T_2(a-b)}]$$

$$= \frac{1}{(a-b)^2}[\{-\alpha(a-b)+\beta\} - \{\beta + (a-b)(\beta T_2 - \alpha)e^{T_2(a-b)}\}]$$

By using Taylor's series approximation, we can obtain

$$\{\frac{K-\alpha}{a-b+c} + \frac{\beta}{(a-b+c)^2}\}\{(a-b+c)T_1 + \frac{(a-b+c)^2 T_1^2}{2}\} - \frac{\beta T_1}{(a-b+c)}$$

$$= \{K-\alpha+\frac{\beta}{(a-b+c)}\}\{T_1 + \frac{(a-b+c)T_1^2}{2}\} - \frac{\beta T_1}{(a-b+c)}$$

$$= (K-\alpha)T_1 + \{(K-\alpha)(a-b+c)+\beta\}\frac{T_1^2}{2}$$

$$(K-\alpha)T_1 + \{(K-\alpha)(a-b+c)+\beta\}\frac{T_1^2}{2}$$

$$= T_2^2\{\alpha(a-b) - 3\beta\} + T_2\{\alpha - \frac{2\beta}{a-b}\}$$

$$T_2 = \frac{-\{\alpha - \frac{2\beta}{a-b}\} \pm \sqrt{\{\alpha - \frac{2\beta}{a-b}\}^2 - 4\{\alpha(a-b)-3\beta\}\{(K-\alpha)T_1 + \{(K-\alpha)(a-b+c)+\beta\}\frac{T_1^2}{2}\}}}{2\{\alpha(a-b)-3\beta\}} \quad ....(15)$$

From Fig. 1, $I_3(T_3) = I_{min} = I_4(0)$; hence from Equation (7) and Equation (8),

$$I_3(T_3) = (1-r)\{\alpha T_3 + \frac{\beta T_3^2}{2}\},$$

$$I_4(0) = (1-r)[\{\frac{K-\alpha}{c} + \frac{\beta}{c^2}\}(1-e^{cT_4}) + \frac{\beta}{c} T_4 e^{cT_4}],$$

$$\alpha T_3 + \frac{\beta T_3^2}{2} = (K-\alpha+\frac{2\beta}{c})T_4 + (\frac{c(K-\alpha)+3\beta}{2})T_4^2$$

$$T_4 = \frac{-(K-\alpha+\frac{2\beta}{c}) \pm \sqrt{(K-\alpha+\frac{2\beta}{c})^2 + 2\{c(K-\alpha)+3\beta\}\ (\alpha T_3 + \frac{\beta T_3^2}{2})}}{c(K-\alpha)+3\beta}$$

By expressing T_2, T_4 in Equation (14) in terms of T_1, T_3 and then simplifying, we have $TCR(T_1,T_3)$ in terms of T_1,T_3.

Our objective is to minimize the total cost function using calculus. The following sufficient conditions for minimizing $TRC(T_1,T_3)$ must be satisfied:

$$\frac{\partial^2 TCR}{\partial^2 T_1} > 0, \frac{\partial^2 TCR}{\partial^2 T_3} > 0, \text{and} \left(\frac{\partial^2 TCR}{\partial^2 T_1}\right)\left(\frac{\partial^2 TCR}{\partial^2 T_3}\right) - \left(\frac{\partial^2 TCR}{\partial T_1 \partial T_3}\right) > 0, \quad ...(17)$$

Since $TCR(T_1,T_3)$ is a non-linear equation, the closed form solution of T_1 and T_3 cannot be obtained. But the above conditions for a particular solution can be proved numerically.

IV. Numerical Example

Let us consider a system with the following parameter values: K=60 unit/month, α=30 unit/ month ,β=0.001, C_h=Rs/ unit, C_{sh}= Rs / unit , C_d= Rs 3/ unit , C_p= Rs 1/ unit, C_s= Rs 2/ unit, a=0.1,b=0.05 , c=0.08 , p =0.5, n =0.01, r =0.6. to illustrate the inventory model for this example.This particular solution can be proved to satisfy the optimal conditions:

$$\frac{\partial^2 TCR}{\partial^2 T_1} > 115, \frac{\partial^2 TCR}{\partial^2 T_3} > 110, \text{and} \left(\frac{\partial^2 TCR}{\partial^2 T_1}\right)\left(\frac{\partial^2 TCR}{\partial^2 T_3}\right) - \left(\frac{\partial^2 TCR}{\partial T_1 \partial T_3}\right) > 12650,$$

The results are shown in Table 1.Hence, the following solution set is optimal for the model. T_1 *= 0.21 month ,T_2 *= 0.20 month ,T_3 *= 0.16 month ,T_4 *= 0.06 month respectively; and the resulting minimum total relevant cost, TCR(T_1,T_3), of the system is Rs 8.07 month. The maximum inventory, I_{max} =6.27 unit.

Table 1. The minimum total relevant cost by computer search

T_1	T_3	T_2	T_4	TRC
10.00	20.00	3.23	6.37	465
10.00	15.00	3.23	5.00	333
10.00	10.00	3.23	3.50	209
5.00	5.00	3.13	1.86	106
5.00	1.00	0.91	0.39	22
0.50	0.50	0.47	0.19	12
0.21	0.16	0.20	0.06	8
0.10	0.10	0.09	0.59	10

V. Conclusion

We have developed a model to analyze an inventory system where the linear demand and declining rate is inventory-level dependent. The inventory can be supported by both recovered products and newly purchased products, and shortage is allowed by partially back-order. The objective is to minimize the total cost and derive the optimum inventory level of recoverable items to initiate the recovery process and to determine the shortage quantity and order quantity for newly procured items. We have proposed a search method to solve the problem because a closed form solution is infeasible.

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