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Construction of Single Sampling Plan Indexed Through Limiting Quality Level

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Abstract: *Acceptance sampling procedures are the practical tools for quality assurance applications involving product control. Acceptance sampling systems are advocated when small sample size are necessary or desirable towards costlier testing for product quality.*
Dodge – Romig plans can be single or double plans. There are two sets of plans. One is based on satisfying a given limiting quality level (LQL) based on a consumers risk β, the target value of which is 0.10 (Dodge and Romig, 1959). The other based on meeting a certain value of the average outgoing limit. For both sets of plans, the objective is to minimize the average total inspection.
If the parameters of the plan under consideration (eg., lot size and process average) are within certain range, the Dodge – Romig tables allow us to determine feasible plans very readily. Determining these plans from basic principles would take much more time. The trade-off is that the Dodge-Romig tables provide a sampling plan for a range at lot sizes and process averages. In this juncture there is a considerable procedure to deal with this problem using Poisson distribution which is often used as a standard probability model for dealing with this kind of single sampling plan. However many data sets in general are not well fitted by a Poisson model, because they consists of more zero counts than are compatible with the Poisson model. Zero inflated models are used to model count data for which proportion of zero counts is greater than expected. For these situations, a zero inflated Poisson (ZIP) model is generally proposed.
It is very logical and encouraging that zero inflated Poisson (ZIP) Distribution provided better fit as compared to Poisson distribution in the data sets because of more number of zeros.
In this paper the procedure for the construction of Single Sampling Plan indexed through Limiting quality Level (LQL) using Zero Inflated Poisson (ZIP) distribution as the base line distribution is presented and a table is also presented using Excel packages for the easy selection of the plans.
Keywords: *Limiting Quality Level, Zero Inflated Poisson (ZIP) distribution, Single Sampling Plan, Operating Characteristic Curve.*

I. Introduction

The Limiting Quality Level (LQL) is used in sampling inspection for a product. It is the proportion of nonconforming items associated with the consumer's risk. It can be regarded as the minimum quality that the customer would not want to accept, even for a single batch. That is the limiting quality level is determined by identifying the proportion of defective outputs from the process that cause the customer to reject entire batches. But the Acceptable Quality Level (AQL) is the minimum acceptable long term average, which is also used in sampling inspection (Ponnuraja and Santhi, 2014). It is the maximum percentage of defectives that is acceptable as a long-term average. It is purely associated with the Producer's Risk**.** Acceptance sampling is a method of measuring random samples of populations called "lots" of materials or products against predetermined standards. Acceptance sampling is a part of operations management or of accounting auditing and services quality supervision. It is important for industrial, but also for business purposes helping decision-making process for the purpose of quality management. Sampling plans are hypothesis tests regarding product that has been submitted for an appraisal and subsequent acceptance or rejection. Acceptance sampling is a major field of statistical quality control. A typical application of acceptance sampling is as follows. A company receives a shipment of product from a vendor. This product is often a component or raw material used in the company's manufacturing process. A sample is taken from the lot and some quality characteristics of the units in the sample inspected. On the basis of the information in this sample a decision is made regarding lot disposition usually this decision is either to accept or to reject the lot. Sometimes we refer to this decision as lot sentencing. Accepted lots are put into productions; rejected lots may return to the vendor or may be subjected to some other lot disposition action. While it is customary to think of acceptance sampling as a receiving inspection activity, there are other uses of sampling methods for example; frequently a manufacturer will sample and inspect its own product at various stages of production. Lots that are accepted are sent forward for further processing while rejected lots may be reworked or scrapped. Acceptance sampling is most likely to useful in the following situation.

- When testing is destructive.
- When the cost of 100% inspection is extremely high.
- When 100% inspection is not technologically feasible or would required so much calendar time that production scheduling would be seriously impacted.
- When there are many items to be inspected and the inspection error rate is sufficiently high than 100% inspection might cause a higher percentage of defectives units to be passed than would occur with the use of sampling plan.
- When the vendor has an excellent quality history, and some reduction in inspection from 100% is desired, but the vendor's process capability ratio is sufficiently low to make no inspection an unsatisfactory alternative.
- When there are potentially serious product liability risks and although the vendor's process is satisfactory a program for continuously monitoring the product is necessary. The products may be grouped into batches or lots or may be single pieces from a continuous operation. A random sample is selected and could be checked for various characteristics. For lots, the entire lot is accepted or rejected in the whole. The decision is based on the pre-specified criteria and the amount of defects or defective units found in the sample. Accepting or rejecting a lot is analogous to not rejecting or rejecting the null hypothesis in a hypothesis test. In the case of continuous production process, a decision may be made to continue sampling or to check subsequent product 100%.

II. Review of Related Literature:

An acceptance-sampling plan is best described in graphical terms on an operating-characteristic curve (OC curve). An OC curve is a plot of the actual number of nonconforming units in a lot (expressed as a percentage) against the probability that the lot will be accepted when sampled according to the plan. The shape of an OC curve is determined primarily by sample size, n, and acceptance number, c, although there is a small effect of lot size, N. (Acceptance number, c, is the largest number of nonconforming units, or non-conformance, that may be found in the sample without causing rejection of the lot.)

Poisson distribution is often used as a standard probability model for count data. For example, a production engineer may count the number of defects in items randomly selected from a production process. Quite often, however, such datasets are not well fit by a Poisson model because they contain more zero counts than are compatible with the Poisson model. An example is again provided by the production process; indeed, according to Ghosh et al. (2006), when some production processes are in a near perfect state, zero defects will occur with a high probability. However, random changes in the manufacturing environment can lead the process to an imperfect state, producing items with defects. The production process can move randomly back and forth between the perfect and the imperfect states. For this type of production process many items will be produced with zero defects, and this excess might be better modeled by a zero-inflated Poisson distribution than a Poisson distribution. Zero-inflated models have been applied to various kinds of count data. Many attempts can be found, in recent years, which developed procedures of testing zero-inflation and over dispersion.

Due to the technological development, production processes are well designed in such a way that the products are in perfect state, so that the number of zero defects will be found more in those cases. However, random fluctuations in the production processes may lead some products to an imperfect state. The appropriate probability distribution to describe such situations is a zero-inflated Poisson (ZIP) distribution. The ZIP distribution can be viewed as a mixture of a distribution which degenerates at zero and a Poisson distribution. ZIP distribution has been used in a wide range of disciplines such as agriculture, epidemiology, econometrics, public health, process control, medicine, manufacturing, etc. Some of the applications of ZIP distribution can be found in Bohning et al. (1999), Lambert (1992), Naya et al. (2008), Ridout et al. (1998), and Yang et al. (2011). Construction of control charts using ZIP distribution are discussed in Sim and Lim (2008) and Xie et al. (2001). Some theoretical aspects of ZIP distributions are mentioned in McLachlan and Peel (2000).

Ghoshet.al.(2006) has rightly pointed out that when some production processes are in a neat perfect state, zero defects will occur with a high probability. However, random changes in the manufacturing environment can lead the process to an imperfect State, producing items with defects. The production process can move randomly. For this type of production process many items will be produced with zero defects and this excess might be better attributed by a zero inflated Poisson model than a Poisson model. A simple way, to model this zero inflated Poisson model than a Poisson model. A simple way to model this zero inflation is given by Jonson et al(1992).Various authors have considered the zero Inflated Poisson(ZIP)as a possible model for biological count data.

Peach and Littauer (1946) has given a table for determining the single sampling plan for a fixed α = 0.05. Burguess (1948) provided graphical method to obtain single sampling plan for a specified (p_1, 1-α) and (p_2, β). Grubs (1949) have given a table, which can be used for selecting a single sampling plan at IQL

(Indifference Quality Level) and LQL (Limiting Quality Level. Cameron (1952) made extension on the work of Peach and Littauer (1946).

Guenther (1969) developed a procedure for constructing a single sampling plan for a specified p_1, p_2 and α based on Binomial, hyper geometric and Poisson Models. Golub Abraham (1953) provided a method and tables for finding the acceptance number c of a single sampling plan involving minimum sum of producer and consumer risk with fixed sample size. Soundarajan and Govindaraju (1983) contributed in designing single sampling plan. Suresh and RamKumar (1996) constructed a single sampling plan indexed through MAAOQ (Maximum Allowable Average Outgoing Quality). Radhakrishnan (2002) continued the work of Suresh and RamKumar (1996) and constructed the various sampling plans including continuous sampling plan.

Govindaraju (1989) using a sampling plan with a given Limiting Quality Level (LQL), and the consumer's risk of 10% or less of the production will be accepted in the long run during the periods of sampling. LQL helps one to plan the manpower requirements for 100 % inspection depending on the level of process quality maintained and the production shift. Stephens (1981) advocated the use of LQL index for consumer's protection. Shankar and Sahu (2002) studied process control plans using AQL, LQL and Average Outgoing Deterioration Limit (AODL). Radhakrishna Rao (1977) suggested the use of Weighted Binomial Distribution in the Construction of Sampling plans. Radhakrishnan and Mohana priya (2008 a,b) constructed Single and Conditional Double Sampling Plans using Weighted Poisson distribution as the base line distribution.

Sampling plans under the conditions of ZIP distribution are very much required to the production processes being monitored well. No attempt is found in literature in this regard. This article attempts to determine SSPs by attributes under the conditions of ZIP distribution.

In this Paper a Singe Sampling Plan is constructed by assuming the probability of acceptance of a lot as 0.10, (the proportion defective corresponding to this probability of acceptance in the OC (Operating Characteristic) curve is termed as Limiting Quality Level using Poisson distribution as the base line distribution.

Conditions for Application

- Lots are submitted sequentially in the order of their Production is steady, so that results of past, present and future lots are broadly indicative of a continuing process.
- Production.
- Inspection is by attributes, with the lot quality defined as the proportion defective.

Glossary of Symbols

ω	-	Parameter
p	-	Proportion Defective / Lot Quality
n	-	Sample Size
α	-	Producer's Risk
$P_a(p)$	-	Probability of acceptance of the lot quality p
c	-	Acceptance number of defective

Operating Characteristic Function

The Operating Characteristic (OC) Function of the single sampling plan (SSP) using Inflated Poisson Distribution, inflated at x = 0 is given by

$$P_a(p) = \begin{cases} \omega_i + (1-\omega_i)e^{-\lambda_i}, when\, x_i = 0 \\ (1-\omega_i)\dfrac{e^{-\lambda_i}\lambda^{x_i}}{x!}, when\, x_i = 1,2,... \\ \qquad 0 \le \omega_i \le 1 \end{cases} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_} \qquad (1)$$

$$P_a(p) = \omega_i + (1-\omega_i)e^{-\lambda_i} + \sum_{x=1}^{c}(1-\omega_i)\frac{e^{-\lambda_i}\lambda^{x_i}}{x!} \quad , where\, \lambda = np$$

When c=0, the lot acceptance probability becomes as

$$P_a(p) = \omega_i + (1-\omega_i)e^{-\lambda_i}$$

In this distribution, ω may be termed as the mixing proportion. ω and λ are the parameters of the ZIP distribution.

III. Construction Of Single Sampling Plan

Construction of single sampling plan indexed through LQL by fixing the probability of acceptance of the lot, $P_a(p)$ as 0.10 and also based on the evidences given in the earlier work like Peach and Littauer (1946) and Dodge and Romig, (1959), for determining the single sampling plan for a fixed α = 0.05. The same concept was used for Inflated Poisson Distribution (IPD) as the basic distribution and it was applied with the above equation (1). The values of the LQL are obtained from the above equation for various combinations of n and c using Visual Basic program and presented in Table 1. The parameters of the Single sampling plan, n and c are recorded for various combinations of LQL.

Table 1: Parameters of SSP for a specified LQL

ω	n	C	LQL		ω	n	C	LQL
0.25	225	1	0.0007		0.25	175	4	0.0001
0.25	225	2	0.00001		0.25	175	5	0.0002
0.25	**225**	**3**	**0.00005**		0.25	150	1	0.0011
0.25	225	4	0.0001		0.25	150	2	0.0007
0.25	225	5	0.0002		0.25	150	3	0.0007
0.25	200	1	0.0008		0.25	150	4	0.0009
0.25	200	2	0.0007		0.25	150	5	0.0004
0.25	200	3	0.0005		0.25	125	1	0.0013
0.25	200	4	0.0001		0.25	125	2	0.0008
0.25	200	5	0.0001		0.25	125	3	0.0007
0.25	175	1	0.0009		0.25	125	4	0.0009
0.25	175	2	0.0007		0.25	125	5	0.0004
0.25	175	3	0.0003					

By choosing the sample size as n and ω value in between 0 to 1, the LQL value will decrease while there is a decrease value of the defects (c). The table 1 gives explanation as for a given LQL is 0.00005, which is the lowest among the group with the lower defectives. Therefore, the value of ω, n and c are treated as optimum results with the arithmetical values as ω=0.25, n = 225 and c = 3. Hence these parameters of Single Sampling Plan are ω =0.25, n = 225 and c = 3 with the specified and best possible LQL is 0.00005. This also replicate in the operating characteristic (OC) curve and for this plan is presented in figure 1. This was conspired as a graph using the above parameters.

The Limiting Quality (LQ) is applied when a lot is considered in isolation. It is a quality level which corresponds to a specified and relatively low probability of acceptance of a lot having a rate of defective items of LQ. In general, the LQ corresponds to the rate of defective items of lots accepted after control in 10 % of the cases. LQ is an indexing device used in ISO 2859-2 (where it is recommended that the LQ is set at least three times the desired AQL, in order to ensure that lots of acceptable quality has a reasonable probability of acceptance). The LQ is generally very low when the plans aim at the control of food safety criteria. It is habitually higher when the plans aim at the control of quality criteria. The LQ is a particular consumers' risk. The users of sampling plans shall mandatory agree on the choice on the AQL or LQL of the plan used for the quality control of the lots.

IV. Operating Characteristic (Oc) Curve

The OC curve reveals the performance of the acceptance sampling plan. We consider two types of OC curves: The first type is for isolated or unique lots. This is a curve showing the probability of accepting a lot as a function of the lot quality. The second type is for a continuous stream of lots. This is a curve showing the probability of accepting a lot as a function of the process average. That is, the second type OC curve will give the proportion of lots accepted as a function of the process average p, naturally the second type of OC curve is the model which we present here. Probability of acceptance is determined with respect to the lot proportion defective.

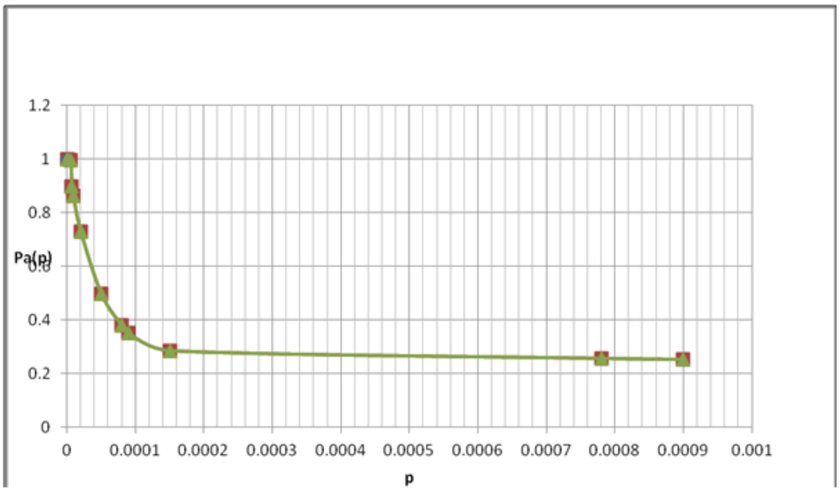
Table -2: Probability of acceptance is determined with respect to the lot proportion defective

ω	n	p	c	$P_a(p)$
0.25	225	0	0	1
0.25	225	0.0005	1	0.995595
0.25	225	0.0007	2	0.898654
0.25	225	0.0009	3	0.863363
0.25	225	0.002	4	0.729038
0.25	225	0.005	5	0.497146
0.25	225	0.008	6	0.379831
0.25	225	0.009	7	0.351738

0.25	225	0.015	8	0.286378
0.25	225	0.078	9	0.257797
0.25	225	0.09	10	0.253847

Table 2 indicates the different values. By choosing **ω = 0.25, n=225,** the proportion of defective **p** value will increase while there is a decrease value of the probability of acceptance of the lot $\mathbf{P_a(p)}$ and same time the probability of acceptance to attain at the minimum value is 0.25 like parameter **ω** value.

Figure 1: OC curve for the plan ω=0.25, n = 225 and c = 3



This also replicates in the operating characteristic (OC) curve for this plan is presented in figure 1 which was constructed based on the table2. This was plotted as a graph using the above parameters.

Practical Application

Suppose a chemical processing company fixes LQL as 0.00005 (It means 5 Non – confirming units out of 100000 items) then inspect a random sample of 225 units taken from a lot of units produced in a given period of time in any scale and count the number of non-confirming units (d). If $d \le 3$, accept the lot of units processed during the period, otherwise reject the lot of units and inform the management for corrective action.

V. Conclusion

In this paper explores a general procedure for constructing a Single Sampling Plan (SSP) indexed through Limiting Quality Level (LQL) using Inflated Poisson Distribution. This was inflated with lowest value for a given x (equal to 0) and a table is also provided for the enhanced selection of the plans. These plans are very useful for commercial companies which have at least one defective unit in their lot. Furthermore this is very much useful for companies which are using for subsequent quality lots.

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References

[1] Burguess AR (1948). A graphical method for determining Single Sampling Plan. Industrial Quality Control. pp.25 - 27.
[2] Cameron JM (1952).Tables for constructing and computing the Operating Characteristics of Single Sampling Plans. Industrial Quality Control. Vol.11, pp.37 - 39.
[3] Dodge, Harold F.,and Harry G.Romig. Sampling Inspection Tables: Single and Double Sampling. New York: John Wiley & Sons, Inc., 1959.
[4] Golub Abraham (1953). Designing Single Sampling Inspection Plan when the sample size is fixed. Journal of the American Statistician Association. Vol.48, pp.278 - 288.
[5] Grubs FE (1949).On designing Single Sampling inspection plan Annals of Mathematical Statistics. Vol. 20, pp. 42 – 256.
[6] Guenther WC (1969). Use of the Binomial, Hyper geometric and Poisson tables to obtain Sampling Plans. Journal of Quality Technology. 1(2), pp. 105 - 109.
[7] John M. Williamson, Hung-Mo Lin, Robert H. Lyles and Allen W (2007). Hightower Power Calculations for ZIP and ZINB Models. Journal of Data Science 5, 519-534
[8] Lambert, D. (1992). Zero-inflated Poisson regression, with an application to defects in manufacturing. Technimetrics 34, 1-14.
[9] Lee, A. H., Wang, K. and Yau, K. K. W. (2001). Analysis of zero-inflated Poisson data incorporating extent of exposure. Biometrical Journal 43, 963-975.
[10] Loganathan.A and Shalini.K. 2012, "Determination of single sampling plans by Attributes under the conditions of zero-Inflated Poisson Distribution". ISSN No: 0361-0918.
[11] Li, C. S., Lu, J. C., Park, J., Brinkley, P. A. and Peterson, J. (1999). Multivariate zero-inflated Poisson models and their applications. Technimetrics 41, 29-38.

[12] Ponnuraja C and Santhi S (2014),"Construction of single sampling plan indexed through Average quality level," International Journal of Scientific research .ISSN NO.2277-8179, Vol.3, pp 60-62.

[13] Radhakrishnan, R., and Pratheeba, S., 2011, "Construction of Single Sampling Plan Indexed through Indifference Quality Level," Global Journal of Mechanical Engineering and Computational Science. ISSN No. 2231-5179, Vol. 1, No.3, pp. 9-11.

[14] Raja.T.A,"On Zero Inflated Poisson Distribution with Reference to Poisson Distribution International, Journal of Modern Mathematical Sciences, 2012, 3(2): 85-90

[15] Shanmugam R (1985). An intervened Poisson distribution and its medical Applications. Biometrics. Vol. 41, pp. 1025 - 1029.

[16] Soundararajan V, Govindaraju K (1983). A note on Designing Single Sampling Plans. IAPQR Transactions. Vol.8, No.1, pp.37 - 41.

[17] Schilling, E.G., Acceptance Sampling in Quality Control (Marcel Dekker, New York. 1982)

[18] Radhakrishnan R (2002). Contribution to the study on selection of certain Acceptance Sampling Plans. Ph.D. Dissertation. Bharathiar University. Coimbatore. India.