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On Expanding $\frac{3}{n}$ Into Three Term Egyptian Fractions

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Abstract: *It is well known that fraction (a/b) can be expressed as the sum of N unit fractions. Such representations are known as Egyptian fractions. In practice, each a/b can be expressed by several different Egyptian fraction expansions. In this paper we present a generalized expression for $3/n$ where $N = 3$ for all positive integers n. Under mind assumptions convergence results has been established.*
Keywords: *Egyptian Fraction, Unit Fraction, Shortest Egyptian Fraction*

I. Introduction

Ancient Egyptian hieroglyphics tell us much about the people of ancient Egypt, including how they did mathematics. The Rhind Mathematical Papyrus, the oldest existing mathematical manuscript, stated that; their basic number system is very similar to ours except in one way – their concept of fractions [2]. The ancient Egyptians had a way of writing numbers to at least 1 million. However, their method of writing fractions was limited. For instance to represent the fraction 1/5, they would simply use the symbol for 5, and place another symbol on top of it [3]. In general, the reciprocal of an integer n was written in the same way. They had no other way of writing fractions, except for a special symbol for 2/3 and perhaps 3/4 [1]. This is not to say that the number 5/6 did not exist in ancient Egypt. They simply had no way of writing it as a single symbol. Instead, they would write 1/2 + 1/3. Thus, Egyptian fraction is a term which now refers to any expression of a rational number as a sum of distinct unit fractions (a unit fraction is a reciprocal of a positive integer). The study of the properties of Egyptian fractions falls into the area of number Theory, and provides many challenging unsolved problems [4, 5]. In this work we proposed to examine some of the problems concerning Egyptian fractions in which we generalized the pattern for 3/n for all positive integers n. This paper has been arranged as follows; we present brief overview and some basic definitions of Egyptian terminologies in section 2, the methodology of this work is given in section 3, and finally conclusion is given in section 4.

II. Preliminaries (Definitions of Terms)

Egyptian Fraction: an **Egyptian fraction** is the sum of distinct unit fractions, such as $\frac{1}{2} + \frac{1}{3} + \frac{1}{16}$. That is, each fraction in the expression has a numerator equal to 1 and a denominator that is a positive integer, and all the denominators differ from each other.

Unit Fraction: a **unit fraction** is a rational number written as a fraction where the numerator is one and the denominator is a positive integer. A unit fraction is therefore the reciprocal of a positive integer, 1/n. Examples are 1/1, 1/2, 1/3, 1/4 etc.

Shortest Egyptian Fraction: a shortest Egyptian fraction for T/B where both T and B are positive integers is the shortest for of expressing T/B as a unit fraction. E.g.
4/19 = 1/5 + 1/95 and
4/13 = 1/4 + 1/18 + 1/468

III. Three Term Egyptian Fraction for 3/n

A fraction $\frac{3}{n}$ is said to be of length three if it can be express as $\frac{3}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ where $a, b, c \in \mathbb{N}$, otherwise if $\frac{3}{n}$ cannot be express in this form then we said that it unit fraction of length three doesn't exist.

In the remaining part of this work, we presented the expression for all categories of $n \in \mathbb{N}$ under consideration;

Now for all even numbers n we have the following

n	$\frac{3}{n}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$
2.	$\frac{3}{2}=\frac{1}{1}+\frac{1}{3}+\frac{1}{6}$
4.	$\frac{3}{4}=\frac{1}{2}+\frac{1}{5}+\frac{1}{20}$
6.	$\frac{3}{6}=\frac{1}{3}+\frac{1}{7}+\frac{1}{42}$
8.	$\frac{3}{8}=\frac{1}{4}+\frac{1}{9}+\frac{1}{72}$
10.	$\frac{3}{10}=\frac{1}{5}+\frac{1}{11}+\frac{1}{110}$
12.	$\frac{3}{12}=\frac{1}{6}+\frac{1}{13}+\frac{1}{156}$
14.	$\frac{3}{14}=\frac{1}{7}+\frac{1}{15}+\frac{1}{210}$

Continue in this manner we present that $\forall n \in \mathbb{N}$, where n is even, then $\frac{3}{n}$ of length three can be express as

$$\frac{3}{n}=\frac{1}{\left(\frac{n}{2}\right)}+\frac{1}{(n+1)}+\frac{1}{n(n+1)}$$

Moreover for $3/n$ where n is odd prime number we investigate using the table below:

n	$\frac{3}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$
3.	$\frac{3}{3}=\frac{1}{2}+\frac{1}{3}+\frac{1}{6}$
5.	$\frac{3}{5}=\frac{1}{3}+\frac{1}{5}+\frac{1}{15}$
7.	$\frac{3}{7}=\frac{1}{4}+\frac{1}{7}+\frac{1}{28}$
9.	$\frac{3}{9}=\frac{1}{5}+\frac{1}{9}+\frac{1}{45}$
11.	$\frac{3}{11}=\frac{1}{6}+\frac{1}{11}+\frac{1}{66}$
13.	$\frac{3}{13}=\frac{1}{7}+\frac{1}{13}+\frac{1}{91}$
15.	$\frac{3}{15}=\frac{1}{8}+\frac{1}{15}+\frac{1}{120}$

Continue in this manner we present that for all $\forall n \in \mathbb{N}$, where n is an odd prime number, then

$$\frac{3}{n}=\frac{1}{\left(\frac{n+1}{2}\right)}+\frac{1}{n}+\frac{1}{n\left(\frac{n+1}{2}\right)}$$

Hence, we present our main result via the below theorem:

Theorem: $\forall \mathbf{n} \in \mathbb{N}, \mathbf{n} \geq \mathbf{2}$ the general pattern of $\frac{3}{n}$ is given by

$$\frac{3}{n}=\delta\left(\frac{1}{\left(\frac{n}{2}\right)}+\frac{1}{(n+1)}+\frac{1}{n(n+1)}\right)+\phi\left(\frac{1}{\left(\frac{n+1}{2}\right)}+\frac{1}{n}+\frac{1}{n\left(\frac{n+1}{2}\right)}\right)$$

Where

$$\delta=1, \phi=0 \text{ if n is even number and}$$
$$\delta=0, \phi=1 \text{ if n is odd prime number.}$$

The Algorithm 1: (Egyptian Fraction for $3/n$)

Step 1: Input n
Step 2: is $n>3$?
If yes go to step 3;
Else go back to step 1;
Step 3: is n mod $2=0$?
If yes set $\delta=1, \phi=0$ and go to step 5;
Else go to step 4;
Step 4: is n mod $2=1$?

If yes set $\delta = 0, \phi = 1$ and go to step 5;
Else go to back to step 1;
Step 5: Compute

$$\frac{3}{n} = \delta\left(\frac{1}{\left(\frac{n}{2}\right)} + \frac{1}{(n+1)} + \frac{1}{n(n+1)}\right) + \phi\left(\frac{1}{\left(\frac{n+1}{2}\right)} + \frac{1}{n} + \frac{1}{n\left(\frac{n+1}{2}\right)}\right)$$

and go to step 6.
Step 6: Output $\frac{3}{n}$

IV. Conclusion

In this paper we presented the general formula for $3/n$ for all positive integers n, our approach is based on even numbers and odd prime numbers. The fact that our formula will converge as n tend to be very large since the denominator is strictly increasing, so we claim that our formula is true for all positive integers n.

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