IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 10, Issue 5 Ver. IV (Sep-Oct. 2014), PP 25-33
www.iosrjournals.org

On Jordan Generalized Higher Reverse Derivations on Γ-rings

Salah Mehdi Salih and Marwa Riyadh Salih
Al-Mustansirya University College of Education Department of Mathematics

Abstract: *In this paper, we study the concepts of generalized higher reverse derivation and Jordan generalized higher reverse derivation and Jordan generalized triple higher reverse derivation on Γ-ring M.*
The aim of this paper is prove that every Jordan generalized higher reverse derivation of Γ-ring M is generalized higher reverse derivation of M.

Mathematics Subject Classification: *16U80, 16W25*
Key word: *Γ-ring, prime Γ-ring, semiprimeΓ-ring, derivation, higher derivation, generalized higher derivation of Γ-ring, reverse derivation of R*

I. Introduction

The concepts of a Γ-ring was first introduced by Nobusause[9] in 1964 this Γ-ring is generalized by W.E.Barnesin [2] a broad sense that served now a day to call a Γ-ring.
Let M and Γ be two additive abelian groups. Suppose that there is a mapping from M ×Γ×M→M (the image of(a,α,b) being denoted by aαb, a,b∈ M and α∈Γ) satisfying for all a,b,c∈ M and α,β∈Γ
i) (a +b) α c = aαc + bαc
a(α + β) c = aαc +aβc
aα(b + c) = aαb + aαc
ii) (aαb)βc = aα(bβc)
Then M is called a Γ-ring.[2]

Throughout this paper M denotes a Γ-ring with center Z(M) [1] ,recall that a -Γring M is called prime if aΓMΓb=(0) implies a=o or b=0[8],and it is called semiprime if aΓMΓa=(0) implies a=o[6], a prim Γ-ring is obviously semiprime and a Γ-ring M is called 2-torision free if 2a=0 implies a=0 for every a ∈ M [5],an additive mapping d from M into itself is called a derivations if d(aαb)=d(a)αb + aαd(b),for all a,b ∈ M ,α∈Γ [7] and d is said to be Jordan derivation of a Γ-ring M if d(aαa)= d(a)αa + aαd(a),for all a ∈ M ,α ∈ Γ [7].A mapping f from M into itself is called generalized derivation of M if there exists derivation d of M such that f(aαb)=f(a)αb +aαd(b), for all a,b∈ M, α∈Γ[4]. And f is said to be Jordan generalized derivation of Γ-ring M if there exists Jordan derivation of M such that f(aαa)= f(a)αa + aαd(a)
for all a ∈ M and α∈ Γ [4].

Bresar and Vukman[3] have introduced the notion of a reverse derivation as an additive mapping d from a ring R into itself satisfying d(xy) = d(y)x +yd(x) for all x,y∈ R.

M. Sammn[10] presented the study between the derivation and reverse derivation in semiprime ring R. Also it is shown that non-commutative prime rings don't admit a non-trivial skew commuting derivation.
We defined in [11] the concepts of higher reverse derivation of Γ-ring M as follow:
Let $D=(d_i)_{i\in N}$ be additive mappings on a ring R then D is called higher reverse derivation of Γ-ring M if

$$d_n(x\alpha y) = \sum_{i+j=n} d_i(y)\alpha d_j(x)$$

For all x,y∈ M ,α∈Γ and n∈N
and Jordan higher reverse derivation of Γ-ring M if

$$d_n(x\alpha x) = \sum_{i+j=n} d_i(x)\alpha d_j(x)$$

and Jordan triple higher reverse derivation of Γ-ring M if

$$d_n(x\alpha y\beta x) = d_n(x)\beta x\alpha y + \sum_{i+j+r=n}^{i<n} d_i(x)\beta d_j(y)\alpha d_r(x)$$

For all x,y∈ M ,α, β∈Γ and n∈N

also we proved that every Jordan higher reverse derivation of a Γ-ring M is higher reverse derivation of M [11], the main object of this paper is present the concepts of generalized higher reverse derivation , Jordan generalized higher reverse derivation of Γ-ring M and we prove that every Jordan generalized higher reverse derivation of Γ-ring M is generalized higher reverse derivation of M.

II. Generalized Higher Reverse Derivation of Γ-Rings

In this section we introduce and study of concepts of generalized higher reverse derivation, Jordan generalized higher reverse derivation and Jordan generalized triple higher reverse derivation of Γ-ring.

Definition2.1:

Let M be a Γ-ring and F = $(fi)_{i\in N}$ be a family of additive mappings of M such that $f_0 = id_M$ then F is called **generalized higher reverse derivation of M** if there exists a higher reverse derivation D =$(di)_{i\in N}$ of M such that for all n∈N we have :

$$f_n(x\alpha y) = \sum_{i+j=n} f_i(y)\alpha d_i(x) \; \ldots . (i)$$

F is called **a Jordan generalized higher reverse derivation of M** if there exists a Jordan higher reverse derivation D = $(di)_{i\in N}$ of M such that for all n∈N we have :

$$f_n(x\alpha x) = \sum_{i+j=n} f_i(x)\alpha d_j(x) \; \ldots \ldots (ii)$$

For every x,y∈ M and α∈Γ

F is said to b**e a Jordan generalized triple higher reverse derivation of M** if there exists Jordan triple higher reverse derivation D = $(di)_{i\in N}$ of M for all n∈N we have:

$$f_n(x\alpha y\beta x) = f_n(x)\beta x\alpha y + \sum_{i+j+r=n}^{i<n} f_i(x)\beta d_j(y)\alpha d_r(x) \; \ldots \ldots (iii)$$

For every x,y∈ M and α,β∈Γ

Example2.2:

Let F =$(f_i)_{i\in N}$ be a generalized higher reverse derivation on a ring R then there exists a higher reverse derivation d =$(f_i)_{i\in N}$ of R such that

$$f_n(xy) = \sum_{i+j=n} f_i(y)d_j(x)$$

We take M = $M_{1X2}(R)$ and Γ=$\{\binom{n}{0}: n \in Z\}$, then M is Γ-ring .

We define D = $(Di)_{i\in N}$ be a family of additive mappings of M such that $D_n(a \;\; b) = (d_n(a) \;\; d_n(b))$ then D is higher reverse derivation of M.

Let F = $(f_i)_{i\in N}$ be a family of additive mappings of M defined by $F_n(a \;\; b) = (f_n(a) \;\; f_n(b))$

Then F is a generalized higher reverse derivation of M .

It is clear that every generalized higher reverse derivation of a Γ-ring M is Jordan generalized Higher reverse derivation of M, But the converse is not true in general.

Lemma 2.3

Let M be a Γ-ring and let F =$(f_i)_{i\in N}$ be a Jordan generalized higher reverse derivation of M then for all x,y,z∈ M , α,β∈Γ and n ∈ N ,the following statements hold :

i) $f_n(x\alpha y + y\alpha x) = \sum_{i+j=n}^{i<n} f_i(y)\alpha d_j(x) + f_i(x)\alpha d_j(y)$

In particular if y ∈Z(M)

ii) $f_n(x\alpha y\beta x + x\beta y\alpha x) = f_n(x)\beta x\alpha y + \sum_{i+j+r=}^{i<n} f_i(x)\beta d_j(y)\alpha d_r(x) + f_n(x)\alpha x\beta y$

$+ \sum_{i+j+r=n}^{i<n} f_i(x)\alpha d_j(y)\beta d_r(x)$

iii) $f_n(x\alpha y\alpha x) = f_n(x)\alpha x\alpha y + \sum_{i+j+r=n}^{i<n} f_i(x)\alpha d_j(y)\alpha d_r(x)$

iv) $f_n(x\alpha y\alpha z + z\alpha y\alpha x) = f_n(z)\alpha x\alpha y + \sum_{i+j+r=n}^{i<n} f_i(z)\alpha d_j(y)\alpha d_r(x) + f_n(x)\alpha z\alpha y + \sum_{i+j+r=}^{i<n} f_i(x)\alpha d_j(y)\alpha d_r(z)$

v) $f_n(x\alpha y\beta z) = f_n(z)\beta x\alpha y + \sum_{i+j+r=n}^{i<n} f_i(z)\beta d_j(y)\alpha d_r(x)$

vi) $f_n(x\alpha y\beta z + z\alpha y\beta x) = f_n(z)\beta x\alpha y + \sum_{i+j+r=n}^{i<n} f_i(z)\beta d_j(y)\alpha d_r(x) + f_n(x)\beta z\alpha y + \sum_{i+j+r=n}^{i<n} f_i(x)\beta d_j(y)\alpha d_r(z)$

Proof:

i)Replace (x + y) for x and y in definition 2.1 (i) we get :

$$f_n\big((x+y)\alpha(x+y)\big) = \sum_{i+j=n} f_i(x+y)\alpha d_j(x+y)$$

$$= \sum_{i+j=n} f_i(x)\alpha d_j(x) + f_i(y)\alpha\, d_j(x) + f_i(x)\alpha d_j(y) + f_i(y)\alpha d_j(y) \qquad \ldots (1)$$

On the other hand:

$$f_n\big((x+y)\alpha(x+y)\big) = f_n(x\,\alpha\,x + x\alpha\,y + y\alpha\,x + y\alpha\,y)$$

$$= f_n(x\alpha\,x + y\alpha\,y) + f_n(x\alpha\,y + y\alpha\,x)$$

$$= \sum_{i+j=n} f_i(x)\alpha d_j(x) + f_i(y)\alpha\, d_j(y) + f_n(x\alpha\,y + y\alpha\,x) \qquad \ldots (2)$$

Compare (1) and (2) we get:

$$f_n(x\alpha\,y + y\alpha\,x) = \sum_{i+j=n} f_i(y)\alpha d_j(x) + f_i(x)\alpha d_j(y)$$

ii) Replacing xβy +yβx for y in 2.3 (i) we get:

$$f_n(x\alpha(x\beta y + y\beta x) + (x\beta y + y\beta x)\alpha\,x)$$

$$= f_n(x\alpha(x\beta y) + x\alpha(y\beta x) + (x\beta y)\alpha\,x + (y\beta x)\alpha\,x)$$

$$= f_n((x\alpha\,x)\beta y + (x\alpha\,y)\beta x + (x\beta y)\alpha\,x + (y\beta x)\alpha\,x$$

$$= \sum_{i+j=n} f_i(y)\beta d_j(x\alpha\,x) + f_i(x)\beta d_j(x\alpha\,y) + f_i(x)\alpha d_i(x\beta\,y) + f_i(x)\alpha d_j(y\beta\,x)$$

$$= \sum_{i+j+r=n} f_i(y)\beta d_j(x)\alpha d_r(x) + f_i(x)\beta d_j(y)\alpha f_i(x) + f_i(x)\alpha d_j(y)\beta d_r(x)$$
$+f_i(x)\alpha d_j(x)\beta d_r(y)$

$$= f_n(y)\beta x\alpha x + \sum_{i+j+r=n}^{i<n} f_i(y)\beta d_j(x)\alpha d_r(x) + f_n(x)\beta x\alpha y + \sum_{i+j+r=n}^{i<n} f_i(x)\beta d_j(y)\alpha d_r(x)$$

$+f_n(x)\alpha\, x\beta\, y + \sum_{i+j+r=n}^{i<n} f_i(x)\alpha\, d_j(y)\beta\, d_r(x) + f_n(x)\alpha\, y\beta\, x + \sum_{i+j+r=n}^{i<n} f_i(x)\alpha d_j(x)\beta d_r(y) \ldots(1)$

On the other hand:

$f_n(x\alpha(x\beta y + y\beta x) + (x\beta y + y\beta x)\alpha\, x) = f_n(x\alpha\, x\beta\, y + x\alpha\, y\beta\, x + x\beta\, y\alpha\, x + y\beta\, x\alpha\, x)$

$$= f_n(y)\beta\, x\alpha\, x + \sum_{i+j+r=n}^{i<n} f_i(y)\beta d_j(x)\alpha d_r(x) + f_n(x)\alpha\, y\beta\, x + \sum_{i+j+r=n}^{i<n} f_i(x)\alpha d_j(x)\beta d_r(y)$$

$$+f_n(x\alpha\, y\beta\, x + x\beta\, y\alpha\, x) \qquad \ldots(2)$$

Compare (1) and (2) we get the require result.

iii) Replacing α for β in 2.3 (ii) we have:

$f_n(x\alpha\, y\alpha\, x + x\alpha\, y\alpha\, x) = 2(f_n(x\alpha\, y\alpha x))$

$= 2(f_n(x)\alpha\, x\alpha\, y + \sum_{i+j+r=n}^{i<n} f_i(x)\alpha d_j(y)\alpha d_r(x))$

Since M is 2- torsion free then we get:

$$f_n(x\alpha\, y\alpha\, x) = f_n(x)\alpha\, x\alpha\, y + \sum_{i+j+r=n}^{i<n} f_i(x)\alpha d_j(y)\alpha d_r(x)$$

iv) Replacing x+z for x in 2.3 (iii) we have:

$$f_n\big((x+y)\alpha y\alpha(x+y)\big) = f_n(x+z)\alpha(x+z)\alpha y + \sum_{i+j+r=n}^{i<n} f_i(x+z)\alpha d_j(y)\alpha d_r(x+z)$$

$$= f_n(x)\alpha x\alpha y + \sum_{i+j+r=n}^{i<n} f_i(x)\alpha d_j(y)\alpha d_r(x)$$

$$+ f_n(z)\alpha x\alpha y + \sum_{i+j+r=n}^{i<n} f_i(z)\alpha d_j(y)\alpha d_r(x)$$

$$+f_n(x)\alpha z\alpha y + \sum_{i+j+r=n}^{i<n} f_i(x)\alpha d_j(y)\alpha d_r(z)$$

$$+ f_n(z)\alpha z\alpha y + \sum_{i+j+r=n}^{i<n} f_i(z)\alpha d_j(y)\alpha d_r(z) \qquad \ldots(1)$$

On the other hand:

$f_n(\,(x+y)\alpha\, y\alpha\,(x+z)\,) = f_n(x\alpha y\alpha x + x\alpha y\alpha z + z\alpha y\alpha x + z\alpha y\alpha z\,)$

$= f_n(x)\alpha\, xy\alpha + \sum_{i+j+r=n}^{i<n} f_i(x)\alpha d_j(y)\alpha d_r(x)$

$+f_n(z)\alpha z\alpha y + \sum_{i+j+r=n}^{i<n} f_i(z)\alpha d_j(y)\alpha d_r(z) + f_n(x\alpha y\alpha z + z\alpha y\alpha x) \qquad \ldots(2)$

Compare (1) and (2) we get the require result.

(v) Replace (x +z) for x in definition 2.1(iii) we have:

$$f_n(\,(x+z)\alpha y\beta(x+z) = f_n(x+z)\beta(x+z)\alpha y + \sum_{i+j+r=n}^{i<n} f_i(x+z)\beta d_j(y)\alpha d_r(x+z)$$

$$= f_n(x)\beta x\alpha y + \sum_{i+j+r=n}^{i<n} f_i(x)\beta d_j(y)\alpha d_r(x) + f_n(z)\beta x\alpha y + \sum_{i+j+r=n}^{i<n} f_i(z)\beta d_j(y)\alpha d_r(x)$$

$$+ f_n(z)\beta x\alpha y + \sum_{i+j+r=n}^{i<n} f_i(z)\beta d_j(y)\alpha d_r(x) + f_n(z)\beta z\alpha y \sum_{i+j+r=n}^{i<n} f_i(z)\beta d_j(y)\alpha d_r(z) \ldots (1)$$

On the other hand:

$f_n(\,(x+z)\alpha\, y\,\beta(x+z)\,) = f_n(x\alpha y\beta x + x\alpha y\beta z + z\alpha y\beta x + z\alpha y\beta z)$

$$= f_n(x\alpha y\beta x + z\alpha y\beta x + z\alpha y\beta z\,) + f_n(x\alpha y\beta z)$$

$$= f_n(x)\beta x\alpha y + \sum_{i+j+r=n}^{i<n} f_i(x)\beta d_j(y)\alpha d_r(x)$$

$$+ f_n(x)\beta z\alpha y + \sum_{i+j+r=n}^{i<n} f_i(x)\beta d_j(y)\alpha d_r(z) + f_n(z)\beta z\alpha y + \sum_{i+j+r=n}^{i<n} f_i(z)\beta d_j(y)\alpha d_r(z)$$

$$+ f_n(x\alpha y\beta z) \qquad \ldots.(2)$$

Compare (1) and (2) we get:

$$f_n(x\alpha y\beta z) = f_n(z)\beta x\alpha y + \sum_{i+j+r=n}^{i<n} f_i(z)\beta d_j(y)\alpha d_r(x)$$

vi)Replace (x + z) for x in definition 2.1(iii) we have:

$$f_n\big((x+z)\alpha\, y\,\beta(x+z)\big) = f_n(x+z)\beta(x+z)\alpha\, y + \sum_{i+j+r=n}^{i<n} f_i(x+z)\beta d_j(y)\alpha d_r(x+z)$$

$$= \big(f_n(x) + f_n(z)\big)\beta(x+z)\alpha y + \sum_{i+j+r=n}^{i<n} \big(f_i(x) + f_i(z)\big)\beta d_j(y)\alpha\,(\,d_r(x) + d_r(z)\,)$$

$$= f_n(x)\beta x\alpha y + f_n(z)\beta x\alpha y + f_n(x)\beta z\alpha y + f_n(z)\beta z\alpha y$$

$$+ \sum_{i+j+r=n}^{i<n} f_i(x)\beta d_j(y)\alpha d_r(x) + f_i(z)\beta d_j(y)\alpha d_r(x) + f_i(x)\beta d_j(y)\alpha d_r(z)$$

$$+ f_i(z)\beta d_j(y)\alpha d_r(z) \ldots\ldots.(1)$$

On the other hand:

$f_n(\,(x+z)\alpha\, y\,\beta(x+z)) = f_n(\,x\alpha y\beta x + x\alpha y\beta z + z\alpha y\beta x + z\alpha y\beta z\,)$

$$= f_n(x\alpha y\beta x + z\alpha y\beta z\,) + f_n(x\alpha y\beta z + z\alpha y\beta x\,)$$

$$= f_n(x)\beta x\alpha y + \sum_{i+j+r=n}^{i<n} f_i(x)\beta d_j(y)\alpha d_r(x)$$

$$+ f_n(z)\beta z\alpha y + \sum_{i+j+r=n}^{i<n} f_i(z)\beta d_j(y)\alpha\, d_r(z) + f_n(x\alpha y\beta z + z\alpha y\beta x\,) \ldots\ldots(2)$$

Compare (1) and (2) we get the require result

Definition 2.4:

Let $F = (f_i)_{i\in N}$ be a Jordan generalized higher reverse derivation of a Γ-ring M,then for all x,y∈ M and α∈ Γ we define:

$$\delta_n(x,y)_\alpha = f_n(x\alpha y) - \sum_{i+j=n} f_i(y)\alpha d_j(x)$$

In the following lemma introduce some properties of $\delta_n(x,y)_\alpha$

Lemma 2.5

If $F = (f_i)_{i\in N}$ is a Jordan generalized higher reverse derivation of Γ-ring M then for all x,y ,z ∈ M , α,β ∈ Γ and n ∈ N:

i. $\delta_n(x,y)_\alpha = -\delta_n(y,x)_\alpha$
ii. $\delta_n(x+y,z)_\alpha = \delta_n(x,z)_\alpha + \delta_n(y,z)_\alpha$
iii. $\delta_n(x,y+z)_\alpha = \delta_n(x,y)_\alpha + \delta_n(x,z)_\alpha$
iv. $\delta_n(x,y)_{\alpha+\beta} = \delta_n(x,y)_\alpha + \delta_n(x,y)_\beta$

Proof:

i. by lemma 2.3 (i) and since f_n is additive mapping of M we get:

$$f_n(x\alpha y + y\alpha x) = \sum_{i+j=n} f_i(y)\alpha d_j(x) + f_i(x)\alpha d_j(y)$$

$$f_n(x\alpha y) + f_n(y\alpha x) = \sum_{i+j=n} f_i(y)\alpha d_j(x) + \sum_{i+j=n} f_i(x)\alpha d_j(y)$$

$$f_n(x\alpha y) - \sum_{i+j=n} f_i(y)\alpha d_j(x) = -f_n(y\alpha x) + \sum_{i+j=n} f_i(x)\alpha d_j(y)$$

$$f_n(x\alpha y) - \sum_{i+j=n} f_i(y)\alpha d_j(x) = -(f_n(y\alpha x) - \sum_{i+j=n} f_i(x)\alpha d_j(y))$$

$$\delta_n(x,y)_\alpha = -\delta_n(y,x)_\alpha.$$

ii.

$$\delta_n(x+y,z)_\alpha = f_n((x+y)\alpha z) - \sum_{i+j=n} f_i(z)\alpha d_j(x+y)$$

$$= f_n(x\alpha z + y\alpha z) - (\sum_{i+j=n} f_i(z)\alpha d_j(x) + f_i(z)\alpha d_j(y))$$

$$= f_n(x\alpha z) - \sum_{i+j=n} f_i(z)\alpha d_j(x) + f_n(y\alpha z) - \sum_{i+j=n} f_i(z)\alpha d_j(y)$$

$=\delta_n(x,z)_\alpha + \delta_n(y,z)_\alpha$ .iii.

$$\delta_n(x,y+z)_\alpha = f_n(x\alpha(y+z)) - \sum_{i+j=n} f_i(y+z)\alpha d_j(x)$$

$$= f_n(x\alpha y + x\alpha z) - \sum_{i+j=n} f_i(y)\alpha d_j(x) - f_i(z)\alpha d_j(x)$$

Since f_n is additive mapping of M then we have:

$$= f_n(x\alpha y) - \sum_{i+j=n} f_i(y)\alpha d_j(x) + f_n(x\alpha z) - \sum_{i+j=n} f_i(z)\alpha d_j(x)$$

$$= \delta_n(x,y)_\alpha + \delta_n(x,z)_\alpha \ .$$

iv.

$$\delta_n(x,y)_{\alpha+\beta} = f_n(x(\alpha+\beta)y) - \sum_{i+j=n} f_i(y)(\alpha+\beta)d_j(x)$$

$$= f_n(x\alpha y + x\beta y) - \sum_{i+j=n} f_i(y)\alpha d_j(x) - f_i(y)\beta d_j(x)$$

Since f_n is additive mapping

$$= f_n(x\alpha y) - \sum_{i+j=n} f_i(y)\alpha d_j(x) + f_n(x\beta y) - \sum_{i+j=n} f_i(y)\beta d_j(x)$$

$$= \delta_n(x,y)_\alpha + \delta_n(x,y)_\beta \ .$$

Remark 2.6:

Note that $F = (f_i)_{i\in N}$ is generalized higher reverse derivation of a Γ-ring M if and only if $\delta_n(x,y)_\alpha = 0$ for all x,y ∈ M ,α ∈ Γ and n ∈ N .

III. The Main Results

In this section we present the main results of this paper.

Theorem 3.1:

Let $F = (f_i)_{i\in N}$ be a Jordan generalized higher reverse derivation of M then $\delta_n(x,y)_\alpha = 0$ for all x, y ∈ M ,α ∈ Γ and n ∈ N .

Proof:

By lemma 2.3 (i) we get:

$$f_n(x\alpha y + y\alpha x) = \sum_{i+j=n} f_i(y)\alpha d_j(x) + f_i(x)\alpha d_j(y) \quad \ldots\ldots..(1)$$

On the other hand:

Since f_n is additive mapping of the Γ-ring M we have:

$$f_n(x\alpha y + y\alpha x) = f_n(x\alpha y) + f_n(y\alpha x)$$

$$= f_n(x\alpha y) + \sum_{i+j=n} f_i(x)\alpha d_j(y) \ \ldots\ldots\ldots\ldots..(2)$$

Compare (1) and (2) we get:

$$f_n(x\alpha y) = \sum_{i+j=n} f_i(y)\alpha d_j(x)$$

$$f_n(x\alpha y) - \sum_{i+j=n} f_i(y)\alpha d_j(x) = 0$$

By definition 2.5 we get:

$$\delta_n(x,y)_\alpha = 0$$

Corollary 3.2:
Every Jordan generalized higher reverse derivation of Γ-ring M is generalized higher reverse derivation of M .

Proof:

By theorem 3.1 we get $\delta_n(x,y)_\alpha = 0$ and by Remark 2.6 we get the require result .

Proposition 3.3
Every Jordan generalized higher reverse derivation of a 2-torision free Γ-ring M such that $x\alpha y\beta z = x\beta y\alpha z$ and $y \in Z(M)$ is Jordan generalized triple higher reverse derivation of M .

Proof:

Let $F = (f_i)_{i\in N}$ be a Jordan generalized higher reverse derivation of M

Replace y by $(x\beta y + y\beta x)$ in lemma 2.3 (i) we get

$f_n(x\alpha(x\beta y + y\beta x) + (x\beta y + y\beta x)\alpha x) = f_n((x\alpha(x\beta y) + x\alpha(y\beta x) + (x\beta y\alpha)x + (y\beta x)\alpha x)$
$= f_n((x\alpha x)\beta y + (x\alpha y)\beta x + (x\beta y)\alpha x + (y\beta x)\alpha x)$

$$= \sum_{i+j=n} f_i(y)\beta d_j(x\alpha x) + f_i(x)\beta d_j(x\alpha y) + f_i(x)\alpha d_j(x\beta y) + f_i(x)\alpha d_j(y\beta x)$$

$$= \sum_{i+j+r=n} f_i(y)\beta d_j(x)\alpha d_r(x) + f_i(x)\beta d_j(y)\alpha d_r(x) + f_i(x)\alpha d_j(y)\beta d_r(x) + f_i(x)\alpha d_j(x)\beta d_r(y)$$

$$= f_n(y)\beta x\alpha x + \sum_{i+j+r=n}^{i<n} f_i(y)\beta d_j(x)\alpha d_r(x) + f_n(x)\beta x\alpha y + \sum_{i=j+r=n}^{i<n} f_i(x)\beta d_j(y)\alpha d_r(x)$$

$$+ f_n(x)\alpha x\beta y + \sum_{i+j+r=n}^{i<n} f_i(x)\alpha d_j(y)\beta d_r(x) + f_n(x)\alpha y\beta x + \sum_{i+j+r=n}^{i<n} f_i(x)\alpha d_j(x)\beta d_r(y) \ldots\ldots(1)$$

On the other hand:

$f_n(x\alpha(x\beta y + y\beta x) + (x\beta y + y\beta x)\alpha x) = f_n(x\alpha x\beta y + x\alpha y\beta x + x\beta y\alpha x + y\beta x\alpha x)$

$= f_n(x\alpha x\beta y + y\beta x\alpha x) + f_n(x\alpha y\beta x + x\beta y\alpha x)$

$$= f_n(y)\beta x\alpha x + \sum_{i+j+r=n}^{i<n} f_i(y)\beta d_j(x)\alpha d_r(x)$$

$$+ f_n(x)\alpha y\beta x + \sum_{i+j+r=n}^{i<n} f_i(x)\alpha d_j(x)\beta d_r(y) + f_n(x\alpha y\beta x + x\beta y\alpha x) \ldots\ldots(2)$$

Compare (1) and (2) and since $x\alpha y\beta z = x\beta y\alpha z$ we get

$f_n(x\alpha y\beta x + x\alpha y\beta x) = 2(f_n(x\alpha y\beta x))$

$$= 2 (f_n(x)\beta x\alpha y + \sum_{i+j+r=n}^{i<n} f_i(x)\beta d_j(y)\alpha d_r(x))$$

Since M is a 2-torision free then we have:

$$f_n(x\alpha y\beta x) = f_n(x)\beta x\alpha y + \sum_{i+j+r=n}^{i<n} f_i(x)\beta d_j(y)\alpha\, d_r(x)$$

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