Two-Phase Multivariate Stratified Sampling with Travel Cost: A Fuzzy Programming Approach

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Abstract: *In sample surveys, stratified sampling is useful if the strata weights are known for each stratum. If they are not known double sampling may be used by selecting a large preliminary sample to estimate the strata weights. Then a stratified sample may be selected independently or from the initial sample. If the problem of non-response is also present then the strata are to be virtually divided into two disjoint and exhaustive groups of respondents and non-respondents. A subsamples from non-respondents is then selected and second more extensive attempt is to be group is to be made for obtaining the required information. The problem of obtaining a compromise allocation for first and second phase of sampling is the formulated as a multi-objective non-linear programming problem that minimizes the sum of variances of the stratified sample mean subject to the non-linear cost constraint. The formulated problem is solved using fuzzy programming and fuzzy goal programming based on piecewise linear approximation. A numerical example is presented to illustrate the computational details.*

Keywords: *Non-response, Multivariate Stratified Sampling, Multiobjective Integer non-linear programming problem, Fuzzy Programming, Travel cost*

I. Introduction

Our society needs a prompt and accurate flow of information on preferences, need and behavior. It returns a good response on the part of government, business and social institutions. The need of statistical information is rapidly growing in our society. Huge amount of data are being collected and used by the states to plan their economic and social activities, e.g. the effect of sales and production in the business and industrial sectors and the development of research and social projects in scientific institution. A sample survey technique may be used where a part (sample) of the population is evaluated, and inference are drawn about the population as a whole on the basis of this sample.

The 'optimum allocation' in stratified random sampling is well known for a univariate population (See Cochran (1977) and Sukhatme et al. (1984)) more than one characteristics are defined on each and every unit of the population, it is not feasible to use the individual optimum allocations to the strata unless there is a strong positive correlation between the characteristics under study. Thus, usually, one has to use an allocation that is optimum in 'some sense' for all the characteristics. Such an allocation is known as a compromise allocation in sampling literature. Various compromise criteria to work out a compromise allocation are available. Geary (1949), Dalenius (1957), Ghosh (1958), Yates (1960), Aoyama (1963), Folks and Antle(1965), Chatterjee (1967, 1968), Kokan and Khan (1967), Ahsan (1975-1967, 1978), Ahsan and Khan(1977), Jahan, Khan and Ahsan (1994,2001), Khan, Khan and Ahsan (2003, 2008), Singh (2003), Díaz-García and Cortez (2006, 2008), Kozak (2006a, 2006b), and many others either suggested new compromise criterion or explored further the existing criteria under various situations.

In surveys of human population the problem of non-response is very common. Hansen and Hurwitz (1946) first considered the problem of non-response in sample surveys.

The method of post stratification is useful only if the relative proportion of each stratum in the population $W_h \left(= N_h / N \right)$ is known for each stratum h. If these proportions are not known, double sampling techniques may be used, with an initial (large) sample used to estimate the unknown population parameter W_h.

Most of the statistician has worked on double sampling problem for stratification where non response is present. Okafor (1994) studied the above problem using a double sampling strategy (DSS) for a univariate stratified population. Najmussehar and Bari (2002) formulated the same problem as a mathematical programming problem and used dynamic programming technique to obtain a solution. Varshney et al (2011) determined a compromise allocation in multivariate stratified random sampling, when strata weights are unknown and non-response is also present. The problem of obtaining a compromise allocation has been formulated as a Multiobjective Integer Nonlinear Programming Problem. The solution is obtained by goal programming technique. Haseen et al. (2012) obtained the solution of this problem by using fuzzy programming techniques.

Generally the real-world decision problems are multiobjective in nature and they conflict with each other regarding optimization of objectives. To resolve the conflict, the goal programming (GP) approach has

been introduced by Charnes and Cooper (1961). The main problem in using GP is that a precise aspiration level needs to be assigned for each of the objectives. But, in a real-life decision situation, it is difficult to set precise target values to objectives due to imprecise nature of human judgments. To overcome such a situation, fuzzy programming (FP) approach has been introduced by Bellman and Zadeh (1970). Zimmermann first proposed the fuzzy linear programming (1978). In FP, membership functions are defined on the basis of assigned aspiration levels and tolerance ranges defined for the fuzzy goals. But, it is difficult to define tolerance ranges in a highly sensitive decision situation. To overcome such difficulties, goal programming approach in fuzzy environment has been first introduced by Narashimann (1980). Thereafter, Fuzzy Goal Programming (FGP) has been studied by many researchers and has been applied in many real life problems. (See Pal B.B. et al (2003), Biswas et al (2005), Parra et al (2001) etc.).

There are many real-world decision problems in different structural optimization areas. It is found that objectives of most of the industrial problems are nonlinear in nature. To solve such problems, different classical approaches were developed and widely circulated in the literature. One of most widely used approaches is the approximation of a nonlinear function by piecewise linear approximation method.

Separable programming is important because it allows a convex nonlinear program to be approximated with arbitrary accuracy with a linear programming model. Separable programming was first introduced by Miller (1963). Thereafter, it has been developed by Cox (1971), Lin and Chen (2002), Chang (2000), and other researchers. But, separable programming approach in the area of FGP is not widely circulated in literature. Generally, separable programming problems are transformed into linear form by using piecewise linear approximation method.

In sample surveys problem when cost of travelling between the selected units of a stratum is also significant, a cost function quadratic in $\sqrt{n}_h$ may be used (See Cochran (1977)). Non linear cost constraint has been studied by many researchers. Shazia Ghufran et al. (2012) worked out the compromise allocation in a multivariate stratified survey using the compromise criterion to minimize the sum of squared coefficients of variation of the estimators subject to the non linear cost. Saman et al. (2013) obtained the compromise solution in presence of non response using travel cost.

In the present manuscript the authors have discussed the problem of Two-Phase sampling (or double sampling) design in presence of non response. The compromise solution of this problem has been obtained by minimizing the variance of double sampling for stratification in presence of non response for fixed travelling cost. The solution obtained using fuzzy programming and fuzzy goal programming technique based on piecewise linear approximation. A numerical example is also given.

II. Non-response in double sampling for stratification

A population of size N, divided into L strata which are homogenous within themselves. The strata sizes are $N_1, N_2, \ldots, N_L$, where $\sum_{h=1}^{L} N_h = N$. The strata weights $W_h \left(= \frac{N_h}{N} \right)$ are used in estimating unbiasedly the mean or the total of the characteristics under study. If these strata weights are not known the technique of double sampling may be used to estimate them that is, a preliminary simple random sample of size n' is selected without replacement, to estimate the strata weights, treating the population as unstratified. Each unit of the sample falling in which stratum is recorded. An unbiased estimate w_h is then given by $w_h = \frac{n'_h}{n'}$.

Subsample of sizes $n_h = v_h n'_h$; $h = 1, 2, \ldots, L$; $0 < v_h \leq 1$ is then drawn out of n'_h units using srswor from each stratum for fixed v_h, to collect information on the characteristics under study, such that $\sum_{h=1}^{L} n_h = n$. Let $W_h \left(= \frac{N_h}{n} \right)$ be proportion of units falling in the hth strata and $w_h \left(= \frac{n'_h}{n'} \right)$ be proportion of first sampling units falling in the hth stratum. The double sampling for stratification estimator of population mean $\bar{Y}_j$ of the jth characteristic measured on each selected unit is given by

$$\bar{y}_{hds} = \sum_{h=1}^{L} w_h \bar{y}_{jh} \qquad (1)$$

$\bar{y}_{jh} = \frac{1}{n_h} \sum_{i=1}^{n_h} y_{jhi}$ is a sample mean of the jth characteristic, where $j = 1, 2, \ldots, p$ based on n_h units for stratum h and 'ds' stand for double sampling.

The sampling variance of $\bar{y}_{jds}$ is then given by

$$V(\bar{y}_{jds}) = \left(\frac{1}{n'} - \frac{1}{N} \right) S_j^2 + \frac{1}{n'} \sum_{h=1}^{L} w_h \left(\frac{1}{v_h} - 1 \right) S_{jh}^2 \qquad (2)$$

where S_j^2 and S_{jh}^2 denote the population and stratum variances of jth characteristic based on N and N_h units.

In the presence of non-response, let n_{h1} units respond at the first attempt and n_{h2} units are the number of non-respondents out of n_h units. Using Hansen and Hurwitz (1946) technique, subsamples are selected from non-respondents group of sizes $m_{h2} = k_h^* n_{h2} ; 0 < k_h^* < 1$ out of n_{h2} units is drawn and interviewed with extra effort. Where k_h^* is a known constant ; $h = 1, 2, \dots, L$.

Let $\bar{y}_{jh1}$ and $\bar{y}_{j m_{h2}}$ denote the sample mean of jth characteristic of the n_{h1} respondents at the first attempt and the m_{h2} subsampled units at the second attempt.

For jth characteristic, an unbiased estimator $\bar{y}_{jds}^*$ for $\bar{Y}_j$ based on sample means from the respondents and the non-respondents group obtained in second attempt is given by

$$\bar{y}_{jds}^* = \sum_{h=1}^{L} w_h \bar{y}_{jh}^* \tag{3}$$

where, $\bar{y}_{jh}^* = \frac{n_{h1} \bar{y}_{jh1} + n_{h2} \bar{y}_{j m_{h2}}}{n_h}$

The variance of $\bar{y}_{jds}^*$ is given by

$$
\begin{aligned}
V(\bar{y}_{jds}^*) &= V(\bar{y}_{jds}) + \frac{1}{n'} \sum_{h=1}^{L} w_{h2} \left(\frac{1 - k_h^*}{k_h^* v_h} \right) S_{jh2}^2 \\
&= \left(\frac{1}{n'} - \frac{1}{N} \right) S_j^2 + \frac{1}{n'} \sum_{h=1}^{L} w_h \left(\frac{1}{v_h} - 1 \right) S_{jh}^2 + \frac{1}{n'} \sum_{h=1}^{L} w_{h2} \left(\frac{1 - k_h^*}{k_h^* v_h} \right) S_{jh2}^2 \\
&= Z_j \; ; \; j = 1, 2, \dots, p
\end{aligned}
\tag{4}
$$

In the above expression $w_{h2} = \frac{n_{h2}}{n_h}$ is the proportion of non-respondents and S_{jh2}^2 is the population variance of jth characteristic, $j = 1, 2, \dots, p$ of the non-respondents in the hth stratum.

A fully generalized cost function draws upon differential calculus and decision theory to minimize variance subject to a given cost (see Cochran, 1963), but a simpler algebraic relationship permits an examination of the effect of sample survey design on survey cost. The cost of carrying out multivariate stratified double sampling in presence of non response are characterized by four major categories as defined in the following.

$$C = c_0 n' + \sum_{h=1}^{L} c_{h1} n_h + \sum_{h=1}^{L} c_{h11} n_{h1} + \sum_{h=1}^{L} c_{h12} m_{h2} \tag{5}$$

where c_0 is the cost of measuring each unit in the preliminary sample.

c_{h1} is the cost of measuring each unit in the first attempt (phase-1).

c_{h11} is the cost of processing the result of all the p characteristic on the n_{h1} selected units from respondents group in the hth stratum at phase-I.

c_{h12} is the cost for measuring and processing the results of all the p characteristic on the m_{h2} units selected from the non-respondents group in the hth stratum at the second attempt(phase-II).

c_{jh11} and c_{jh12} are the per unit cost of measuring the characteristic at phase-I and phase-II respectively.

Since n_{h1} is not known until the first attempt has been made, the quantity $w_{h1} n_h$ may be used as its estimated value. The total expected cost of $\hat{C}$ the survey is thus given by

$$\hat{C} = c_0 n' + \sum_{h=1}^{L} (c_{h1} + w_{h1} c_{h11}) n_h + \sum_{h=1}^{L} c_{h12} m_{h2} \tag{6}$$

In linear cost function the cost associated with the non travel activity of the survey include drawing samples, preparing survey materials, locating identifying and interviewing respondents and coding data. Sometimes, it is not fulfill our requirement. If the cost of travelling between selected units of a stratum is substantial it should also be considered while estimating the total cost of the survey. Then the total cost includes both the travel and the measurement costs. Let the per unit travel cost be t_{h1} and t_{h12} for first and second attempt respectively. An adequate approximation to the total expected cost $\hat{C}$ of the double sampling stratified survey in presence of non response will then be

$$\hat{C} = c_0 n' + \sum_{h=1}^{L} (c_{h1} + w_{h1} c_{h11}) n_h + \sum_{h=1}^{L} c_{h12} m_{h2} + \sum_{h=1}^{L} t_{h1} \sqrt{n_h} + \sum_{h=1}^{L} t_{h12} \sqrt{m_{h2}} \tag{7}$$

III. Formulation of the above problem

In phase-I, the problem is to find the optimum sizes of the second samples $n_h ; h = 1, 2, \dots, L$ which may be obtained by minimizing $Z_j ; j = 1, 2, \dots, p$ given by (4) for the cost given in (7). In phase-II the optimum subsample of size $m_{h2} ; h = 1, 2, \dots, L$ from non response are obtained for a fixed cost of the survey.

The multi-objective formulation of the problem at phase-I may be given as

Minimize $\quad [Z_1, Z_2, \ldots, Z_p] \qquad$ simultaneously

Subject to $\quad \sum_{h=1}^{L}(c_{h1} + w_{h1}c_{h11})n_h + \sum_{h=1}^{L} c_{h12}m_{h2} + \sum_{h=1}^{L} t_{h1}\sqrt{n_h} + \sum_{h=1}^{L} t_{h12}\sqrt{m_{h2}} \leq \hat{C} - c_0 n' \left.\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\right\}$ (8)

$$2 \leq n_h \leq n_h'$$

$\qquad$ and $\qquad\qquad n_h$ integers $; h = 1, 2, \ldots, L$

where $Z_j ; j = 1, 2, \ldots, p$ are as defined in (4).

After ignoring the terms which are independent of n_h in (4) and rearranging the cost constraints for first phase, the MINLPP (8) becomes

Minimize $\quad V_j = \sum_{h=1}^{L} \frac{a_{jh}}{n_h}$ $\left.\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\right\}$

Subject to $\quad \sum_{h=1}^{L}(c_{h1} + w_{h1}c_{h11})n_h + \sum_{h=1}^{L} t_{h1}\sqrt{n_h} \leq \hat{C}_0$ $\qquad$ (9)

$$2 \leq n_h \leq n_h'$$

$\qquad$ and $\qquad n_h$ integers $; h = 1, 2, \ldots, L$

where $\quad a_{jh} = \frac{1}{n'}\left(\sum_{h=1}^{L} w_h n_h' S_{jh}^2 + w_{h2}\left(\frac{(1-k_h^*)}{k_h^*}\right)n_h' S_{jh2}^2\right)$

and $\sum_{h=1}^{L}(c_{h1} + w_{h1}c_{h11})n_h \leq \hat{C}_0$

where $\hat{C}_0 = \hat{C} - c_0 n' - \sum_{h=1}^{L} c_{h12}m_{h2} + \sum_{h=1}^{L} t_{h12}\sqrt{m_{h2}}$

Ignoring the terms independent of m_{h2} in the R.H.S of (4), at phase-II, the problem is to work out the optimum values of m_{h2} which minimize V_j', $j = 1, 2, \ldots, p$ for a given cost in (7).

Substituting $k_h^* = m_{h2}/n_{h2}$ and $v_h = \frac{n_h}{n_h}$, and rearranging the cost constraints for the phase-II, problem (8) may be stated as

Minimize $V_j' = \sum_{h=1}^{L} \frac{b_{jh}}{m_{h2}}$ $\left.\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\right\}$

Subject to $\quad \sum_{h=1}^{L} c_{h12}m_{h2} + \sum_{h=1}^{L} t_{h12}\sqrt{m_{h2}} \leq \hat{C}_0'$ $\qquad$ (10)

$$2 \leq m_{h2} \leq n_{h2}$$

$\qquad$ and $\qquad m_{h2}$ integers $; h = 1, 2, \ldots, L ; j = 1, 2, \ldots, p$

where $b_{jh} = \frac{1}{n'}\sum_{h=1}^{L} \frac{w_{h2}n_{h2}n_h' S_{jh2}^2}{n_h}$

and $\hat{C}_0' = \hat{C} - c_0 n' - \sum_{h=1}^{L}(c_{h1} + w_{h1}c_{h11})n_h + \sum_{h=1}^{L} t_{h1}\sqrt{n_h}$.

IV. The Solution

In the following two approaches to solve to solve the formulated problems in phase-I and phase-II are discussed. The first approach is through Fuzzy Programming Technique while the second approach is through Fuzzy Goal Programming Technique using piecewise linear approximation.

4.1 Fuzzy Programming

Let V_j^* is the optimal value of V_j obtained by solving the MINLPP (9)

Further let

$$\tilde{V}_j = \tilde{V}_j(n_1, n_2, \ldots, n_h, \ldots, n_L) \qquad (11)$$

denote the value of the variances under the compromise allocation, where $n_h ; h = 1, 2, \ldots, L$ are to be worked out.

Obviously $\tilde{V}_j \geq V_j^*$ and $\tilde{V}_j - V_j^* \geq 0; j = 1, 2, \dots, p$ will give the increase in the variance due to not using the individual optimum allocation for j^{th} characteristic.

To obtain a fuzzy solution, we first compute the maximum value U_j and the minimum value L_j, for each $j = 1, 2, \dots, p$.

Now,

$$L_j = \min_j V\left(n_{h,j}^*\right) \qquad\qquad U_j = \max_j V(n_{h,j}^*) \qquad\qquad (12)$$

where $n_{h,j}^*$ denote the optimum allocation for the jth characteristic in four strata.

The differences of the maximum and minimum values of V_j are denoted by $d_j = U_j - L_j, j = 1, 2, \dots, p$.

The fuzzy programming formulation of the MINLPP in (9) is given by the following INLPP:

$$
\left.
\begin{aligned}
&\text{Minimize} && \delta \\
&\text{Subject to} && \sum_{h=1}^{L} \frac{a_{jh}}{n_h} - \delta d_k \leq V_j^* \\
& && \sum_{h=1}^{L} (c_{h1} + w_{h1}c_{h11})n_h + \sum_{h=1}^{L} t_{h1}\sqrt{n_h} \leq \hat{C}_0 \\
& && 2 \leq n_h \leq n_h' \;\; n_h \geq 0 \;, \delta \geq 0 \;, \\
&\text{and} && n_h \; are \; integers; \qquad j = 1,2,\dots p; \;\; h = 1,2,\dots,L
\end{aligned}
\right\} \qquad (13)
$$

where $\delta \geq 0$ is the decision variable representing the worst deviation level.

The fuzzy programming (FPP) may be solved using the optimization software LINGO-13.

Similarly, for Phase II the MINLPP (10) FPP becomes

$$
\left.
\begin{aligned}
&\text{Minimize} && \delta' \\
&\text{Subject to} && \sum_{h=1}^{L} \frac{b_{jh}}{m_{h2}} - \delta' d_k \leq V_j'^* \\
& && \sum_{h=1}^{L} c_{h12}m_{h2} + \sum_{h=1}^{L} t_{h12}\sqrt{m_{h2}} \leq \hat{C}_0' \\
& && 2 \leq m_{h2} \leq n_{h2} \;, \delta' \geq 0 \;, \\
&\text{and} && m_{h2} \; are \; integers; \qquad j = 1,2,\dots p; \;\; h = 1,2,\dots,L
\end{aligned}
\right\} \qquad (14)
$$

4.2 Fuzzy Goal Programming using piecewise linear approximation

First, we formulate the fuzzy programming model of NLLP in (8) by transforming the objective function V_j into fuzzy goals by means of assigning an imprecise aspiration level for each objective. Let V_j^* be the optimal solution of the objective function. Then the fuzzy goals appear in the form

$$V_1 \gtrsim V_1^*$$
$$\vdots$$
$$V_j \gtrsim V_j^*$$

Using the individual best solution, we find the upper and lower tolerance limit U_j and L_j for each objective function already described in section (4.1).

The fuzzy goals are characterized by their membership functions. The membership function of objective function for each characteristic is given below

$$
\mu_j(\bar{n}) = \left\{
\begin{aligned}
&1 && if \; V_j(\bar{n}) \leq L \\
&\frac{U_j - V_j(\bar{n})}{U_j - L_j} && if \; L_j \leq V_j(\bar{n}) \leq U_j, \\
&0 && if \; V_j(\bar{n}) \geq U_j,
\end{aligned}
\right\} j = 1,2,\dots,p \qquad (15)
$$

In the fuzzy goal programming (FGP) formulation, the defined membership function in equation (15) for each characteristics are transformed into membership goals by introducing under and over deviational variables and assigning the highest membership value means unity (one) as the aspiration level to each of them. Under the above circumstances, the membership goals are

$$\frac{U_j - V_j(\bar{n})}{U_j - L_j} + d_j = 1 \; ; j = 1,2,\dots,p \qquad (16)$$

Where $d_j \geq 0$ represent the under and over deviational variables.

We transform the non linear membership function $\mu_j(\bar{n})$ into equivalent linear membership function by using piecewise linear approximation:

The goal in (16) can be written as

$$\frac{U_j - \sum_{h=1}^{L} f_{jh}(n_h)}{U_j - L_j} + d_j = 1 \; ; j = 1,2,\dots,p \quad h = 1,2,\dots,L \tag{17}$$

For linearizing the non linear function $f_{jh}(n_h)$ in (17), the grid points for the variable n_h $(h = 1,2,\dots L)$ are chosen as a_{hr} $(r = 0,1,\dots,r_h)$ Further, we introduced a new variable y_{hr} $(r = 0,1,\dots,r_h)$, so that n_h can be expressed as

$$n_h = \sum_{r=0}^{r_h} a_{hr}\, y_{hr} \tag{18}$$

Where $\sum_{r=0}^{r_h} y_{hr} = 1$ $(y_{hr} \geq 0)$ with grid points $a_{h0}, a_{h1}, \dots, a_{hr_h}$.

So, the piecewise linear form of nonlinear function $f_{jh}(n_h)$ nominated as F_{jh} can be written as

$$F_{jh} = \sum_{r=0}^{r_h} y_{hr}\, f_{jh}(a_{hr}) \tag{19}$$

Then, the linear FGP model by using expression of (18) and (19) in MINLPP (9) for Phase I can
Minimize $Z = \sum_{j=1}^{p} d_j$

so as to satisfy

$$
\left.
\begin{aligned}
&\frac{1}{U_j - L_j}\left[U_j - \sum_{h=1}^{L} F_{jh}\right] + d_j = 1 \\
&\quad where\ F_{jh} = \sum_{r=0}^{r_h} y_{hr}\, f_{jh}(a_{hr}) \\
Subject\ to\quad &\sum_{h=1}^{L}(c_{h1} + w_{h1}c_{h11})\sum_{r=0}^{r_h} a_{hr}\, y_{hr} + \sum_{h=1}^{L} t_{h1}\sqrt{\sum_{r=0}^{r_h} a_{hr}\, y_{hr}} \leq \hat{C}_0 \\
&\sum_{r=0}^{r_k} \gamma_{kr} = 1\ (\gamma_{kr} \geq 0) \\
&\quad h = 1,2,\dots L \\
&\quad r = 0,1,\dots,r_k \\
&\quad j = 1,2,\dots,p
\end{aligned}
\right\} \tag{20}
$$

Simlarly, for Phase- II FGPP for MINLPP (10) can be written as

Minimize $Z = \sum_{j=1}^{p} d_j'$

so as to satisfy

$$
\left.
\begin{aligned}
&\frac{1}{U_j - L_j}\left[U_j - \sum_{h=1}^{L} F_{jh}'\right] + d_j' = 1 \\
where\quad &F_{jh}' = \sum_{r=0}^{r_h} y_{hr}\, f_{jh}(a_{hr})_{jh}' \\
Subject\ to\quad &\sum_{h=1}^{L} c_{h12}\sum_{r=0}^{r_h} a_{hr}\, y_{hr} + \sum_{h=1}^{L} t_{h12}\sqrt{\sum_{r=0}^{r_h} a_{hr}\, y_{hr}} \leq \hat{C}_0' \\
&\sum_{r=0}^{r_k} \gamma_{kr} = 1\ (\gamma_{kr} \geq 0) \\
&\quad h = 1,2,\dots L \\
&\quad r = 0,1,\dots,r_k \\
&\quad j = 1,2,\dots,p
\end{aligned}
\right\} \tag{21}
$$

In above expression $m_{h2} = \sum_{r=0}^{r_h} a_{hr}\, y_{hr}$

V. Numerical Example

An application of the above technique is given in the following using a numerical data of Haseen et al (2011). The travelling costs t_{h1} and t_{h12} are assumed by authors.

A population of size $N = 3850$ is divided into four strata. Two characteristics are defined on each unit of the population. It is assumed that the estimation of population means of the two characteristics is of interest.

Table 1 shows the required information. Each stratum is further subdivided into respondents and non-respondents groups as given in Table2. It is assumed that v_h and k_h^* are known and the preliminary sample size $n' = 1000$.

Table 1 Data for four strata and two characteristics

h	w_h	S_{1h}^2	S_{2h}^2	v_h	k_h^*	c_{h1}	c_{h11}	c_{h12}	t_{h1}	t_{h12}
1	0.32	784	1444	0.4	0.5	1	2	3	0.5	2
2	0.21	576	676	0.5	0.6	1	3	4	0.5	2.5
3	0.27	1024	1936	0.6	0.7	1	4	5	0.5	3
4	0.20	2916	6084	0.65	0.75	1	5	6	0.5	4.5

Table 2 Subdivided data as respondents and non-respondents groups for four strata with two characteristics

h	Group	S_{1h}^2	S_{2h}^2	$\dfrac{w_{hk}}{k=1,2}$
1	Respondents	361.06	767.82	$w_{11} = 0.70$
	Non-respondents	310.55	454.76	$w_{12} = 0.30$
2	Respondents	373.79	449.92	$w_{21} = 0.80$
	Non-respondents	326.29	353.81	$w_{22} = 0.20$
3	Respondents	930.15	1272.88	$w_{31} = 0.75$
	Non-respondents	560.28	1165.97	$w_{32} = 0.25$
4	Respondents	2355.98	2690.53	$w_{41} = 0.72$
	Non-respondents	1013.08	2403.55	$w_{42} = 0.28$

In the last column of the Table 2, $k = 1$ is for respondents group and $k = 2$ is for non-respondents group. Further the total amount available for the survey be $C = 3000$ units. Out of these 3000 units 750 units are earmarked for the preliminary sample of size n', 1,900 units are earmarked for phase-I and 350 units are earmarked for phase-II.

Using estimated values of strata weights and the size of selected preliminary sample, the values of $n_h' = w_h n'$; $h = 1,2,\dots,L$ are obtained as

$$n_1' = 320, \quad n_2' = 210, \quad n_3' = 270, \quad n_4' = 200 \quad with \quad \sum_{h=1}^{4} n_h' = 1,000.$$

5.1 Solution using Fuzzy programming

We obtain the individual best solution for each of the objective in **Phase I**. After putting the values from Table 1 and Table 2, the MINLPP (9) becomes

For $j = 1$

$$\text{Minimize} \quad V_1 = \frac{110.09440}{n_1} + \frac{45.95787}{n_2} + \frac{162.89370}{n_3} + \frac{286.83744}{n_4}$$

$$\text{Subject to} \quad 2.4n_1 + 3.4n_2 + 4n_3 + 4.6n_4 + 0.5\sqrt{n_1} + 0.5\sqrt{n_2} + 0.5\sqrt{n_3} + 0.5\sqrt{n_4} \le 1900$$

$$2 \le n_1 \le 320$$
$$2 \le n_2 \le 210$$
$$2 \le n_3 \le 270$$
$$2 \le n_4 \le 200$$

and $\qquad n_h \ integers; \ h = 1,2,\dots,4.$

The optimum solution of the above problem using software LINGO-13 is

$$n_{11}^* = 140, \ n_{21}^* = 75, \ n_{31}^* = 133, n_{41}^* = 164 \quad with \ V_1^* = 4.372934$$

Similarly, for $j = 2$

$$\text{Minimize} \quad V_1 = \frac{191.52260}{n_1} + \frac{52.10163}{n_2} + \frac{324.7747}{n_3} + \frac{647.15640}{n_4}$$

$$\text{Subject to} \quad 2.4n_1 + 3.4n_2 + 4n_3 + 4.6n_4 + 0.5\sqrt{n_1} + 0.5\sqrt{n_2} + 0.5\sqrt{n_3} + 0.5\sqrt{n_4} \le 1900$$

$$2 \le n_1 \le 320$$
$$2 \le n_2 \le 210$$
$$2 \le n_3 \le 270$$
$$2 \le n_4 \le 200$$

and $\qquad n_h \ integers; \ h = 1,2,\dots,4.$

The optimum solution using software LINGO-13 is

$$n_{11}^* = 136, \ n_{21}^* = 58, \ n_{31}^* = 135, n_{41}^* = 177 \quad with \ V_2^* = 8.368548$$

Further the value of V_1 at the point n_{h2}^* is 4.429065 and the value of V_2 at the point n_{h1}^* is 8.450698. Thus,

$$L_1 = 4.372934, \qquad U_1 = 4.429065$$

$$L_2 = 8.368548, \qquad U_2 = 8.450698$$
$$d_1 = 0.056131, \qquad d_2 = 0.08215$$

After computing the optimum allocation and optimal variances for the two characteristics the compromising optimal solution for the above problem can be obtained by solving the given FPP (13)

Minimize $\quad \delta$

Subject to $\dfrac{110.09440}{n_1} + \dfrac{45.95787}{n_2} + \dfrac{162.89370}{n_3} + \dfrac{286.83744}{n_4} - \delta(0.056131) \le 4.372934$

$\dfrac{191.52260}{n_1} + \dfrac{52.10163}{n_2} + \dfrac{324.7747}{n_3} + \dfrac{647.15640}{n_4} - \delta(0.08215) \le 8.368548$

$2.4n_1 + 3.4n_2 + 4n_3 + 4.6n_4 + 0.5\sqrt{n_1} + 0.5\sqrt{n_2} + 0.5\sqrt{n_3} + 0.5\sqrt{n_4} \le 1900$

$$2 \le n_1 \le 320$$
$$2 \le n_2 \le 210$$
$$2 \le n_3 \le 270$$
$$2 \le n_4 \le 200$$

and $\qquad\qquad n_h \ integers; \ h = 1, 2, \dots, 4.$

Now the fuzzy compromise solution using LINGO-13 is

$$n_1^* = 136, \ n_2^* = 67, \ n_3^* = 133, n_4^* = 172 \quad with \ \delta = 0.2662736$$
$$V_1 = 4.387880, \quad V_2 = 8.390343$$

Similarly for **Phase II** the MINLPP (10)

For $j = 1$

Minimize $\ V_1' = \dfrac{8.94384}{m_{12}} + \dfrac{2.740836}{m_{22}} + \dfrac{9.454725}{m_{32}} + \dfrac{15.88509}{m_{42}}$

Subject to $\quad 3m_{12} + 4m_{22} + 5m_{32} + 6m_{42} + 2\sqrt{m_{12}} + 2.5\sqrt{m_{22}} + 3\sqrt{m_{32}} + 4.5\sqrt{m_{42}} \le 350$

$$2 \le m_{12} \le 41$$
$$2 \le m_{22} \le 12$$
$$2 \le m_{32} \le 34$$
$$2 \le m_{42} \le 50$$

and $\qquad\qquad m_{h2} \ integers; \ h = 1, 2, \dots, 4.$

The optimum solution of the above problem using LINGO-13 is

$$m_{1,12}^* = 20, m_{1,22}^* = 9, m_{1,32}^* = 17, m_{1,42}^* = 20 \quad with \ V_1'^* = 2.102144$$

Similarly, for $j = 2$

Minimize $\ V_2' = \dfrac{191.52260}{m_{12}} + \dfrac{52.10163}{m_{22}} + \dfrac{324.7747}{m_{32}} + \dfrac{647.15640}{m_{42}}$

Subject to $\quad 3m_{12} + 4m_{22} + 5m_{32} + 6m_{42} + 2\sqrt{m_{12}} + 2.5\sqrt{m_{22}} + 3\sqrt{m_{32}} + 4.5\sqrt{m_{42}} \le 350$

$$2 \le m_{12} \le 41$$
$$2 \le m_{22} \le 12$$
$$2 \le m_{32} \le 34$$
$$2 \le m_{42} \le 50$$

and $\qquad\qquad m_{h2} \ integers; \ h = 1, 2, \dots, 4.$

The optimum solution by using software LINGO-13 is

$$m_{2,12}^* = 17, m_{2,22}^* = 7, m_{2,32}^* = 18, m_{2\ 42}^* = 22 \quad with \ V_2'^* = 4.001161$$

Further the value of V_1' at the point $m_{2,h2}^*$ is 2.164968 and the value of V_2' at the point $m_{2,h2}^*$ is 4.026857. Thus,

$$L_1 = 2.102144, \qquad U_1 = 2.164968$$
$$L_2 = 4.001161, \qquad U_2 = 4.026857$$
$$d_1 = 0.062824, \qquad d_2 = 0.025696$$

After computing the optimum allocation and optimal variances for the two characteristics the compromising optimal solution for the above problem can be obtained by solving the given FPP (14)

$$\text{Minimize} \quad \delta'$$

$$\text{Subject to} \quad \frac{8.94384}{m_{12}} + \frac{2.740836}{m_{22}} + \frac{9.454725}{m_{32}} + \frac{15.88509}{m_{42}}$$

$$\delta'(0.0628241) \leq 2.102144$$

$$\frac{191.52260}{m_{12}} + \frac{52.10163}{m_{22}} + \frac{324.7747}{m_{32}} + \frac{647.15640}{m_{42}}$$

$$\delta'(0.025696) \leq 4.001161$$

$$3m_{12} + 4m_{22} + 5m_{32} + 6m_{42} +$$

$$2\sqrt{m_{12}} + 2.5\sqrt{m_{22}} + 3\sqrt{m_{32}} + 4.5\sqrt{m_{42}} \leq 350$$

$$2 \leq m_{12} \leq 41$$

$$2 \leq m_{22} \leq 12$$

$$2 \leq m_{32} \leq 34$$

$$2 \leq m_{42} \leq 50$$

$$\text{and} \quad m_{h2} \ integers; \ \ h = 1, 2, \dots, 4$$

Now the fuzzy compromise solution by using software LINGO 13 is

$$m_{12}^* = 18, m_{22}^* = 9, m_{32}^* = 17, m_{42}^* = 21 \quad with \ \ \delta' = 0.3395363$$

$$V_1' = 2.114010, \ \ V_2' = 4.009886$$

5.2 Fuzzy goal programming using piecewise linear approximation

For **Phase-I**, we calculate the membership function for each characteristic

$$\mu_1(\bar{n}) = \frac{1}{0.056131}[4.429065 - V_1(\bar{n})]$$

$$and \quad \mu_2(\bar{n}) = \frac{1}{0.08215}[8.450698 - V_2(\bar{n})]$$

Each of the objective function can be expressed as the sum of the separable functions which are shown in the table 3.

Table 3 Separable functions associated with the each objective

$f_{11}(n_1)$	$110.09440/n_1$
$f_{12}(n_2)$	$45.95787/n_2$
$f_{13}(n_3)$	$162.89370/n_3$
$f_{14}(n_4)$	$286.83744/n_4$
$f_{21}(n_1)$	$191.52260/n_1$
$f_{22}(n_2)$	$52.10163/n_2$
$f_{23}(n_3)$	$324.7747/n_3$
$f_{24}(n_4)$	$647.15640/n_4$

After introducing the deviational variables the nonlinear membership goals are

$$\frac{1}{0.056131}[4.429065 - f_{11}(n_1) - f_{12}(n_2) - f_{13}(n_3) - f_{14}(n_4)] + d_1 = 1$$

$$and \ \frac{1}{0.08215}[8.450698 - f_{21}(n_1) - f_{22}(n_2) - f_{23}(n_3) - f_{24}(n_4)] + d_2 = 1$$

These non linear membership goals are approximated with linear form. Let the grid points for each variables n_1, n_2, n_3, n_4 are

$$a_{10} = 2, \ a_{11} = 81.5, \ a_{12} = 161, \ a_{13} = 240.5, a_{14} = 320$$

$$a_{20} = 2, a_{21} = 54, a_{22} = 106, a_{23} = 158, a_{24} = 210$$

$$a_{30} = 2, \ a_{31} = 69, \ a_{32} = 136, \ a_{33} = 203, \ a_{34} = 270$$

$$a_{40} = 2, a_{41} = 51.5, \ a_{42} = 101, \ a_{43} = 150.5, \ a_{44} = 200$$

The piecewise linear approximation to the function given in table 3 are

$$F_{11} = (55.04720)y_{10} + (1.350852)y_{11} + (0.6838161) y_{12} + (0.4577730) y_{13} + (0.3440450) y_{14}$$

$$F_{12} = (22.97893)y_{20} + (0.8510717)y_{21} + (0.4335648) y_{22} + (0.2908726)y_{23} + (0.2188470)y_{24}$$

$$F_{13} = (81.44685)y_{30} + (2.360778)y_{31} + (1.197748) y_{32} + (0.8004604)y_{33} + (0.6033100) y_{34}$$

$$F_{14} = (143.4187)y_{40} + (5.569659)y_{41} + (2.839975) y_{42} + (1.905897)y_{43} + (1.434187)y_{44}$$

$$F_{21} = (95.76130)y_{10} + (2.349971)y_{11} + (1.189581) y_{12} + (0.7963518) y_{13} + (0.5985081) y_{14}$$

$$F_{22} = (26.050821)y_{20} + (0.9648450)y_{21} + (0.4915248) y_{22} + (0.3297572)y_{23} + (0.2481030)y_{24}$$

$$F_{23} = (162.3873)y_{30} + (4.706880)y_{31} + (2.388049) y_{32} + (1.595944)y_{33} + (1.202869) y_{34}$$

$$F_{24} = (323.5782)y_{40} + (12.56614)y_{41} + (6.407489) y_{42} + (4.300043)y_{43} + (3.235782)y_{44}$$

$$g_1 = 2y_{10} + 81.5y_{11} + 161y_{12} + 240.5 y_{13} + 320y_{14}$$

$$g_2 = 2y_{20} + 54y_{21} + 106y_{22} + 158\,y_{23} + 210y_{24}$$
$$g_3 = 2y_{30} + 69y_{31} + 136y_{32} + 203\,y_{33} + 270y_{34}$$
$$g_4 = 2y_{40} + 51.5y_{41} + 101y_{42} + 150.5\,y_{43} + 200y_{44}$$

Using these values the proposed linear FGP at **Phase-I** becomes

$$\text{Minimize} \quad V = d_1 + d_2$$

$$\text{Subject to} \quad \frac{1}{0.056131}[4.429065 - F_{11} - F_{12} - F_{13} - F_{14}] + d_1 = 1$$

$$\frac{1}{0.08215}[8.450698 - F_{21} - F_{22} - F_{23} - F_{24}] + d_2 = 1$$

$$2.4g_1 + 3.4g_2 + 4g_3 + 4.6g_4 + 0.5\sqrt{g_1} + 0.5\sqrt{g_2} + 0.5\sqrt{g_3} + 0.5\sqrt{g_4} \le 1900$$

$$\gamma_{10} + \gamma_{11} + \gamma_{12} + \gamma_{13} + \gamma_{14} = 1$$
$$\gamma_{20} + \gamma_{21} + \gamma_{22} + \gamma_{23} + \gamma_{24} = 1$$
$$\gamma_{30} + \gamma_{31} + \gamma_{32} + \gamma_{33} + \gamma_{34} = 1$$
$$\gamma_{40} + \gamma_{41} + \gamma_{42} + \gamma_{43} + \gamma_{44} = 1$$
$$\gamma_{kr} \ge 0; k = 1,2,3,4 \text{ and } r = 0,1,2,3,4,$$

Using the software LINGO-13 the problem is solved and the obtained solution is
$$n_1^* = 161, \, n_2^* = 54, \, n_3^* = 136, n_4^* = 163 \quad with\, V_1 = 4.492375 \text{ and } V_2 = 8.512760$$
$$C = 1886.033$$

Similarly, **Phase-II,** we calculate the membership function for each characteristic

$$\mu_1(\bar{m}) = \frac{1}{0.062824}[2.164968 - V_1'(\bar{m})]$$

and $$\mu_2(\bar{m}) = \frac{1}{0.025696}[4.026857 - V_2'(\bar{m})]$$

Each of the objective function can be expressed as the sum of the separable functions which are shown in the table 4.

Table 4 Separable functions associated with the each objective

$f_{11}'(m_{12})$	$8.94384/m_{12}$
$f_{12}'(m_{22})$	$2.740836/m_{22}$
$f_{13}'(m_{32})$	$9.454725/m_{32}$
$f_{14}'(m_{42})$	$15.88509/m_{42}$
$f_{21}'(m_{12})$	$13.09709/m_{12}$
$f_{22}'(m_{22})$	$2.972004/m_{22}$
$f_{23}'(m_{32})$	$19.67574/m_{32}$
$f_{24}'(m_{42})$	$37.68766/m_{42}$

After introducing the deviational variables the nonlinear membership goals are

$$\frac{1}{0.062824}[2.164968 - f_{11}'(m_{12}) - f_{12}'(m_{22}) - f_{13}'(m_{32}) - f_{14}'(m_{42})] + d_1' = 1$$

and $$\frac{1}{0.025696}[4.026857 - f_{21}'(m_{12}) - f_{22}'(m_{22}) - f_{23}'(m_{32}) - f_{24}'(m_{42})] + d_2' = 1$$

These non linear membership goals are approximated with linear form. Let the grid points for each subsample $m_{12}, m_{22}, m_{32}, m_{42}$ are

$$a_{10} = 2, \, a_{11} = 11.75, \, a_{12} = 21.5, \, a_{13} = 31.25, a_{14} = 41$$
$$a_{20} = 2, a_{21} = 4.5, a_{22} = 7, \, a_{23} = 9.5, \, a_{24} = 12$$
$$a_{30} = 2, a_{31} = 10, \, a_{32} = 18, \, a_{33} = 26, \, a_{34} = 34$$
$$a_{40} = 2, a_{41} = 14, \, a_{42} = 26, \, a_{43} = 38, \, a_{44} = 50$$

The piecewise linear approximation to the function given in table 4 are

$$F_{11}' = (4.471920)y_{10} + (0.7611779)y_{11} + (0.4159926)\,y_{12} + (0.2862029)\,y_{13} + (0.2181424)\,y_{14}$$
$$F_{12}' = (1.370418)y_{20} + (0.6090747)y_{21} + (0.3915480)\,y_{22} + (0.2885091)y_{23} + (0.2284030)\,y_{24}$$
$$F_{13}' = (4.727362)y_{30} + (0.9454725)y_{31} + (0.5252625)\,y_{32} + (0.3636433)y_{33} + (0.2626312)\,y_{34}$$
$$F_{14}' = (7.942545)y_{40} + (1.134649)y_{41} + (0.6109650)\,y_{42} + (0.4180287)\,y_{43} + (0.3177018)\,y_{44}$$
$$F_{21}' = (6.548545)y_{10} + (1.114646)y_{11} + (0.6091670)\,y_{12} + (0.4191069)\,y_{13} + (0.3194412)\,y_{14}$$
$$F_{22}' = (1.486002)y_{20} + (0.6604453)y_{21} + (0.4245720)\,y_{22} + (0.3128425)y_{23} + (0.2476670)\,y_{24}$$
$$F_{23}' = (9.837870)y_{30} + (1.967574)y_{31} + (1.093097)\,y_{32} + (0.7567592)y_{33} + (0.5465483)y_{34}$$
$$F_{24}' = (18.84383)y_{40} + (2.691976)y_{41} + (1.449525)\,y_{42} + (0.9917805)y_{43} + (0.7537532)\,y_{44}$$

$$g_1' = 2y_{10} + 11.75y_{11} + 21.5y_{12} + 31.25\,y_{13} + 41y_{14}$$
$$g_2' = 2y_{20} + 4.5y_{21} + 7y_{22} + 9.5\,y_{23} + 12y_{24}$$
$$g_3' = 2y_{30} + 10y_{31} + 18y_{32} + 26\,y_{33} + 34y_{34}$$
$$g_4' = 2y_{40} + 14y_{41} + 26y_{42} + 38\,y_{43} + 50y_{44}$$

Using these values the proposed linear FGP at **Phase-II** becomes

$$\text{Minimize} \quad V' = d_1' + d_2'$$
$$\text{Subject to} \quad \frac{1}{0.062824}[2.164968 - F_{11}' - F_{12}' - F_{13}' - F_{14}'] + d_1' = 1$$
$$\frac{1}{0.025696}[4.026857 - F_{21} - F_{22} - F_{23} - F_{24}] + d_2' = 1$$
$$3g_1' + 4g_2' + 5g_3' + 6g_4 + 2\sqrt{g_1'} + 2.5\sqrt{g_2'} + 3\sqrt{g_3'} + 4.5\sqrt{g_4'} \leq 350$$
$$\gamma_{10} + \gamma_{11} + \gamma_{12} + \gamma_{13} + \gamma_{14} = 1$$
$$\gamma_{20} + \gamma_{21} + \gamma_{22} + \gamma_{23} + \gamma_{24} = 1$$
$$\gamma_{30} + \gamma_{31} + \gamma_{32} + \gamma_{33} + \gamma_{34} = 1$$
$$\gamma_{40} + \gamma_{41} + \gamma_{42} + \gamma_{43} + \gamma_{44} = 1$$
$$\gamma_{kr} \geq 0; k = 1,2,3,4 \ and \ r = 0,1,2,3,4,$$

Using the software LINGO-13 the problem is solved and the obtained solution is
$$m_{12}^* = 12,\ m_{22}^* = 7,\ m_{32}^* = 18, m_{42}^* = 24 \quad with\ V_1 = 2.324009\ and\ V_2 = 4.179412$$
$$and\ C = 346.3159$$

Table 5 Solution at Phase-I

Techniques	Allocations				Variances		Cost
	n_1	n_2	n_3	n_4	V_1	V_2	incurred
FP	136	67	133	172	4.387880	8.390343	1900
FGP	161	54	136	163	4.492375	8.512760	1886

Table 6 Solution at Phase-II

Techniques	Allocations				Variances		Cost
	m_{12}	m_{22}	m_{32}	m_{42}	V_1'	V_2'	incurred
FP	18	9	17	21	2.114010	4.009886	350
FGP	12	7	18	24	2.324009	4.179412	346

VI.   Conclusion

Tables 5 and 6 show that none of the two methods is uniformly better than the other. At Phase-I the Fuzzy Goal Programming (FGP) technique gives a slightly better result as compared to the Fuzzy Programming (FP) technique in terms of the Trace (See Sukhatme et al. (1984)). The trace while FP technique is used is 4.387880+8.390343=12.778223 and that for FGP technique is 4.492375+8.512760=12.619975. Which gives a relative increase in Trace as $\frac{12.778223-12.619975}{12.619975} \times 100\% \cong 1.25\%$ when FP technique is used.

At Phase-II the relative increase in the trace is $\frac{6.50342-6.123896}{6.123896} \times 100\% \cong 6.20\%$ when FGP technique is use.

Thus we conclude that the use of FP technique is more advisable because it gives a comparative large gain at Phase-II at the cost of a small loss in precision at Phase-I.

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