Stochastic Analysis of Manpower System with Production and Sales

K.Hari Kumar[1], P.Sekar[2]

[1]Asst.Professor, SRM University,Kattankulathur,Tamilnadu.
[2]Principal, C.Kandaswami Naidu College, Chennai-102

Abstract: *In this paper, Manpower System of an organization with two groups is considered. Breakdown occurs in the two groups of the Manpower System due to attrition process. Group A consists of employees other than top management level executives; group B consists of top management level executives. In this model group A is exposed to Cumulative Shortage Process (CSP) due to attrition and group B has an Erlang phase 2 distribution. Joint Laplace transform of Time to Recruit , Recruitment time and sales time has been found. Their expectations are presented with numerical illustrations.*
Keywords: *Manpower system, attrition, shortage, cumulative shortage process, Erlang Phase two distributions.*

I. Introduction

In an organization, the total flow out of the Manpower System (MPS) is termed as shortage, The flow out of the MPS of an organization happens due to resignation, dismissal and death. The shortages that have occurred due to the outflow of manpower should be compensated by recruitment. But recruitment cannot be made frequently since it involves cost. Therefore, the MPS is allowed to undergo Cumulative Shortage Process (CSP). The basic idea is that accumulating random amount of shortages due to successive attritions leads to the breakdown of the system when the total shortage crosses a random threshold level. The breakdown point or the threshold can also be interpreted as that point at which immediate recruitment is necessitated.

The shortage of MPS depends on individual propensity to leave the organization, which in turn depends on various factors as discussed before. Such models have been discussed by Grinold and Marshall [5], Bartholomew and Forbes [2]. For statistical approach one may refer to Bartholomew [1]. Lesson [6] has given methods to compute shortages (Resignations, Dismissals, Deaths) and promotion intensities, which produce the proportions corresponding to some descried planning probpsals. Markovian models are designed for shortage and promotion in MPS by Vassiliou [13]. Subramanian. V [11] has made an attempt to provide optimal policy for recruitment, training, promotion and shortages in manpower planning models with special provisions such as time bound promotions, cost of training and voluntary retirement schemes.

Esary et al. [3] have discussed that any component or decice, when exposed to shocks which cause damage to the device or system, is likely to fail when the total accumulated damage exceeds a level called threshold. R. Ramanarayanan and G.Sankaranarayanan [10], in their paper "On Correlated Life and Repair times" have studied the same using Gaver D.P. [4]. Linton D.G. and Saw J.G. [7] hav done Reliability analysis of k-out-of-n : F system. In manpower planning, threshold having SCBZ (setting the clock back to zero) property has been discussed and the expected time to recruitment has been found by Sathiyamoorthy R. and Parthasarathy. S. [12]. For MPS having threshold that follows Exponential distribution, expected time to recruitment and variance have been obtained by S. Parthasarathy and R. Vinoth [9]. Probabilistic Analysis of Time to Recruit and Recruitment Time in Manpower System with two groups by S.Mythilli, R.Ramanarayanan [8].

In this paper, two groups of the MPS are exposed to shortage processes.
Group A consists of employees of MPS other than top level management executives. Group B consists of top level management executives. We assume that after the threshold of MPS recruitment takes places one by one. Here group A is exposed to CSP and group B has Erlang Phase 2 life distribution. We find the joint Laplace Transforms of Time to recruit , Recruitment time, sales time and their expectation. Numerical cases are presented.

II. Model – 1

2.1. Model Description

1. MPS has two groups A and B. Group A has employees of MPS other than top level management executives. Group B consists of top level management executives. Group A is exposed to CSP and group B has Erlang distribution of order 2 with parameter λ. The time to breakdown of MPS is T = min $\{T_1,T_2\}$ where T_1 and T_2

are the times to breakdown of group A and B respectively. We assume that the recruitment time R of a shortage is independent of the shortage magnitude.

2.   Shortages occur in group A in accordance with their occurrence time distribution F(x) and $\int_0^\infty x dF(x) < \infty$.

3.   N is the random number of shortages required for breakdown of group A. The group survives k shortages with an arbitrary but known probability P_k (=P(N >k)) for k = 0, 1, 2, …, {P_k} is assumed to be a decreasing sequence of numbers in the unit interval subject only to $\sum_{k=0}^\infty P_k < \infty, \quad P_0 = 1$.

4.   Let p_k =($P_{k-1} - P_k$), the probability that group A has a breakdown on the kth shortage, be given by the generating function $\emptyset(S), 0 \leq S \leq 1$. T_2 has an Erlang Phase E_2 distribution with parameter μ.

5.   At time 0, the MPS comprising of the two groups A and B is free of shortage due to manpower loss and put into operation. Groups A and B function independently during time to recruit.

6.   When the breakdown of MPS occurs, recruitment begins immediately. Productive work does not happen in group B when the system breakdown occurs due to group A. When it occurs due to group B, productive work does not happen in group A and recruitment takes place for group A.

7.   Recruitment time of the ith shortage is R_i. R_i's are independent and identically distributed r.v. with distribution function R(y), independent of the shortage magnitudes. Recruitment time for group B is assumed to be R independent of R_i's but with distribution function R(y), $\int_0^\infty y dR(y) < \infty$.

8.   K is the number of projects completed or project produced before the breakdown of MPS.

9.   Sales Time of each product S whose distribution function is G(z) and produced are sold one by one.

2.2 Analysis

As per additive recruitment policy, recruitments are done for all the shortages one at a time after time T. $R_T = R_1 + R_2 + … + R_i$ where i is the number of shortages occurred during T. When the MPS breakdown occurs due to group A, we have to recruit for N shortages. Hence, $R_T = R_1 + R_2 + … + R_N$.

In the other case, when the MPS breakdown occurs due to group B, we have $R_T = R_1 + R_2 + … + R_{n+1}$ as we have to rejuvenate group B also with the n (< N) shortages that occurred in group A during T. Considering also the numbr of products produced and its sales time we find the joint distribution of T, R_T, K and S_K is given as

$$P\left(T \leq x, R_T \leq y, K = k, S_K \leq z\right) = \sum_{n=1}^\infty p_n \int_0^x (e^{-\lambda u} + \lambda u e^{-\lambda u}) R_n(y) e^{-\mu u} \frac{(\mu u)^k}{k!} dF_n(u) G_k(z) +$$

$$\sum_{n=0}^\infty P_n R_{n+1}(y) \int_0^x \lambda(\lambda u) e^{-\lambda u} [F_n(u) - F_{n+1}(u)] du \; e^{-\mu x} \frac{(\mu x)^k}{k!} G_k(z)$$

Here $R_n(y)$ and $F_n(u)$ are the n fold convolutions of R(y) and F(u) respectively.

The joint probability density function is given by

$$\frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial z} f\left(x, y, k, z\right) = \sum_{n=1}^\infty p_n (e^{-\lambda x} + \lambda x e^{-\lambda x}) f_n(x) e^{-\mu x} \frac{(\mu x)^k}{k!} g_k(z) r_n(y) +$$

$$\sum_{n=0}^\infty P_n r_{n+1}(y) \lambda(\lambda x) e^{-\lambda x} [F_n(x) - F_{n+1}(x)] e^{-\mu x} \frac{(\mu x)^k}{k!} g_k(z) \quad ……(1)$$

The term under the first summation symbol of the right side of (1) is the joint probability that the MPS breakdown occurs due to group A on the nth shortage during (0, x) and $\sum_{i=1}^n R_i \leq y$. The term under the second summation symbol is the joint probability that MPS breakdown occurs due to group B during (0, x), group A survives the n shortages occurred to it before threshold level and $\sum_{i=1}^{n+1} R_i \leq y$.

Applying the Laplace-Stiljies Transform in equation(1), we get

$$E[e^{-\varepsilon T} e^{-\eta R_T} \theta^K e^{-\chi S_K}] = \sum_{k=0}^\infty \int_0^\infty \int_0^\infty \int_0^\infty f\left(x, y, n, z\right) e^{-\varepsilon x} e^{-\eta y} \theta^k e^{-\chi z} dx dy dz \quad ………(2)$$

$$= \sum_{k=0}^{\infty} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \left[\begin{array}{l} \sum_{n=1}^{\infty} p_n (e^{-\lambda x} + \lambda x e^{-\lambda x}) f_n(x) e^{-\mu x} \dfrac{(\mu x)^k}{k!} g_k(z) r_n(y) + \\[2mm] \sum_{n=0}^{\infty} P_n r_{n+1}(y) \lambda(\lambda x) e^{-\lambda x} [F_n(x) - F_{n+1}(x)] e^{-\mu x} \dfrac{(\mu x)^k}{k!} g_k(z) \end{array} \right] e^{-\varepsilon x} e^{-\eta y} \theta^k e^{-\chi z} \, dx\,dy\,dz$$

On simplication, we get

$$E\left[e^{-\varepsilon T} e^{-\eta R_T} \theta^K e^{-\chi S_K} \right] = \phi\left(r^*(\eta) f^*(A) \right) - \lambda \frac{d}{dA}\left[\phi\left(r^*(\eta) f^*(A) \right) \right] +$$

$$\frac{\lambda^2}{(A)^2} r^*(\eta)\left(1 - f^*(A) \right)\left(\frac{1 - \phi(r^*(n) f^*(A))}{1 - r^*(n) f^*(A)} \right) - \frac{\lambda^2}{A} r^*(\eta) \; \frac{d}{dA}\left[\left(1 - f^*(A) \right)\left(\frac{1 - \phi(r^*(n) f^*(A))}{1 - r^*(n) f^*(A)} \right) \right] \quad ......(3)$$

$$\text{Here } A = \varepsilon + \lambda + \mu - \mu\theta g^*(\chi)$$

Here * denotes Laplace-Stieltjes transform.
Using differentiation of equation (3), we get

$$E[T] = \frac{2\left(1 - \phi\left(f^*(\lambda) \right) \right)}{\lambda} + \phi'\left(f^*(\lambda) \right) f^{*'}(\lambda) \qquad ....(4)$$

Using differentiation in equation(3),we get

$$E[R_T] = \left(\frac{1 - \phi\left(f^*(\lambda) \right)}{1 - f^*(\lambda)} - \lambda \frac{d}{d\lambda}\left(\frac{1 - \phi\left(f^*(\lambda) \right)}{1 - f^*(\lambda)} \right) \right) E[R_{T_1}] \qquad ....(5)$$

Assume that f(.) as exponential density function with parameter δ
Using differentiation in equation(3), we get

$$E[S_K] = \mu E[T] E[S_1] \qquad ....(6)$$

and

$$E[K] = \mu E[T] \qquad ....(7)$$

Numerical Illustration

We assume that $\mathrm{p}_1 = \dfrac{1}{4}, \mathrm{p}_2 = \dfrac{1}{4}, \mathrm{p}_3 = \dfrac{1}{2}$ *and* $E[R_{T_1}] = 20$, here $\phi(\mathrm{r}) = \sum_{i=0}^{\infty} p_i r^i$

For the different values of δ (say δ=10,20,30,40,50,60) and the varying parameter is λ=5,10,15,20,25,30 in the equation (5) and (6), we get
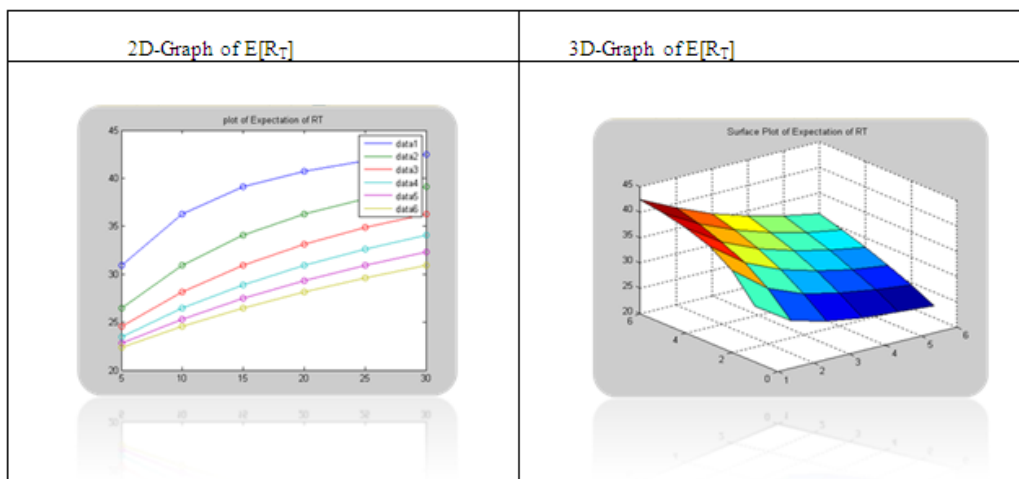
Table of E[T]

$\delta \rightarrow$ λ	10	20	30	40	50	60
5	0.1611	0.0903	0.0624	0.0476	0.0385	0.0323
10	0.1281	0.0806	0.0580	0.0452	0.0369	0.0312
15	0.1042	0.0717	0.0537	0.0427	0.0353	0.0301
20	0.0870	0.0641	0.0497	0.0403	0.0338	0.0290
25	0.0743	0.0576	0.0460	0.0380	0.0322	0.0279
30	0.0646	0.0521	0.0427	0.0358	0.0307	0.0269

2D-Graph of E[T]	3D-Graph of E[T]
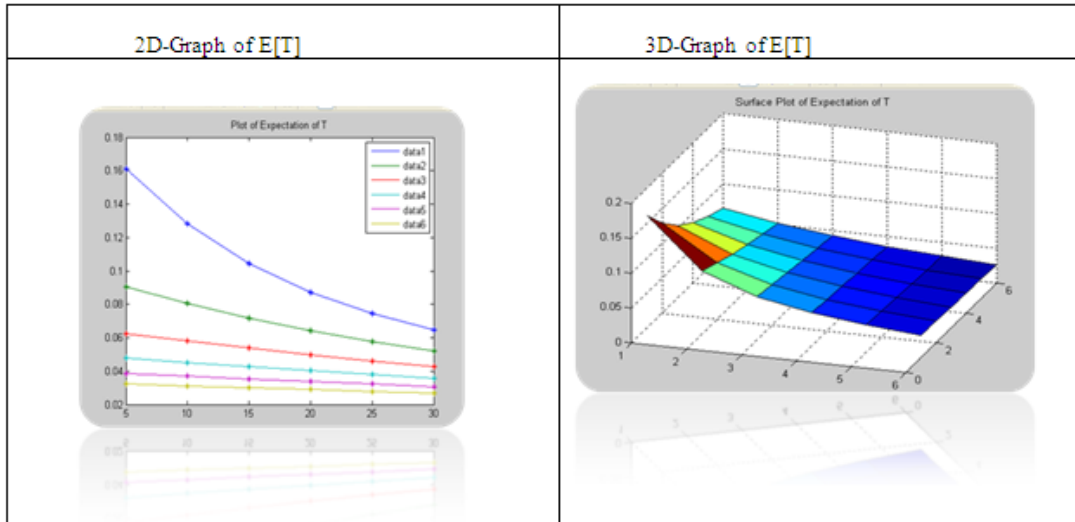
From the above table, as the value of δ increases, E[T] decreases and the value of λ increases, E[T] is also decreases.

Table of E[R_T]

$\dfrac{\delta \rightarrow}{\lambda}$	10	20	30	40	50	60
5	30.9259	26.4400	24.5335	23.4911	22.8362	22.3873
10	36.2500	30.9259	28.1250	26.4400	25.3241	24.5335
15	39.0800	34.0379	30.9259	28.8918	27.4761	26.4400
20	40.7407	36.2500	33.1200	30.9259	29.3294	28.1250
25	41.7930	37.8669	34.8573	32.6195	30.9259	29.6123
30	42.5000	39.0800	36.2500	34.0379	32.3047	30.9259

2D-Graph of E[R_T]	3D-Graph of E[R_T]

From the above table, as the value of δ increases, E[R_T] decreases and the value of λ increases, E[R_T] increases.

Table of E[K]

$\dfrac{\delta \rightarrow}{\lambda}$	10	20	30	40	50	60
5	0.3222	0.1806	0.1248	0.0952	0.0770	0.0646
10	0.2562	0.1611	0.1160	0.0903	0.07738	0.0624
15	0.2085	0.1434	0.1074	0.0854	0.0707	0.0602
20	0.1741	0.1281	0.0994	0.0806	0.0675	0.0580
25	0.1486	0.1152	0.0920	0.0760	0.0644	0.0558
30	0.1293	0.1042	0.0854	0.0717	0.0615	0.0537

From the above table, as the value of δ increases, E[K] decreases and the value of λ increases, E[K] decreases.

Table of E[S$_K$]

$\delta \rightarrow$ / λ	10	20	30	40	50	60
5	0.6444	0.3613	0.2496	0.1905	0.1535	0.1291
10	0.5125	0.3222	0.2320	0.1806	0.1477	0.1248
15	0.4170	0.2868	0.2148	0.1707	0.1413	0.1204
20	0.3481	0.2562	0.1987	0.1611	0.1350	0.1160
25	0.2973	0.2304	0.1840	0.1520	0.1289	0.1117
30	0.2586	0.2085	0.1708	0.1434	0.1230	0.1074

From the above table, as the value of δ increases, E[S$_K$] decreases and the value of λ increases, E[S$_K$] decreases.

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