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The Degree of an Edge in Alpha Product, Beta Product and Gamma Product of Two Fuzzy Graphs

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Abstract: *A fuzzy graph can be obtained from two given fuzzy graphs using alpha product, beta product and gamma product. In this paper, we find the degree of an edge in fuzzy graphs formed by these operations in terms of the degree of edges and vertices in the given fuzzy graphs in some particular cases.*
Key Words: *Alpha product, Beta product, Gamma product, Degree of a vertex, Degree of an edge.*
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I. Introduction

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975 [9]. Mordeson. J. N and Peng. C. S introduced the concept of operations on fuzzy graphs [2]. The degree of a vertex in fuzzy graphs which are obtained from two given fuzzy graphs using the operations of alpha product, beta product and gamma product was discussed by Nagoor Gani. A and Fathima Kani. B [3]. Radha. K and Kumaravel. N introduced the concept of degree of an edge and total degree of an edge in fuzzy graphs [8]. We study about the degree of an edge in fuzzy graphs which are obtained from two given fuzzy graphs using the operations of alpha, beta and gamma product. In general, the degree of an edge in alpha, beta and gamma product of two fuzzy graphs G_1 and G_2 cannot be expressed in terms of these in G_1 and G_2. In this paper, we find the degree of an edge in alpha, beta and gamma product of two fuzzy graphs G_1 and G_2 in terms of the degree of edges of G_1 and G_2 in some particular cases. First we go through some basic concepts from [1] – [10].

A fuzzy subset of a set V is a mapping σ from V to [0, 1]. A fuzzy graph G is a pair of functions $G:(\sigma,\mu)$ where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ, (i.e.) $\mu(xy)\le\sigma(x)\wedge\sigma(y)$ for all $x,y\in V$. The underlying crisp graph of $G:(\sigma,\mu)$ is denoted by $G^*:(V,E)$ where $E\subseteq V\times V$. Throughout this paper, $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ denote two fuzzy graphs with underlying crisp graphs $G_1^*:(V_1,E_1)$ and $G_2^*:(V_2,E_2)$ with $|V_i|=p_i, i=1,2$. Also $d_{G_i^*}(u_i)$ denotes the degree of u_i in G_i^* and $d_{\overline{G_i^*}}(u_i)$ denotes the degree of u_i in $\overline{G_i}^*$, where $\overline{G_i}^*$ is the complement of G_i^*. Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. The degree of a vertex u is $d_G(u)=\sum_{u\neq v}\mu(uv)$. The minimum degree of G is $\delta(G)=\wedge\{d_G(v),\forall\ v\in V\}$ and the maximum degree of G is $\Delta(G)=\vee\{d_G(v),\forall\ v\in V\}$. The total degree of a vertex $u\in V$ is defined by $td_G(u)=\sum_{u\neq v}\mu(uv)+\sigma(u)$. The order and size of a fuzzy graph G are defined by $O(G)=\sum_{u\in V}\sigma(u)$ and $S(G)=\sum_{uv\in E}\mu(uv)$. Let $G^*:(V,E)$ be a graph and let $e=uv$ be an edge in G^*. Then the degree of an edge $e=uv\in E$ is defined by $d_{G^*}(uv)=d_{G^*}(u)+d_{G^*}(v)-2$. Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. The degree of an edge uv is $d_G(uv)=d_G(u)+d_G(v)-2\mu(uv)$. This is equivalent to $d_G(uv)=\sum_{\substack{uw\in E\\ w\neq v}}\mu(uw)+\sum_{\substack{wv\in E\\ w\neq u}}\mu(wv)$. The total degree of an edge $uv\in E$ is defined by $td_G(uv)=d_G(u)+d_G(v)-\mu(uv)$. This is equivalent to $td_G(uv)=\sum_{\substack{uw\in E\\ w\neq v}}\mu(uw)+\sum_{\substack{wv\in E\\ w\neq u}}\mu(wv)+\mu(uv)=$

$d_G(uv)+\mu(uv)$. The minimum edge degree and maximum edge degree of G are $\delta_E(G)=\wedge\{d_G(uv),\forall\, uv\in E\}$ and $\Delta_E(G)=\vee\{d_G(uv),\forall\, uv\in E\}$.

Definition 1.1 [3]: Let $G^*=G_1^*\underset{\alpha}{\times}G_2^*=(V,E)$ be the alpha product of two graphs G_1^* and G_2^*, where $V=V_1\times V_2$ and $E=\{(u_1,u_2)(v_1,v_2):u_1=v_1,u_2v_2\in E_2\ \ (or)\ \ u_1v_1\in E_1,u_2=v_2\ \ (or)\ \ u_1v_1\in E_1,u_2v_2\notin E_2\ \ (or)\ \ u_1v_1\notin E_1,u_2v_2\in E_2\}$. Then the alpha product of two fuzzy graphs G_1 and G_2 is a fuzzy graph $G=G_1\underset{\alpha}{\times}G_2=G_1\underset{\alpha}{\times}G_2:(\sigma_1\underset{\alpha}{\times}\sigma_2,\mu_1\underset{\alpha}{\times}\mu_2)$ defined by

$(\sigma_1\underset{\alpha}{\times}\sigma_2)(u_1,u_2)=\sigma_1(u_1)\wedge\sigma_2(u_2),\forall(u_1,u_2)\in V$ and

$$(\mu_1\underset{\alpha}{\times}\mu_2)((u_1,u_2)(v_1,v_2))=\begin{cases}\sigma_1(u_1)\wedge\mu_2(u_2v_2), & if\ u_1=v_1,u_2v_2\in E_2\\ \mu_1(u_1v_1)\wedge\sigma_2(u_2), & if\ u_1v_1\in E_1,u_2=v_2\\ \mu_1(u_1v_1)\wedge\sigma_2(u_2)\wedge\sigma_2(v_2), & if\ u_1v_1\in E_1,u_2v_2\notin E_2\\ \sigma_1(u_1)\wedge\sigma_1(v_1)\wedge\mu_2(u_2v_2), & if\ u_1v_1\notin E_1,u_2v_2\in E_2\end{cases}.$$

Definition 1.2 [4]: Let $G^*=G_1^*\underset{\beta}{\times}G_2^*=(V,E)$ be the beta product of two graphs G_1^* and G_2^*, where $V=V_1\times V_2$ and $E=\{(u_1,u_2)(v_1,v_2):u_1v_1\in E_1,u_2v_2\in E_2\ \ (or)\ \ u_1v_1\in E_1,u_2v_2\notin E_2\ \ (or)\ \ u_1v_1\notin E_1,u_2v_2\in E_2\}$. Then the beta product of two fuzzy graphs G_1 and G_2 is a fuzzy graph $G=G_1\underset{\beta}{\times}G_2=G_1\underset{\beta}{\times}G_2:(\sigma_1\underset{\beta}{\times}\sigma_2,\mu_1\underset{\beta}{\times}\mu_2)$ defined by

$(\sigma_1\underset{\beta}{\times}\sigma_2)(u_1,u_2)=\sigma_1(u_1)\wedge\sigma_2(u_2),\forall(u_1,u_2)\in V$ and

$$(\mu_1\underset{\beta}{\times}\mu_2)((u_1,u_2)(v_1,v_2))=\begin{cases}\mu_1(u_1v_1)\wedge\mu_2(u_2v_2), & if\ u_1v_1\in E_1,\forall u_2v_2\in E_2\\ \mu_1(u_1v_1)\wedge\sigma_2(u_2)\wedge\sigma_2(v_2), & if\ u_1v_1\in E_1,u_2v_2\notin E_2\\ \sigma_1(u_1)\wedge\sigma_1(v_1)\wedge\mu_2(u_2v_2), & if\ u_1v_1\notin E_1,u_2v_2\in E_2\end{cases}.$$

Definition 1.3 [4]: Let $G^*=G_1^*\underset{\gamma}{\times}G_2^*=(V,E)$ be the gamma product of two graphs G_1^* and G_2^*, where $V=V_1\times V_2$ and $E=\{(u_1,u_2)(v_1,v_2):u_1=v_1,u_2v_2\in E_2\ \ (or)\ \ u_1v_1\in E_1,u_2=v_2\ \ (or)\ \ u_1v_1\in E_1,u_2v_2\notin E_2\ \ (or)\ \ u_1v_1\notin E_1,u_2v_2\in E_2\ \ (or)\ \ u_1v_1\in E_1,u_2v_2\in E_2\}$. Then the gamma product of two fuzzy graphs G_1 and G_2 is a fuzzy graph $G=G_1\underset{\gamma}{\times}G_2=G_1\underset{\gamma}{\times}G_2:(\sigma_1\underset{\gamma}{\times}\sigma_2,\mu_1\underset{\gamma}{\times}\mu_2)$ defined by

$(\sigma_1\underset{\gamma}{\times}\sigma_2)(u_1,u_2)=\sigma_1(u_1)\wedge\sigma_2(u_2),\forall(u_1,u_2)\in V$ and

$$(\mu_1\underset{\gamma}{\times}\mu_2)((u_1,u_2)(v_1,v_2))=\begin{cases}\sigma_1(u_1)\wedge\mu_2(u_2v_2), & if\ u_1=v_1,u_2v_2\in E_2\\ \mu_1(u_1v_1)\wedge\sigma_2(u_2), & if\ u_1v_1\in E_1,u_2=v_2\\ \mu_1(u_1v_1)\wedge\sigma_2(u_2)\wedge\sigma_2(v_2), & if\ u_1v_1\in E_1,u_2v_2\notin E_2\\ \sigma_1(u_1)\wedge\sigma_1(v_1)\wedge\mu_2(u_2v_2), & if\ u_1v_1\notin E_1,u_2v_2\in E_2\\ \mu_1(u_1v_1)\wedge\mu_2(u_2v_2), & if\ u_1v_1\in E_1,u_2v_2\in E_2\end{cases}.$$

Theorem 1.4 [5]: If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1\le\mu_2$, then $\sigma_2\ge\mu_1$ and vice versa.

Theorem 1.5: If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs.

(1). If $\sigma_1 \le \mu_2$, then $\mu_1 \le \mu_2$.
(2). If $\sigma_2 \le \mu_1$, then $\mu_2 \le \mu_1$.

Proof:

(1). By the definition of fuzzy graphs, $\mu_i(uv) = \sigma_i(u) \wedge \sigma_i(v)$ for $uv \in E_i, i = 1,2.$

Therefore, $\mu_i \le \max \sigma_i, i = 1,2$.

Since $\sigma_1 \le \mu_2$, $\max \sigma_1 \le \min \mu_2$.

Hence $\mu_1 \le \max \sigma_1 \le \min \mu_2$.

Thus $\mu_1 \le \min \mu_2$.

Hence $\mu_1 \le \mu_2$.

(2). Proof is similar to the proof of (1).

II. Degree of an Edge in Alpha Product

By definition, for any $((u_1,u_2)(v_1,v_2)) \in E$,

$$d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,v_2)) = \sum_{(u_1,u_2)(w_1,w_2)\in E} (\mu_1 \underset{\alpha}{\times} \mu_2)((u_1,u_2)(w_1,w_2)) + \sum_{(w_1,w_2)(v_1,v_2)\in E} (\mu_1 \underset{\alpha}{\times} \mu_2)((w_1,w_2)(v_1,v_2))$$

$$-2(\mu_1 \underset{\alpha}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$

$$= \sum_{u_2w_2\in E_2, u_1=w_1} \sigma_1(u_1) \wedge \mu_2(u_2w_2) + \sum_{u_1w_1\in E_1, u_2=w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2)$$

$$+ \sum_{u_1w_1\in E_1, u_2w_2\notin E_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{u_1w_1\notin E_1, u_2w_2\in E_2} \sigma_1(u_1) \wedge \sigma_1(w_1) \wedge \mu_2(u_2w_2)$$

$$+ \sum_{w_2v_2\in E_2, w_1=u_1} \sigma_1(v_1) \wedge \mu_2(w_2v_2) + \sum_{w_1v_1\in E_1, w_2=v_2} \mu_1(w_1v_1) \wedge \sigma_2(v_2) + \sum_{w_1v_1\in E_1, w_2v_2\notin E_2} \mu_1(w_1v_1) \wedge \sigma_2(w_2) \wedge \sigma_2(v_2)$$

$$+ \sum_{w_1v_1\notin E_1, w_2v_2\in E_2} \sigma_1(w_1) \wedge \sigma_1(v_1) \wedge \mu_2(w_2v_2) - 2(\mu_1 \underset{\alpha}{\times} \mu_2)((u_1,u_2)(v_1,v_2)) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2.1)}$$

Theorem 2.1:

Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs.

Suppose that $\sigma_1 \ge \mu_2$ and $\sigma_2 \ge \mu_1$. Then for any $(u_1,u_2)(v_1,v_2) \in E$,

(1). When $u_1 = v_1, u_2v_2 \in E_2$,

$$d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(u_1,v_2)) = d_{G_2}(u_2v_2) + d_{G_1}(u_1)(d_{\overline{G_2^*}}(u_2) + d_{\overline{G_2^*}}(v_2) + 2)$$
$$+ d_{\overline{G_1^*}}(u_1)(d_{G_2}(u_2) + d_{G_2}(v_2)),$$

(2). When $u_2 = v_2, u_1v_1 \in E_1$,

$$d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,u_2)) = d_{G_1}(u_1v_1) + d_{G_2}(u_2)(d_{\overline{G_1^*}}(u_1) + d_{\overline{G_1^*}}(v_1) + 2) + d_{\overline{G_2^*}}(u_2)(d_{G_1}(u_1) + d_{G_1}(v_1)),$$

(3). $d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1v_1) + d_{G_2}(u_2)[1 + d_{\overline{G_1^*}}(u_1)] + d_{G_2}(v_2)[1 + d_{\overline{G_1^*}}(v_1)]$

$+ d_{\overline{G_2^*}}(u_2) d_{G_1}(u_1) + d_{\overline{G_2^*}}(v_2) d_{G_1}(v_1)$, when $u_1v_1 \in E_1, u_2v_2 \notin E_2$ and

$d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_2}(u_2v_2) + d_{G_1}(u_1)[1 + d_{\overline{G_2^*}}(u_2)] + d_{G_1}(v_1)[1 + d_{\overline{G_2^*}}(v_2)]$

$+ d_{\overline{G_1^*}}(u_1) d_{G_2}(u_2) + d_{\overline{G_1^*}}(v_1) d_{G_2}(v_2)$, when $u_1v_1 \notin E_1, u_2v_2 \in E_2$.

Proof:

1. We have $\sigma_1 \ge \mu_2$ and $\sigma_2 \ge \mu_1$.

(1). From (2.1), for any $(u_1,u_2)(u_1,v_2) \in E$,

$$d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(u_1,v_2)) = \sum_{u_2w_2\in E_2, u_1=w_1} \sigma_1(u_1) \wedge \mu_2(u_2w_2) + \sum_{u_1w_1\in E_1, u_2=w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2)$$

$$+\sum_{u_1w_1\in E_1,u_2w_2\notin E_2}\mu_1(u_1w_1)\wedge\sigma_2(u_2)\wedge\sigma_2(w_2)+\sum_{u_1w_1\notin E_1,u_2w_2\in E_2}\sigma_1(u_1)\wedge\sigma_1(w_1)\wedge\mu_2(u_2w_2)$$

$$+\sum_{w_2v_2\in E_2,w_1=u_1}\sigma_1(u_1)\wedge\mu_2(w_2v_2)+\sum_{w_1u_1\in E_1,w_2=v_2}\mu_1(w_1u_1)\wedge\sigma_2(v_2)+\sum_{w_1u_1\in E_1,w_2v_2\notin E_2}\mu_1(w_1u_1)\wedge\sigma_2(w_2)\wedge\sigma_2(v_2)$$

$$+\sum_{w_1u_1\notin E_1,w_2v_2\in E_2}\sigma_1(w_1)\wedge\sigma_1(u_1)\wedge\mu_2(w_2v_2)-2(\mu_1\underset{\alpha}{\times}\mu_2)((u_1,u_2)(u_1,v_2))$$

$$=\sum_{u_2w_2\in E_2}\mu_2(u_2w_2)+\sum_{u_1w_1\in E_1}\mu_1(u_1w_1)+\sum_{u_1w_1\in E_1,u_2w_2\notin E_2}\mu_1(u_1w_1)+\sum_{u_1w_1\notin E_1,u_2w_2\in E_2}\mu_2(u_2w_2)+\sum_{w_2v_2\in E_2}\mu_2(w_2v_2)$$

$$+\sum_{w_1u_1\in E_1}\mu_1(w_1u_1)+\sum_{w_1u_1\in E_1,w_2v_2\notin E_2}\mu_1(w_1u_1)+\sum_{w_1u_1\notin E_1,w_2v_2\in E_2}\mu_2(w_2v_2)-2(\sigma_1(u_1)\wedge\mu_2(u_2v_2))$$

$$=\sum_{u_2w_2\in E_2}\mu_2(u_2w_2)+\sum_{u_1w_1\in E_1}\mu_1(u_1w_1)+d_{\overline{G}_2^*}(u_2)\sum_{u_1w_1\in E_1}\mu_1(u_1w_1)+d_{\overline{G}_1^*}(u_1)\sum_{u_2w_2\in E_2}\mu_2(u_2w_2)+\sum_{w_2v_2\in E_2}\mu_2(w_2v_2)$$

$$+\sum_{w_1u_1\in E_1}\mu_1(w_1u_1)+d_{\overline{G}_2^*}(v_2)\sum_{w_1u_1\in E_1}\mu_1(w_1u_1)+d_{\overline{G}_1^*}(u_1)\sum_{w_2v_2\in E_2}\mu_2(w_2v_2)-2\mu_2(u_2v_2)$$

$$=\sum_{u_2w_2\in E_2}\mu_2(u_2w_2)+\sum_{w_2v_2\in E_2}\mu_2(w_2v_2)-2\mu_2(u_2v_2)+d_{G_1}(u_1)+d_{G_1}(u_1)d_{\overline{G}_2^*}(u_2)+d_{\overline{G}_1^*}(u_1)d_{G_2}(u_2)$$

$$+d_{G_1}(u_1)+d_{G_1}(u_1)d_{\overline{G}_2^*}(v_2)+d_{\overline{G}_1^*}(u_1)d_{G_2}(v_2)$$

$$\therefore d_{G_1\underset{\alpha}{\times}G_2}((u_1,u_2)(u_1,v_2))=d_{G_2}(u_2v_2)+d_{G_1}(u_1)(d_{\overline{G}_2^*}(u_2)+d_{\overline{G}_2^*}(v_2)+2)$$

$$+d_{\overline{G}_1^*}(u_1)(d_{G_2}(u_2)+d_{G_2}(v_2)).$$

(2). Proof is similar to the proof of (1).

(3). From (2.1), for any $(u_1,u_2)(v_1,v_2)\in E$,

$$d_{G_1\underset{\alpha}{\times}G_2}((u_1,u_2)(v_1,v_2))=\sum_{u_2w_2\in E_2,u_1=w_1}\sigma_1(u_1)\wedge\mu_2(u_2w_2)+\sum_{u_1w_1\in E_1,u_2=w_2}\mu_1(u_1w_1)\wedge\sigma_2(u_2)$$

$$+\sum_{u_1w_1\in E_1,u_2w_2\notin E_2}\mu_1(u_1w_1)\wedge\sigma_2(u_2)\wedge\sigma_2(w_2)+\sum_{u_1w_1\notin E_1,u_2w_2\in E_2}\sigma_1(u_1)\wedge\sigma_1(w_1)\wedge\mu_2(u_2w_2)$$

$$+\sum_{w_2v_2\in E_2,w_1=u_1}\sigma_1(v_1)\wedge\mu_2(w_2v_2)+\sum_{w_1v_1\in E_1,w_2=v_2}\mu_1(w_1v_1)\wedge\sigma_2(v_2)+\sum_{w_1v_1\in E_1,w_2v_2\notin E_2}\mu_1(w_1v_1)\wedge\sigma_2(w_2)\wedge\sigma_2(v_2)$$

$$+\sum_{w_1v_1\notin E_1,w_2v_2\in E_2}\sigma_1(w_1)\wedge\sigma_1(v_1)\wedge\mu_2(w_2v_2)-2(\mu_1\underset{\alpha}{\times}\mu_2)((u_1,u_2)(v_1,v_2))$$

$$=\sum_{u_2w_2\in E_2}\mu_2(u_2w_2)+\sum_{u_1w_1\in E_1}\mu_1(u_1w_1)+\sum_{u_1w_1\in E_1,u_2w_2\notin E_2}\mu_1(u_1w_1)+\sum_{u_1w_1\notin E_1,u_2w_2\in E_2}\mu_2(u_2w_2)+\sum_{w_2v_2\in E_2}\mu_2(w_2v_2)$$

$$+\sum_{w_1v_1\in E_1}\mu_1(w_1v_1)+\sum_{w_1v_1\in E_1,w_2v_2\notin E_2}\mu_1(w_1v_1)+\sum_{w_1v_1\notin E_1,w_2v_2\in E_2}\mu_2(w_2v_2)-2(\mu_1\underset{\alpha}{\times}\mu_2)((u_1,u_2)(v_1,v_2))$$

$$=d_{G_2}(u_2)+d_{G_1}(u_1)+\sum_{u_1w_1\in E_1,u_2w_2\notin E_2}\mu_1(u_1w_1)+\sum_{u_1w_1\notin E_1,u_2w_2\in E_2}\mu_2(u_2w_2)+d_{G_2}(v_2)+d_{G_1}(v_1)+\sum_{w_1v_1\in E_1,w_2v_2\notin E_2}\mu_1(w_1v_1)$$

$$+\sum_{w_1v_1\notin E_1,w_2v_2\in E_2}\mu_2(w_2v_2)-2(\mu_1\underset{\alpha}{\times}\mu_2)((u_1,u_2)(v_1,v_2))$$

$$=d_{G_2}(u_2)+d_{G_2}(v_2)+d_{G_1}(u_1)+d_{G_1}(v_1)+d_{\overline{G}_2^*}(u_2)\sum_{u_1w_1\in E_1}\mu_1(u_1w_1)+d_{\overline{G}_1^*}(u_1)\sum_{u_2w_2\in E_2}\mu_2(u_2w_2)$$

$$+d_{\overline{G}_2^*}(v_2)\sum_{w_1v_1\in E_1}\mu_1(w_1v_1)+d_{\overline{G}_1^*}(v_1)\sum_{w_2v_2\in E_2}\mu_2(w_2v_2)-2(\mu_1\underset{\alpha}{\times}\mu_2)((u_1,u_2)(v_1,v_2))$$

$$\therefore d_{G_1\underset{\alpha}{\times}G_2}((u_1,u_2)(v_1,v_2))=d_{G_1}(u_1)+d_{G_1}(v_1)+d_{G_2}(u_2)+d_{G_2}(v_2)+d_{\overline{G}_2^*}(u_2)d_{G_1}(u_1)$$

$$+d_{\overline{G}_1^*}(u_1)d_{G_2}(u_2)+d_{\overline{G}_2^*}(v_2)d_{G_1}(v_1)+d_{\overline{G}_1^*}(v_1)d_{G_2}(v_2)-2(\mu_1\underset{\alpha}{\times}\mu_2)((u_1,u_2)(v_1,v_2)).$$

Now, for any $(u_1,u_2)(v_1,v_2)\in E$, we have to consider two cases:

$u_1v_1 \in E_1, u_2v_2 \notin E_2$ (or) $u_1v_1 \notin E_1, u_2v_2 \in E_2$.

Case 1: $u_1v_1 \in E_1, u_2v_2 \notin E_2$.

$$\therefore d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1) + d_{G_1}(v_1) + d_{G_2}(u_2) + d_{G_2}(v_2) + d_{\overline{G_2^*}}(u_2)d_{G_1}(u_1)$$
$$+ d_{\overline{G_1^*}}(u_1)d_{G_2}(u_2) + d_{\overline{G_2^*}}(v_2)d_{G_1}(v_1) + d_{\overline{G_1^*}}(v_1)d_{G_2}(v_2) - 2(\mu_1(u_1v_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2))$$
$$= d_{G_1}(u_1) + d_{G_1}(v_1) + d_{G_2}(u_2) + d_{G_2}(v_2) + d_{\overline{G_2^*}}(u_2)d_{G_1}(u_1)$$
$$+ d_{\overline{G_1^*}}(u_1)d_{G_2}(u_2) + d_{\overline{G_2^*}}(v_2)d_{G_1}(v_1) + d_{\overline{G_1^*}}(v_1)d_{G_2}(v_2) - 2\mu_1(u_1v_1)$$
$$\therefore d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1v_1) + d_{G_2}(u_2)[1 + d_{\overline{G_1^*}}(u_1)] + d_{G_2}(v_2)[1 + d_{\overline{G_1^*}}(v_1)]$$
$$+ d_{\overline{G_2^*}}(u_2)d_{G_1}(u_1) + d_{\overline{G_2^*}}(v_2)d_{G_1}(v_1).$$

Case 2: $u_1v_1 \notin E_1, u_2v_2 \in E_2$.

Proof is similar to the proof of case 1.

Theorem 2.2:

Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs.

1. If $\sigma_1 \le \mu_2$ and σ_1 is a constant function with $\sigma_1(u) = c_1$ for all $u \in V_1$, then for any $(u_1,u_2)(v_1,v_2) \in E$,

(a). When $u_1 = v_1, u_2v_2 \in E_2$,

$$d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(u_1,v_2)) = c_1(d_{G_2^*}(u_2) + d_{G_2^*}(v_2))(d_{\overline{G_1^*}}(u_1) + 1) - 2c_1$$
$$+ d_{G_1}(u_1)(d_{\overline{G_2^*}}(u_2) + d_{\overline{G_2^*}}(v_2) + 2),$$

(b). When $u_2 = v_2, u_1v_1 \in E_1$,

$$d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,u_2)) = d_{G_1}(u_1v_1) + c_1 d_{G_2^*}(u_2)(2 + d_{\overline{G_1^*}}(u_1) + d_{\overline{G_1^*}}(v_1))$$
$$+ d_{\overline{G_2^*}}(u_2)(d_{G_1}(u_1) + d_{G_1}(v_1)),$$

(c). $d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1v_1) + c_1 d_{G_2^*}(u_2)(1 + d_{\overline{G_1^*}}(u_1)) + c_1 d_{G_2^*}(v_2)(1 + d_{\overline{G_1^*}}(v_1))$

$+ d_{G_1}(u_1)d_{\overline{G_2^*}}(u_2) + d_{G_1}(v_1)d_{\overline{G_2^*}}(v_2)$, when $u_1v_1 \in E_1, u_2v_2 \notin E_2$.

(d). $d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1) + d_{G_1}(v_1) + c_1 d_{G_2^*}(u_2)(1 + d_{\overline{G_1^*}}(u_1)) + c_1 d_{G_2^*}(v_2)(1 + d_{\overline{G_1^*}}(v_1))$

$+ d_{G_1}(u_1)d_{\overline{G_2^*}}(u_2) + d_{G_1}(v_1)d_{\overline{G_2^*}}(v_2) - 2c_1$, when $u_1v_1 \notin E_1, u_2v_2 \in E_2$.

2. If $\sigma_2 \le \mu_1$ and σ_2 is a constant function with $\sigma_2(u) = c_2$ for all $u \in V_2$, then for any $(u_1,u_2)(v_1,v_2) \in E$,

(a). When $u_1 = v_1, u_2v_2 \in E_2$,

$$d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(u_1,v_2)) = d_{G_2}(u_2v_2) + c_2 d_{G_1^*}(u_1)(2 + d_{\overline{G_2^*}}(u_2) + d_{\overline{G_2^*}}(v_2))$$
$$+ d_{\overline{G_1^*}}(u_1)(d_{G_2}(u_2) + d_{G_2}(v_2)),$$

(b). When $u_2 = v_2, u_1v_1 \in E_1$,

$$d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,u_2)) = c_2(d_{G_1^*}(u_1) + d_{G_1^*}(v_1))(d_{\overline{G_2^*}}(u_2) + 1) - 2c_2$$
$$+ d_{G_2}(u_2)(d_{\overline{G_1^*}}(u_1) + d_{\overline{G_1^*}}(v_1) + 2),$$

(c). $d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_2}(u_2) + d_{G_2}(v_2) + c_2 d_{G_1^*}(u_1)(1 + d_{\overline{G_2^*}}(u_2)) + c_2 d_{G_1^*}(v_1)(1 + d_{\overline{G_2^*}}(v_2))$

$+ d_{\overline{G_1^*}}(u_1)d_{G_2}(u_2) + d_{\overline{G_1^*}}(v_1)d_{G_2}(v_2) - 2c_2$, when $u_1v_1 \in E_1, u_2v_2 \notin E_2$.

(d). $d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_2}(u_2v_2) + c_2 d_{G_1^*}(u_1)(1+d_{\overline{G}_2^*}(u_2)) + c_2 d_{G_1^*}(v_1)(1+d_{\overline{G}_2^*}(v_2))$

$+ d_{\overline{G}_1^*}(u_1) d_{G_2}(u_2) + d_{\overline{G}_1^*}(v_1) d_{G_2}(v_2)$, when $u_1v_1 \notin E_1, u_2v_2 \in E_2$.

Proof:

1. We have $\sigma_1 \le \mu_2$. Then by theorem 1.4, $\sigma_2 \ge \mu_1$.

(a). From (2.1), for any $(u_1,u_2)(u_1,v_2) \in E$,

$$d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(u_1,v_2)) = \sum_{u_2w_2 \in E_2, u_1=w_1} \sigma_1(u_1) \wedge \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1, u_2=w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2)$$

$$+ \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \sigma_1(u_1) \wedge \sigma_1(w_1) \wedge \mu_2(u_2w_2)$$

$$+ \sum_{w_2v_2 \in E_2, w_1=u_1} \sigma_1(u_1) \wedge \mu_2(w_2v_2) + \sum_{w_1u_1 \in E_1, w_2=v_2} \mu_1(w_1u_1) \wedge \sigma_2(v_2) + \sum_{w_1u_1 \in E_1, w_2v_2 \notin E_2} \mu_1(w_1u_1) \wedge \sigma_2(w_2) \wedge \sigma_2(v_2)$$

$$+ \sum_{w_1u_1 \notin E_1, w_2v_2 \in E_2} \sigma_1(w_1) \wedge \sigma_1(u_1) \wedge \mu_2(w_2v_2) - 2(\mu_1 \underset{\alpha}{\times} \mu_2)((u_1,u_2)(u_1,v_2))$$

$$= \sum_{u_2w_2 \in E_2} \sigma_1(u_1) + \sum_{u_1w_1 \in E_1} \mu_1(u_1w_1) + \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \sigma_1(u_1) \wedge \sigma_1(w_1) + \sum_{w_2v_2 \in E_2} \sigma_1(u_1)$$

$$+ \sum_{w_1u_1 \in E_1} \mu_1(w_1u_1) + \sum_{w_1u_1 \in E_1, w_2v_2 \notin E_2} \mu_1(w_1u_1) + \sum_{w_1u_1 \notin E_1, w_2v_2 \in E_2} \sigma_1(w_1) \wedge \sigma_1(u_1) - 2(\sigma_1(u_1) \wedge \mu_2(u_2v_2))$$

$$= c_1 d_{G_2^*}(u_2) + d_{G_1}(u_1) + d_{\overline{G}_2^*}(u_2) \sum_{u_1w_1 \in E_1} \mu_1(u_1w_1) + d_{\overline{G}_1^*}(u_1) d_{G_2^*}(u_2) c_1 + c_1 d_{G_2^*}(v_2) + d_{G_1}(u_1)$$

$$+ d_{\overline{G}_2^*}(v_2) \sum_{w_1u_1 \in E_1} \mu_1(w_1u_1) + d_{\overline{G}_1^*}(u_1) d_{G_2^*}(v_2) c_1 - 2\sigma_1(u_1)$$

$$= c_1 d_{G_2^*}(u_2) + d_{G_1}(u_1) + d_{\overline{G}_2^*}(u_2) d_{G_1}(u_1) + d_{\overline{G}_1^*}(u_1) d_{G_2^*}(u_2) c_1 + c_1 d_{G_2^*}(v_2) + d_{G_1}(u_1) + d_{\overline{G}_2^*}(v_2) d_{G_1}(u_1)$$

$$+ d_{\overline{G}_1^*}(u_1) d_{G_2^*}(v_2) c_1 - 2c_1$$

$$= c_1(d_{G_2^*}(u_2) + d_{G_2^*}(v_2) - 2) + 2d_{G_1}(u_1) + d_{G_1}(u_1)(d_{\overline{G}_2^*}(u_2) + d_{\overline{G}_2^*}(v_2))$$

$$+ c_1 d_{\overline{G}_1^*}(u_1)(d_{G_2^*}(u_2) + d_{G_2^*}(v_2))$$

$$= c_1(d_{G_2^*}(u_2) + d_{G_2^*}(v_2))(1 + d_{\overline{G}_1^*}(u_1)) - 2c_1 + d_{G_1}(u_1)(d_{\overline{G}_2^*}(u_2) + d_{\overline{G}_2^*}(v_2) + 2)$$

$$\therefore d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(u_1,v_2)) = c_1(d_{G_2^*}(u_2) + d_{G_2^*}(v_2))(d_{\overline{G}_1^*}(u_1) + 1) - 2c_1$$

$$+ d_{G_1}(u_1)(d_{\overline{G}_2^*}(u_2) + d_{\overline{G}_2^*}(v_2) + 2).$$

(b). From (2.1), for any $(u_1,u_2)(v_1,u_2) \in E$,

$$d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,u_2)) = \sum_{u_2w_2 \in E_2, u_1=w_1} \sigma_1(u_1) \wedge \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1, u_2=w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2)$$

$$+ \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \sigma_1(u_1) \wedge \sigma_1(w_1) \wedge \mu_2(u_2w_2)$$

$$+ \sum_{w_2u_2 \in E_2, w_1=u_1} \sigma_1(v_1) \wedge \mu_2(w_2u_2) + \sum_{w_1v_1 \in E_1, w_2=u_2} \mu_1(w_1v_1) \wedge \sigma_2(u_2) + \sum_{w_1v_1 \in E_1, w_2u_2 \notin E_2} \mu_1(w_1v_1) \wedge \sigma_2(w_2) \wedge \sigma_2(u_2)$$

$$+ \sum_{w_1v_1 \notin E_1, w_2u_2 \in E_2} \sigma_1(w_1) \wedge \sigma_1(v_1) \wedge \mu_2(w_2u_2) - 2(\mu_1 \underset{\alpha}{\times} \mu_2)((u_1,u_2)(v_1,u_2))$$

$$= \sum_{u_2w_2 \in E_{21}} \sigma_1(u_1) + \sum_{u_1w_1 \in E_1} \mu_1(u_1w_1) + \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \sigma_1(u_1) \wedge \sigma_1(w_1) + \sum_{w_2u_2 \in E_2} \sigma_1(v_1)$$

$$+ \sum_{w_1v_1 \in E_1} \mu_1(w_1v_1) + \sum_{w_1v_1 \in E_1, w_2u_2 \notin E_2} \mu_1(w_1v_1) + \sum_{w_1v_1 \notin E_1, w_2u_2 \in E_2} \sigma_1(w_1) \wedge \sigma_1(v_1) - 2(\mu_1(u_1v_1) \wedge \sigma_2(u_2))$$

$$= c_1 d_{G_2^*}(u_2) + \sum_{u_1w_1\in E_1}\mu_1(u_1w_1) + \sum_{w_1v_1\in E_1}\mu_1(w_1v_1) + d_{\overline{G_2^*}}(u_2)\sum_{u_1w_1\in E_1}\mu_1(u_1w_1) + \sum_{u_1w_1\notin E_1, u_2w_2\in E_2} c_1$$

$$+ c_1 d_{G_2^*}(u_2) + d_{\overline{G_2^*}}(u_2)\sum_{w_1v_1\in E_1}\mu_1(w_1v_1) + \sum_{w_1v_1\notin E_1, w_2u_2\in E_2} c_1 \ -2\mu_1(u_1v_1)$$

$$= 2c_1 d_{G_2^*}(u_2) + d_{G_1}(u_1v_1) + d_{\overline{G_2^*}}(u_2) d_{G_1}(u_1) + c_1 d_{\overline{G_1^*}}(u_1) d_{G_2^*}(u_2)$$

$$+ d_{\overline{G_2^*}}(u_2) d_{G_1}(v_1) + c_1 d_{\overline{G_1^*}}(v_1) d_{G_2^*}(u_2)$$

$$\therefore d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,u_2)) = d_{G_1}(u_1v_1) + c_1 d_{G_2^*}(u_2)(2 + d_{\overline{G_1^*}}(u_1) + d_{\overline{G_1^*}}(v_1))$$

$$+ d_{\overline{G_2^*}}(u_2)(d_{G_1}(u_1) + d_{G_1}(v_1)).$$

From (2.1), for any $(u_1,u_2)(v_1,v_2)\in E$,

$$d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,v_2)) = \sum_{u_2w_2\in E_2, u_1=w_1}\sigma_1(u_1)\wedge\mu_2(u_2w_2) + \sum_{u_1w_1\in E_1, u_2=w_2}\mu_1(u_1w_1)\wedge\sigma_2(u_2)$$

$$+ \sum_{u_1w_1\in E_1, u_2w_2\notin E_2}\mu_1(u_1w_1)\wedge\sigma_2(u_2)\wedge\sigma_2(w_2) + \sum_{u_1w_1\notin E_1, u_2w_2\in E_2}\sigma_1(u_1)\wedge\sigma_1(w_1)\wedge\mu_2(u_2w_2)$$

$$+ \sum_{w_2v_2\in E_2, w_1=u_1}\sigma_1(v_1)\wedge\mu_2(w_2v_2) + \sum_{w_1v_1\in E_1, w_2=v_2}\mu_1(w_1v_1)\wedge\sigma_2(v_2) + \sum_{w_1v_1\in E_1, w_2v_2\notin E_2}\mu_1(w_1v_1)\wedge\sigma_2(w_2)\wedge\sigma_2(v_2)$$

$$+ \sum_{w_1v_1\notin E_1, w_2v_2\in E_2}\sigma_1(w_1)\wedge\sigma_1(v_1)\wedge\mu_2(w_2v_2) - 2(\mu_1 \underset{\alpha}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$

$$= \sum_{u_2w_2\in E_2}\sigma_1(u_1) + \sum_{u_1w_1\in E_1}\mu_1(u_1w_1) + \sum_{u_1w_1\in E_1, u_2w_2\notin E_2}\mu_1(u_1w_1) + \sum_{u_1w_1\notin E_1, u_2w_2\in E_2}\sigma_1(u_1)\wedge\sigma_1(w_1) + \sum_{w_2v_2\in E_2}\sigma_1(v_1)$$

$$+ \sum_{w_1v_1\in E_1}\mu_1(w_1v_1) + \sum_{w_1v_1\in E_1, w_2v_2\notin E_2}\mu_1(w_1v_1) + \sum_{w_1v_1\notin E_1, w_2v_2\in E_2}\sigma_1(w_1)\wedge\sigma_1(v_1) - 2(\mu_1 \underset{\alpha}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$

$$= c_1 d_{G_2^*}(u_2) + d_{G_1}(u_1) + d_{\overline{G_2^*}}(u_2)\sum_{u_1w_1\in E_1}\mu_1(u_1w_1) + \sum_{u_1w_1\notin E_1, u_2w_2\in E_2} c_1 + c_1 d_{G_2^*}(v_2) + d_{G_1}(v_1)$$

$$+ d_{\overline{G_2^*}}(v_2)\sum_{w_1v_1\in E_1}\mu_1(w_1v_1) + \sum_{w_1v_1\notin E_1, w_2v_2\in E_2} c_1 \ -2(\mu_1 \underset{\alpha}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$

$$= c_1(d_{G_2^*}(u_2) + d_{G_2^*}(v_2)) + d_{G_1}(u_1) + d_{G_1}(v_1) + d_{\overline{G_2^*}}(u_2) d_{G_1}(u_1) + c_1 d_{\overline{G_1^*}}(u_1) d_{G_2^*}(u_2)$$

$$+ d_{\overline{G_2^*}}(v_2) d_{G_1}(v_1) + c_1 d_{\overline{G_1^*}}(v_1) d_{G_2^*}(v_2) - 2(\mu_1 \underset{\alpha}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$

$$\therefore d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1) + d_{G_1}(v_1) + c_1 d_{G_2^*}(u_2)(1 + d_{\overline{G_1^*}}(u_1)) + c_1 d_{G_2^*}(v_2)(1 + d_{\overline{G_1^*}}(v_1))$$

$$+ d_{G_1}(u_1) d_{\overline{G_2^*}}(u_2) + d_{G_1}(v_1) d_{\overline{G_2^*}}(v_2) - 2(\mu_1 \underset{\alpha}{\times} \mu_2)((u_1,u_2)(v_1,v_2)).$$

Now, for any $(u_1,u_2)(v_1,v_2)\in E$, we have to consider two cases:

$u_1v_1 \in E_1, u_2v_2 \notin E_2$ (or) $u_1v_1 \notin E_1, u_2v_2 \in E_2$.

(c). Case 1: $u_1v_1 \in E_1, u_2v_2 \notin E_2$.

$$\therefore d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1) + d_{G_1}(v_1) + c_1 d_{G_2^*}(u_2)(1 + d_{\overline{G_1^*}}(u_1)) + c_1 d_{G_2^*}(v_2)(1 + d_{\overline{G_1^*}}(v_1))$$

$$+ d_{G_1}(u_1) d_{\overline{G_2^*}}(u_2) + d_{G_1}(v_1) d_{\overline{G_2^*}}(v_2) - 2(\mu_1(u_1v_1)\wedge\sigma_2(u_2)\wedge\sigma_2(v_2)).$$

$$= d_{G_1}(u_1) + d_{G_1}(v_1) + c_1 d_{G_2^*}(u_2)(1 + d_{\overline{G_1^*}}(u_1)) + c_1 d_{G_2^*}(v_2)(1 + d_{\overline{G_1^*}}(v_1))$$

$$+ d_{G_1}(u_1) d_{\overline{G_2^*}}(u_2) + d_{G_1}(v_1) d_{\overline{G_2^*}}(v_2) - 2\mu_1(u_1v_1).$$

$$\therefore d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1v_1) + c_1 d_{G_2^*}(u_2)(1 + d_{\overline{G_1^*}}(u_1)) + c_1 d_{G_2^*}(v_2)(1 + d_{\overline{G_1^*}}(v_1))$$

$$+ d_{G_1}(u_1) d_{\overline{G_2^*}}(u_2) + d_{G_1}(v_1) d_{\overline{G_2^*}}(v_2).$$

(d). Case 2: $u_1v_1 \notin E_1, u_2v_2 \in E_2$.

$$\therefore d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1) + d_{G_1}(v_1) + c_1 d_{G_2^*}(u_2)(1 + d_{\overline{G}_1^*}(u_1)) + c_1 d_{G_2^*}(v_2)(1 + d_{\overline{G}_1^*}(v_1))$$
$$+ d_{G_1}(u_1) d_{\overline{G}_2^*}(u_2) + d_{G_1}(v_1) d_{\overline{G}_2^*}(v_2) - 2(\sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \mu_2(u_2 v_2)).$$
$$\therefore d_{G_1 \underset{\alpha}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1) + d_{G_1}(v_1) + c_1 d_{G_2^*}(u_2)(1 + d_{\overline{G}_1^*}(u_1)) + c_1 d_{G_2^*}(v_2)(1 + d_{\overline{G}_1^*}(v_1))$$
$$+ d_{G_1}(u_1) d_{\overline{G}_2^*}(u_2) + d_{G_1}(v_1) d_{\overline{G}_2^*}(v_2) - 2c_1.$$

2. Proof is similar to the proof of (1).

III. Degree of an Edge in Beta Product

By definition, for any $((u_1,u_2)(v_1,v_2)) \in E$,

$$d_{G_1 \underset{\beta}{\times} G_2}((u_1,u_2)(v_1,v_2)) = \sum_{\substack{(u_1,u_2)(w_1,w_2)\in E,\\ (w_1,w_2)\neq(v_1,v_2)}} (\mu_1 \underset{\beta}{\times} \mu_2)((u_1,u_2)(w_1,w_2)) + \sum_{\substack{(w_1,w_2)(v_1,v_2)\in E,\\ (w_1,w_2)\neq(u_1,u_2)}} (\mu_1 \underset{\beta}{\times} \mu_2)((w_1,w_2)(v_1,v_2))$$
$$-2(\mu_1 \underset{\beta}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$
$$= \sum_{u_1w_1\in E_1, u_2w_2\notin E_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{u_1w_1\notin E_1, u_2w_2\in E_2} \sigma_1(u_1) \wedge \sigma_1(w_1) \wedge \mu_2(u_2w_2)$$
$$+ \sum_{u_1w_1\in E_1, u_2w_2\in E_2} \mu_1(u_1w_1) \wedge \mu_2(u_2w_2) + \sum_{w_1v_1\in E_1, w_2v_2\notin E_2} \mu_1(w_1v_1) \wedge \sigma_2(w_2) \wedge \sigma_2(v_2)$$
$$+ \sum_{w_1v_1\notin E_1, w_2v_2\in E_2} \sigma_1(w_1) \wedge \sigma_1(v_1) \wedge \mu_2(w_2v_2) + \sum_{w_1v_1\in E_1, w_2v_2\in E_2} \mu_1(w_1v_1) \wedge \mu_2(w_2v_2) - 2(\mu_1 \underset{\beta}{\times} \mu_2)((u_1,u_2)(v_1,v_2)). \quad (3.1)$$

Theorem 3.1:

Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs.

1. If $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_1 \geq \mu_2$, then for any $(u_1,u_2)(v_1,v_2) \in E$,

(a). $d_{G_1 \underset{\beta}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{\overline{G}_2^*}(u_2) d_{G_1}(u_1) + d_{\overline{G}_2^*}(v_2) d_{G_1}(v_1) + (p_1 - 1)(d_{G_2}(u_2) + d_{G_2}(v_2))$ $- 2\mu_1(u_1v_1)$, when $u_1v_1 \in E_1, u_2v_2 \notin E_2$ and

(b). $d_{G_1 \underset{\beta}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{\overline{G}_2^*}(u_2) d_{G_1}(u_1) + d_{\overline{G}_2^*}(v_2) d_{G_1}(v_1) + (p_1 - 2)(d_{G_2}(u_2) + d_{G_2}(v_2))$ $+ d_{G_2}(u_2v_2)$, when $u_1v_1 \notin E_1, u_2v_2 \in E_2$ & $u_1v_1 \in E_1, u_2v_2 \in E_2$.

2. If $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_2 \geq \mu_1$, then for any $(u_1,u_2)(v_1,v_2) \in E$,

(a). $d_{G_1 \underset{\beta}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{\overline{G}_1^*}(u_1) d_{G_2}(u_2) + d_{\overline{G}_1^*}(v_1) d_{G_2}(v_2) + (p_2 - 2)(d_{G_1}(u_1) + d_{G_1}(v_1))$ $+ d_{G_1}(u_1v_1)$, when $u_1v_1 \in E_1, u_2v_2 \notin E_2$ & $u_1v_1 \in E_1, u_2v_2 \in E_2$ and

(b). $d_{G_1 \underset{\beta}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{\overline{G}_1^*}(u_1) d_{G_2}(u_2) + d_{\overline{G}_1^*}(v_1) d_{G_2}(v_2) + (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1))$ $- 2\mu_2(u_2v_2)$, when $u_1v_1 \notin E_1, u_2v_2 \in E_2$.

Proof:

1. We have $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_1 \geq \mu_2$.

From (3.1), for any $(u_1,u_2)(v_1,v_2) \in E$, $d_{G_1 \underset{\beta}{\times} G_2}((u_1,u_2)(v_1,v_2))$

$$= \sum_{u_1w_1\in E_1, u_2w_2\notin E_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{u_1w_1\notin E_1, u_2w_2\in E_2} \sigma_1(u_1) \wedge \sigma_1(w_1) \wedge \mu_2(u_2w_2)$$
$$+ \sum_{u_1w_1\in E_1, u_2w_2\in E_2} \mu_1(u_1w_1) \wedge \mu_2(u_2w_2) + \sum_{w_1v_1\in E_1, w_2v_2\notin E_2} \mu_1(w_1v_1) \wedge \sigma_2(w_2) \wedge \sigma_2(v_2)$$
$$+ \sum_{w_1v_1\notin E_1, w_2v_2\in E_2} \sigma_1(w_1) \wedge \sigma_1(v_1) \wedge \mu_2(w_2v_2) + \sum_{w_1v_1\in E_1, w_2v_2\in E_2} \mu_1(w_1v_1) \wedge \mu_2(w_2v_2) - 2(\mu_1 \underset{\beta}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$

$$= \sum_{u_1w_1\in E_1, u_2w_2\notin E_2} \mu_1(u_1w_1) + \sum_{u_1w_1\notin E_1, u_2w_2\in E_2} \mu_2(u_2w_2) + \sum_{u_1w_1\in E_1, u_2w_2\in E_2} \mu_2(u_2w_2) + \sum_{w_1v_1\in E_1, w_2v_2\notin E_2} \mu_1(w_1v_1) + \sum_{w_1v_1\notin E_1, w_2v_2\in E_2} \mu_2(w_2v_2)$$
$$+ \sum_{w_1v_1\in E_1, w_2v_2\in E_2} \mu_2(w_2v_2) - 2(\mu_1 \underset{\beta}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$
$$= d_{\overline{G}_2^*}(u_2) \sum_{u_1w_1\in E_1} \mu_1(u_1w_1) + d_{\overline{G}_1^*}(u_1) \sum_{u_2w_2\in E_2} \mu_2(u_2w_2) + d_{G_1^*}(u_1) \sum_{u_2w_2\in E_2} \mu_2(u_2w_2) + d_{\overline{G}_2^*}(v_2) \sum_{w_1v_1\in E_1} \mu_1(w_1v_1)$$
$$+ d_{\overline{G}_1^*}(v_1) \sum_{w_2v_2\in E_2} \mu_2(w_2v_2) + d_{G_1^*}(v_1) \sum_{w_2v_2\in E_2} \mu_2(w_2v_2) - 2(\mu_1 \underset{\beta}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$
$$= d_{\overline{G}_2^*}(u_2) d_{G_1}(u_1) + d_{\overline{G}_1^*}(u_1) d_{G_2}(u_2) + d_{G_1^*}(u_1) d_{G_2}(u_2) + d_{\overline{G}_2^*}(v_2) d_{G_1}(v_1)$$
$$+ d_{\overline{G}_1^*}(v_1) d_{G_2}(v_2) + d_{G_1^*}(v_1) d_{G_2}(v_2) - 2(\mu_1 \underset{\beta}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$
$$= d_{\overline{G}_2^*}(u_2) d_{G_1}(u_1) + d_{G_2}(u_2)(d_{\overline{G}_1^*}(u_1) + d_{G_1^*}(u_1)) + d_{\overline{G}_2^*}(v_2) d_{G_1}(v_1) + d_{G_2}(v_2)(d_{\overline{G}_1^*}(v_1) + d_{G_1^*}(v_1))$$
$$- 2(\mu_1 \underset{\beta}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$
$$= d_{\overline{G}_2^*}(u_2) d_{G_1}(u_1) + d_{G_2}(u_2)(p_1 - 1) + d_{\overline{G}_2^*}(v_2) d_{G_1}(v_1) + d_{G_2}(v_2)(p_1 - 1)$$
$$- 2(\mu_1 \underset{\beta}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$
$$\therefore d_{G_1 \underset{\beta}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{\overline{G}_2^*}(u_2) d_{G_1}(u_1) + d_{\overline{G}_2^*}(v_2) d_{G_1}(v_1) + (p_1 - 1)(d_{G_2}(u_2) + d_{G_2}(v_2))$$
$$- 2(\mu_1 \underset{\beta}{\times} \mu_2)((u_1,u_2)(v_1,v_2)).$$

Now, for any $(u_1,u_2)(v_1,v_2) \in E$, we have to consider three cases:

$u_1v_1 \in E_1, u_2v_2 \notin E_2$ (or) $u_1v_1 \notin E_1, u_2v_2 \in E_2$ (or) $u_1v_1 \in E_1, u_2v_2 \in E_2$.

(a). When $u_1v_1 \in E_1, u_2v_2 \notin E_2$,

$$(\mu_1 \underset{\beta}{\times} \mu_2)((u_1,u_2)(v_1,v_2)) = \mu_1(u_1v_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2)$$
$$= \mu_1(u_1v_1).$$
$$\therefore d_{G_1 \underset{\beta}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{\overline{G}_2^*}(u_2) d_{G_1}(u_1) + d_{\overline{G}_2^*}(v_2) d_{G_1}(v_1) + (p_1 - 1)(d_{G_2}(u_2) + d_{G_2}(v_2))$$
$$- 2\mu_1(u_1v_1).$$

(b). When $u_1v_1 \notin E_1, u_2v_2 \in E_2$,

$$(\mu_1 \underset{\beta}{\times} \mu_2)((u_1,u_2)(v_1,v_2)) = \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \mu_2(u_2v_2)$$
$$= \mu_2(u_2v_2).$$

When $u_1v_1 \in E_1, u_2v_2 \in E_2$,

$$(\mu_1 \underset{\beta}{\times} \mu_2)((u_1,u_2)(v_1,v_2)) = \mu_1(u_1v_1) \wedge \mu_2(u_2v_2)$$
$$= \mu_2(u_2v_2).$$
$$\therefore d_{G_1 \underset{\beta}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{\overline{G}_2^*}(u_2) d_{G_1}(u_1) + d_{\overline{G}_2^*}(v_2) d_{G_1}(v_1) + (p_1 - 1)(d_{G_2}(u_2) + d_{G_2}(v_2))$$
$$- 2\mu_2(u_2v_2).$$
$$\therefore d_{G_1 \underset{\beta}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{\overline{G}_2^*}(u_2) d_{G_1}(u_1) + d_{\overline{G}_2^*}(v_2) d_{G_1}(v_1) + (p_1 - 2)(d_{G_2}(u_2) + d_{G_2}(v_2))$$
$$+ d_{G_2}(u_2v_2).$$

2. Proof is similar to the proof of (1).

Theorem 3.2:

Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs.

1. If $\sigma_1 \le \mu_2$ and σ_1 is a constant function with $\sigma_1(u) = c_1$ for all $u \in V_1$, then for any $(u_1, u_2)(v_1, v_2) \in E$,

(a). $d_{G_1 \times_\beta G_2}((u_1,u_2)(v_1,v_2)) = (p_2 - 2)(d_{G_1}(u_1) + d_{G_1}(v_1)) + c_1(d_{\overline{G}_1^*}(u_1)d_{G_2^*}(u_2) + d_{\overline{G}_1^*}(v_1)d_{G_2^*}(v_2))$
$+ d_{G_1}(u_1v_1)$, when $u_1v_1 \in E_1, u_2v_2 \notin E_2$ & $u_1v_1 \in E_1, u_2v_2 \in E_2$ and

(b). $d_{G_1 \times_\beta G_2}((u_1,u_2)(v_1,v_2)) = (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1)) + c_1(d_{\overline{G}_1^*}(u_1)d_{G_2^*}(u_2))$
$+ d_{\overline{G}_1^*}(v_1)d_{G_2^*}(v_2) - 2$, when $u_1v_1 \notin E_1, u_2v_2 \in E_2$.

2. If $\sigma_2 \le \mu_1$ and σ_2 is a constant function with $\sigma_2(u) = c_2$ for all $u \in V_2$, then for any $(u_1, u_2)(v_1, v_2) \in E$,

(a). $d_{G_1 \times_\beta G_2}((u_1,u_2)(v_1,v_2)) = (p_1 - 1)(d_{G_2}(u_2) + d_{G_2}(v_2)) + c_2(d_{G_1^*}(u_1)d_{\overline{G}_2^*}(u_2))$
$+ d_{G_1^*}(v_1)d_{\overline{G}_2^*}(v_2) - 2$, when $u_1v_1 \in E_1, u_2v_2 \notin E_2$ and

(b). $d_{G_1 \times_\beta G_2}((u_1,u_2)(v_1,v_2)) = (p_1 - 2)(d_{G_2}(u_2) + d_{G_2}(v_2)) + c_2(d_{G_1^*}(u_1)d_{\overline{G}_2^*}(u_2) + d_{G_1^*}(v_1)d_{\overline{G}_2^*}(v_2))$
$+ d_{G_2}(u_2v_2)$, when $u_1v_1 \notin E_1, u_2v_2 \in E_2$ & $u_1v_1 \in E_1, u_2v_2 \in E_2$.

Proof:

1. We have $\sigma_1 \le \mu_2$. Then by theorem 1.4 and theorem 1.5, $\sigma_2 \ge \mu_1$ and $\mu_1 \le \mu_2$.

From (3.1), for any $(u_1, u_2)(v_1, v_2) \in E$, $d_{G_1 \times_\beta G_2}((u_1,u_2)(v_1,v_2))$

$$= \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \sigma_1(u_1) \wedge \sigma_1(w_1) \wedge \mu_2(u_2w_2)$$
$$+ \sum_{u_1w_1 \in E_1, u_2w_2 \in E_2} \mu_1(u_1w_1) \wedge \mu_2(u_2w_2) + \sum_{w_1v_1 \in E_1, w_2v_2 \notin E_2} \mu_1(w_1v_1) \wedge \sigma_2(w_2) \wedge \sigma_2(v_2)$$
$$+ \sum_{w_1v_1 \notin E_1, w_2v_2 \in E_2} \sigma_1(w_1) \wedge \sigma_1(v_1) \wedge \mu_2(w_2v_2) + \sum_{w_1v_1 \in E_1, w_2v_2 \in E_2} \mu_1(w_1v_1) \wedge \mu_2(w_2v_2) - 2(\mu_1 \times_\beta \mu_2)((u_1,u_2)(v_1,v_2))$$
$$= \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \sigma_1(u_1) \wedge \sigma_1(w_1) + \sum_{u_1w_1 \in E_1, u_2w_2 \in E_2} \mu_1(u_1w_1) + \sum_{w_1v_1 \in E_1, w_2v_2 \notin E_2} \mu_1(w_1v_1)$$
$$+ \sum_{w_1v_1 \notin E_1, w_2v_2 \in E_2} \sigma_1(w_1) \wedge \sigma_1(v_1) + \sum_{w_1v_1 \in E_1, w_2v_2 \in E_2} \mu_1(w_1v_1) - 2(\mu_1 \times_\beta \mu_2)((u_1,u_2)(v_1,v_2))$$
$$= d_{\overline{G}_2^*}(u_2) \sum_{u_1w_1 \in E_1} \mu_1(u_1w_1) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} c_1 + d_{G_2^*}(u_2) \sum_{u_1w_1 \in E_1} \mu_1(u_1w_1) + d_{\overline{G}_2^*}(v_2) \sum_{w_1v_1 \in E_1} \mu_1(w_1v_1) + \sum_{w_1v_1 \notin E_1, w_2v_2 \in E_2} c_1$$
$$+ d_{G_2^*}(v_2) \sum_{w_1v_1 \in E_1} \mu_1(w_1v_1) - 2(\mu_1 \times_\beta \mu_2)((u_1,u_2)(v_1,v_2))$$
$$= d_{\overline{G}_2^*}(u_2)d_{G_1}(u_1) + c_1 d_{\overline{G}_1^*}(u_1)d_{G_2^*}(u_2) + d_{G_2^*}(u_2)d_{G_1}(u_1) + d_{\overline{G}_2^*}(v_2)d_{G_1}(v_1) + c_1 d_{\overline{G}_1^*}(v_1)d_{G_2^*}(v_2)$$
$$+ d_{G_2^*}(v_2)d_{G_1}(v_1) - 2(\mu_1 \times_\beta \mu_2)((u_1,u_2)(v_1,v_2))$$
$$= d_{G_1}(u_1)(d_{\overline{G}_2^*}(u_2) + d_{G_2^*}(u_2)) + c_1 d_{\overline{G}_1^*}(u_1)d_{G_2^*}(u_2) + d_{G_1}(v_1)(d_{\overline{G}_2^*}(v_2) + d_{G_2^*}(v_2)) + c_1 d_{\overline{G}_1^*}(v_1)d_{G_2^*}(v_2)$$
$$- 2(\mu_1 \times_\beta \mu_2)((u_1,u_2)(v_1,v_2))$$
$$= d_{G_1}(u_1)(p_2 - 1) + d_{G_1}(v_1)(p_2 - 1) + c_1(d_{\overline{G}_1^*}(u_1)d_{G_2^*}(u_2) + d_{\overline{G}_1^*}(v_1)d_{G_2^*}(v_2))$$
$$- 2(\mu_1 \times_\beta \mu_2)((u_1,u_2)(v_1,v_2))$$
$$\therefore d_{G_1 \times_\beta G_2}((u_1,u_2)(v_1,v_2)) = (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1)) + c_1(d_{\overline{G}_1^*}(u_1)d_{G_2^*}(u_2) + d_{\overline{G}_1^*}(v_1)d_{G_2^*}(v_2))$$
$$- 2(\mu_1 \times_\beta \mu_2)((u_1,u_2)(v_1,v_2)).$$

Now, for any $(u_1, u_2)(v_1, v_2) \in E$, we have to consider three cases:

$u_1v_1 \in E_1, u_2v_2 \notin E_2$ (or) $u_1v_1 \notin E_1, u_2v_2 \in E_2$ (or) $u_1v_1 \in E_1, u_2v_2 \in E_2$.

(a). When $u_1v_1 \in E_1, u_2v_2 \notin E_2$,

$$(\mu_1 \underset{\beta}{\times} \mu_2)((u_1,u_2)(v_1,v_2)) = \mu_1(u_1v_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2)$$
$$= \mu_1(u_1v_1).$$

When $u_1v_1 \in E_1, u_2v_2 \in E_2$,

$$(\mu_1 \underset{\beta}{\times} \mu_2)((u_1,u_2)(v_1,v_2)) = \mu_1(u_1v_1) \wedge \mu_2(u_2v_2)$$
$$= \mu_1(u_1v_1).$$

$$\therefore d_{G_1 \underset{\beta}{\times} G_2}((u_1,u_2)(v_1,v_2)) = (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1)) + c_1(d_{\overline{G}_1^*}(u_1)d_{G_2^*}(u_2) + d_{\overline{G}_1^*}(v_1)d_{G_2^*}(v_2))$$
$$-2\mu_1(u_1v_1)$$

$$\therefore d_{G_1 \underset{\beta}{\times} G_2}((u_1,u_2)(v_1,v_2)) = (p_2 - 2)(d_{G_1}(u_1) + d_{G_1}(v_1)) + c_1(d_{\overline{G}_1^*}(u_1)d_{G_2^*}(u_2) + d_{\overline{G}_1^*}(v_1)d_{G_2^*}(v_2))$$
$$+ d_{G_1}(u_1v_1).$$

(b). When $u_1v_1 \notin E_1, u_2v_2 \in E_2$,

$$(\mu_1 \underset{\beta}{\times} \mu_2)((u_1,u_2)(v_1,v_2)) = \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \mu_2(u_2v_2)$$
$$= \sigma_1(u_1) \wedge \sigma_1(v_1)$$
$$= c_1.$$

$$\therefore d_{G_1 \underset{\beta}{\times} G_2}((u_1,u_2)(v_1,v_2)) = (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1)) + c_1(d_{\overline{G}_1^*}(u_1)d_{G_2^*}(u_2) + d_{\overline{G}_1^*}(v_1)d_{G_2^*}(v_2))$$
$$-2c_1.$$

$$\therefore d_{G_1 \underset{\beta}{\times} G_2}((u_1,u_2)(v_1,v_2)) = (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1)) + c_1(d_{\overline{G}_1^*}(u_1)d_{G_2^*}(u_2) + d_{\overline{G}_1^*}(v_1)d_{G_2^*}(v_2) - 2).$$

2. Proof is similar to the proof of (1).

IV. Degree of an Edge in Gamma Product

By definition, for any $((u_1,u_2)(v_1,v_2)) \in E$,

$$d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = \sum_{\substack{(u_1,u_2)(w_1,w_2) \in E, \\ (w_1,w_2) \neq (v_1,v_2)}} (\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(w_1,w_2)) + \sum_{\substack{(w_1,w_2)(v_1,u_2) \in E, \\ (w_1,w_2) \neq (u_1,u_2)}} (\mu_1 \underset{\gamma}{\times} \mu_2)((w_1,w_2)(v_1,v_2))$$
$$-2(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$
$$= \sum_{u_2w_2 \in E_2, u_1 = w_1} \sigma_1(u_1) \wedge \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1, u_2 = w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2)$$
$$+ \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \sigma_1(u_1) \wedge \sigma_1(w_1) \wedge \mu_2(u_2w_2)$$
$$+ \sum_{u_1w_1 \in E_1, u_2w_2 \in E_2} \mu_1(u_1w_1) \wedge \mu_2(u_2w_2) + \sum_{w_2v_2 \in E_2, w_1 = u_1} \sigma_1(v_1) \wedge \mu_2(w_2v_2) + \sum_{w_1v_1 \in E_1, w_2 = v_2} \mu_1(w_1v_1) \wedge \sigma_2(v_2)$$
$$+ \sum_{w_1v_1 \in E_1, w_2v_2 \notin E_2} \mu_1(w_1v_1) \wedge \sigma_2(w_2) \wedge \sigma_2(v_2) + \sum_{w_1v_1 \notin E_1, w_2v_2 \in E_2} \sigma_1(w_1) \wedge \sigma_1(v_1) \wedge \mu_2(w_2v_2)$$
$$+ \sum_{w_1v_1 \in E_1, w_2v_2 \in E_2} \mu_1(w_1v_1) \wedge \mu_2(w_2v_2) - 2(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2)) \quad \text{.......................................} \quad (4.1)$$

Theorem 4.1:

Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs.

1. If $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_1 \geq \mu_2$, then for any $(u_1,u_2)(v_1,v_2) \in E$,

(i). When $u_1 = v_1, u_2v_2 \in E_2$,

$d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(u_1,v_2)) = d_{G_2}(u_2v_2) + d_{G_1}(u_1)(d_{\overline{G_2^*}}(u_2) + d_{\overline{G_2^*}}(v_2) + 2) + (p_1 - 1)(d_{G_2}(u_2)$
$+ d_{G_2}(v_2))$,

(ii). When $u_2 = v_2, u_1v_1 \in E_1$,

$d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,u_2)) = d_{G_1}(u_1v_1) + 2p_1 d_{G_2}(u_2) + d_{\overline{G_2^*}}(u_2)(d_{G_1}(u_1) + d_{G_1}(v_1))$,

(iii). $d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1v_1) + d_{G_2}(u_2) + d_{G_2}(v_2) + d_{\overline{G_2^*}}(u_2)d_{G_1}(u_1) + d_{\overline{G_2^*}}(v_2)d_{G_1}(v_1)$
$+ (p_1 - 1)(d_{G_2}(u_2) + d_{G_2}(v_2))$, when $u_1v_1 \in E_1, u_2v_2 \notin E_2$ and

(iv). $d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1) + d_{G_1}(v_1) + d_{G_2}(u_2v_2) + d_{\overline{G_2^*}}(u_2)d_{G_1}(u_1) + d_{\overline{G_2^*}}(v_2)d_{G_1}(v_1)$
$+ (p_1 - 1)(d_{G_2}(u_2) + d_{G_2}(v_2))$, when $u_1v_1 \notin E_1, u_2v_2 \in E_2$ & $u_1v_1 \in E_1, u_2v_2 \in E_2$.

2. If $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_2 \geq \mu_1$, then for any $(u_1,u_2)(v_1,v_2) \in E$,

(i). When $u_1 = v_1, u_2v_2 \in E_2$,

$d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(u_1,v_2)) = d_{G_2}(u_2v_2) + 2p_2 d_{G_1}(u_1) + d_{\overline{G_1^*}}(u_1)(d_{G_2}(u_2) + d_{G_2}(v_2))$,

(ii). When $u_2 = v_2, u_1v_1 \in E_1$,

$d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,u_2)) = d_{G_1}(u_1v_1) + d_{G_2}(u_2)(d_{\overline{G_1^*}}(u_1) + d_{\overline{G_1^*}}(v_1) + 2) + (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1))$,

(iii). $d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1v_1) + d_{G_2}(u_2) + d_{G_2}(v_2) + d_{\overline{G_1^*}}(u_1)d_{G_2}(u_2) + d_{\overline{G_1^*}}(v_1)d_{G_2}(v_2)$
$+ (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1))$, when $u_1v_1 \in E_1, u_2v_2 \notin E_2$ & $u_1v_1 \in E_1, u_2v_2 \in E_2$ and

(iv). $d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1) + d_{G_1}(v_1) + d_{G_2}(u_2v_2) + d_{\overline{G_1^*}}(u_1)d_{G_2}(u_2) + d_{\overline{G_1^*}}(v_1)d_{G_2}(v_2)$
$+ (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1))$, when $u_1v_1 \notin E_1, u_2v_2 \in E_2$.

Proof:

1. We have $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_1 \geq \mu_2$.

(i). From (4.1), when $u_1 = v_1, u_2v_2 \in E_2$,

$$d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(u_1,v_2)) = \sum_{u_2w_2 \in E_2, u_1 = w_1} \sigma_1(u_1) \wedge \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1, u_2 = w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2)$$
$$+ \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \sigma_1(u_1) \wedge \sigma_1(w_1) \wedge \mu_2(u_2w_2)$$
$$+ \sum_{u_1w_1 \in E_1, u_2w_2 \in E_2} \mu_1(u_1w_1) \wedge \mu_2(u_2w_2) + \sum_{w_2v_2 \in E_2, w_1 = u_1} \sigma_1(u_1) \wedge \mu_2(w_2v_2) + \sum_{w_1u_1 \in E_1, w_2 = v_2} \mu_1(w_1u_1) \wedge \sigma_2(v_2)$$
$$+ \sum_{w_1u_1 \in E_1, w_2v_2 \notin E_2} \mu_1(w_1u_1) \wedge \sigma_2(w_2) \wedge \sigma_2(v_2) + \sum_{w_1u_1 \notin E_1, w_2v_2 \in E_2} \sigma_1(w_1) \wedge \sigma_1(u_1) \wedge \mu_2(w_2v_2)$$
$$+ \sum_{w_1u_1 \in E_1, w_2v_2 \in E_2} \mu_1(w_1u_1) \wedge \mu_2(w_2v_2) - 2(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(u_1,v_2))$$
$$= \sum_{u_2w_2 \in E_2} \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1} \mu_1(u_1w_1) + \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1, u_2w_2 \in E_2} \mu_2(u_2w_2)$$
$$+ \sum_{w_2v_2 \in E_2} \mu_2(w_2v_2) + \sum_{w_1u_1 \in E_1} \mu_1(w_1u_1) + \sum_{w_1u_1 \in E_1, w_2v_2 \notin E_2} \mu_1(w_1u_1) + \sum_{w_1u_1 \notin E_1, w_2v_2 \in E_2} \mu_2(w_2v_2) + \sum_{w_1u_1 \in E_1, w_2v_2 \in E_2} \mu_2(w_2v_2)$$
$$- 2(\sigma_1(u_1) \wedge \mu_2(u_2v_2))$$
$$= \sum_{u_2w_2 \in E_2} \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1} \mu_1(u_1w_1) + d_{\overline{G_2^*}}(u_2) \sum_{u_1w_1 \in E_1} \mu_1(u_1w_1) + d_{\overline{G_1^*}}(u_1) \sum_{u_2w_2 \in E_2} \mu_2(u_2w_2)$$
$$+ d_{G_1^*}(u_1) \sum_{u_2w_2 \in E_2} \mu_2(u_2w_2) + \sum_{w_2v_2 \in E_2} \mu_2(w_2v_2) + \sum_{w_1u_1 \in E_1} \mu_1(w_1u_1) + d_{\overline{G_2^*}}(v_2) \sum_{w_1u_1 \in E_1} \mu_1(w_1u_1)$$

$+ d_{\overline{G}_1^*}(u_1) \sum_{w_2v_2 \in E_2} \mu_2(w_2v_2) + d_{G_1^*}(u_1) \sum_{w_2v_2 \in E_2} \mu_2(w_2v_2) - 2\mu_2(u_2v_2)$

$= \sum_{u_2w_2 \in E_2} \mu_2(u_2w_2) + \sum_{w_2v_2 \in E_2} \mu_2(w_2v_2) - 2\mu_2(u_2v_2) + d_{G_1}(u_1) + d_{G_1}(u_1)d_{\overline{G}_2^*}(u_2) + d_{\overline{G}_1^*}(u_1)d_{G_2}(u_2)$

$+ d_{G_1^*}(u_1)d_{G_2}(u_2) + d_{G_1}(u_1) + d_{G_1}(u_1)d_{\overline{G}_2^*}(v_2) + d_{\overline{G}_1^*}(u_1)d_{G_2}(v_2) + d_{G_1^*}(u_1)d_{G_2}(v_2)$

$= d_{G_2}(u_2v_2) + d_{G_1}(u_1)(d_{\overline{G}_2^*}(u_2) + d_{\overline{G}_2^*}(v_2) + 2) + d_{\overline{G}_1^*}(u_1)(d_{G_2}(u_2) + d_{G_2}(v_2))$

$+ d_{G_1^*}(u_1)(d_{G_2}(u_2) + d_{G_2}(v_2))$

$= d_{G_2}(u_2v_2) + d_{G_1}(u_1)(d_{\overline{G}_2^*}(u_2) + d_{\overline{G}_2^*}(v_2) + 2) + (d_{\overline{G}_1^*}(u_1) + d_{G_1^*}(u_1))(d_{G_2}(u_2) + d_{G_2}(v_2))$

$\therefore d_{G_1 \times_\gamma G_2}((u_1,u_2)(u_1,v_2)) = d_{G_2}(u_2v_2) + d_{G_1}(u_1)(d_{\overline{G}_2^*}(u_2) + d_{\overline{G}_2^*}(v_2) + 2)$

$+ (p_1 - 1)(d_{G_2}(u_2) + d_{G_2}(v_2))$.

(ii). From (4.1), when $u_2 = v_2, u_1v_1 \in E_1$,

$d_{G_1 \times_\gamma G_2}((u_1,u_2)(v_1,u_2)) = \sum_{u_2w_2 \in E_2, u_1 = w_1} \sigma_1(u_1) \wedge \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1, u_2 = w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2)$

$+ \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \sigma_1(u_1) \wedge \sigma_1(w_1) \wedge \mu_2(u_2w_2)$

$+ \sum_{u_1w_1 \in E_1, u_2w_2 \in E_2} \mu_1(u_1w_1) \wedge \mu_2(u_2w_2) + \sum_{w_2u_2 \in E_2, w_1 = u_1} \sigma_1(v_1) \wedge \mu_2(w_2u_2) + \sum_{w_1v_1 \in E_1, w_2 = u_2} \mu_1(w_1v_1) \wedge \sigma_2(u_2)$

$+ \sum_{w_1v_1 \in E_1, w_2u_2 \notin E_2} \mu_1(w_1v_1) \wedge \sigma_2(w_2) \wedge \sigma_2(u_2) + \sum_{w_1v_1 \notin E_1, w_2u_2 \in E_2} \sigma_1(w_1) \wedge \sigma_1(v_1) \wedge \mu_2(w_2u_2)$

$+ \sum_{w_1v_1 \in E_1, w_2u_2 \in E_2} \mu_1(w_1v_1) \wedge \mu_2(w_2u_2) - 2(\mu_1 \times_\gamma \mu_2)((u_1,u_2)(v_1,u_2))$

$= \sum_{u_2w_2 \in E_2} \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1} \mu_1(u_1w_1) + \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1, u_2w_2 \in E_2} \mu_2(u_2w_2)$

$+ \sum_{w_2u_2 \in E_2} \mu_2(w_2u_2) + \sum_{w_1v_1 \in E_1} \mu_1(w_1v_1) + \sum_{w_1v_1 \in E_1, w_2u_2 \notin E_2} \mu_1(w_1v_1) + \sum_{w_1v_1 \notin E_1, w_2u_2 \in E_2} \mu_2(w_2u_2) + \sum_{w_1v_1 \in E_1, w_2u_2 \in E_2} \mu_2(w_2u_2)$

$- 2(\mu_1(u_1v_1) \wedge \sigma_2(u_2))$

$= \sum_{u_2w_2 \in E_2} \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1} \mu_1(u_1w_1) + d_{\overline{G}_2^*}(u_2) \sum_{u_1w_1 \in E_1} \mu_1(u_1w_1) + d_{\overline{G}_1^*}(u_1) \sum_{u_2w_2 \in E_2} \mu_2(u_2w_2)$

$+ d_{G_1^*}(u_1) \sum_{u_2w_2 \in E_2} \mu_2(u_2w_2) + \sum_{w_2u_2 \in E_2} \mu_2(w_2u_2) + \sum_{w_1v_1 \in E_1} \mu_1(w_1v_1) + d_{\overline{G}_2^*}(u_2) \sum_{w_1v_1 \in E_1} \mu_2(w_1v_1)$

$+ d_{\overline{G}_1^*}(v_1) \sum_{w_2u_2 \in E_2} \mu_2(w_2u_2) + d_{G_1^*}(v_1) \sum_{w_2u_2 \in E_2} \mu_2(w_2u_2) - 2\mu_1(u_1v_1)$

$= d_{G_2}(u_2) + d_{G_1}(u_1v_1) + d_{\overline{G}_2^*}(u_2)d_{G_1}(u_1) + d_{\overline{G}_1^*}(u_1)d_{G_2}(u_2) + d_{G_1^*}(u_1)d_{G_2}(u_2) + d_{G_2}(u_2)$

$+ d_{\overline{G}_2^*}(u_2)d_{G_1}(v_1) + d_{\overline{G}_1^*}(v_1)d_{G_2}(u_2) + d_{G_1^*}(v_1)d_{G_2}(u_2)$

$= d_{G_1}(u_1v_1) + 2d_{G_2}(u_2) + d_{G_2}(u_2)(d_{\overline{G}_1^*}(u_1) + d_{\overline{G}_1^*}(v_1) + d_{G_1^*}(u_1) + d_{G_1^*}(v_1))$

$+ d_{\overline{G}_2^*}(u_2)(d_{G_1}(u_1) + d_{G_1}(v_1))$

$= d_{G_1}(u_1v_1) + d_{G_2}(u_2)(d_{\overline{G}_1^*}(u_1) + d_{\overline{G}_1^*}(v_1) + d_{G_1^*}(u_1) + d_{G_1^*}(v_1) + 2) + d_{\overline{G}_2^*}(u_2)(d_{G_1}(u_1) + d_{G_1}(v_1))$

$= d_{G_1}(u_1v_1) + d_{G_2}(u_2)(p_1 - 1 + p_1 - 1 + 2) + d_{\overline{G}_2^*}(u_2)(d_{G_1}(u_1) + d_{G_1}(v_1))$

$\therefore d_{G_1 \times_\gamma G_2}((u_1,u_2)(v_1,u_2)) = d_{G_1}(u_1v_1) + 2p_1d_{G_2}(u_2) + d_{\overline{G}_2^*}(u_2)(d_{G_1}(u_1) + d_{G_1}(v_1))$.

From (4.1), for any $(u_1,u_2)(v_1,v_2) \in E$,

$$d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = \sum_{u_2w_2 \in E_2, u_1=w_1} \sigma_1(u_1) \wedge \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1, u_2=w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2)$$

$$+ \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \sigma_1(u_1) \wedge \sigma_1(w_1) \wedge \mu_2(u_2w_2)$$

$$+ \sum_{u_1w_1 \in E_1, u_2w_2 \in E_2} \mu_1(u_1w_1) \wedge \mu_2(u_2w_2) + \sum_{w_2v_2 \in E_2, w_1=u_1} \sigma_1(v_1) \wedge \mu_2(w_2v_2) + \sum_{w_1v_1 \in E_1, w_2=v_2} \mu_1(w_1v_1) \wedge \sigma_2(v_2)$$

$$+ \sum_{w_1v_1 \in E_1, w_2v_2 \notin E_2} \mu_1(w_1v_1) \wedge \sigma_2(w_2) \wedge \sigma_2(v_2) + \sum_{w_1v_1 \notin E_1, w_2v_2 \in E_2} \sigma_1(w_1) \wedge \sigma_1(v_1) \wedge \mu_2(w_2v_2)$$

$$+ \sum_{w_1v_1 \in E_1, w_2v_2 \in E_2} \mu_1(w_1v_1) \wedge \mu_2(w_2v_2) - 2(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$

$$= \sum_{u_2w_2 \in E_2} \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1} \mu_1(u_1w_1) + \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1, u_2w_2 \in E_2} \mu_2(u_2w_2)$$

$$+ \sum_{w_2v_2 \in E_2} \mu_2(w_2v_2) + \sum_{w_1v_1 \in E_1} \mu_1(w_1v_1) + \sum_{w_1v_1 \in E_1, w_2v_2 \notin E_2} \mu_1(w_1v_1) + \sum_{w_1v_1 \notin E_1, w_2v_2 \in E_2} \mu_2(w_2v_2) + \sum_{w_1v_1 \in E_1, w_2v_2 \in E_2} \mu_2(w_2v_2)$$

$$- 2(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$

$$= d_{G_2}(u_2) + d_{G_1}(u_1) + \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1, u_2w_2 \in E_2} \mu_2(u_2w_2) + d_{G_2}(v_2) + d_{G_1}(v_1)$$

$$+ \sum_{w_1v_1 \in E_1, w_2v_2 \notin E_2} \mu_1(w_1v_1) + \sum_{w_1v_1 \notin E_1, w_2v_2 \in E_2} \mu_2(w_2v_2) + \sum_{w_1v_1 \in E_1, w_2v_2 \in E_2} \mu_2(w_2v_2) - 2(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$

$$= d_{G_2}(u_2) + d_{G_2}(v_2) + d_{G_1}(u_1) + d_{G_1}(v_1) + d_{\overline{G_2^*}}(u_2) \sum_{w_1 \in V_1} \mu_1(u_1w_1) + d_{\overline{G_1^*}}(u_1) \sum_{w_2 \in V_2} \mu_2(u_2w_2)$$

$$+ d_{G_1^*}(u_1) \sum_{w_2 \in V_2} \mu_2(u_2w_2) + d_{\overline{G_2^*}}(v_2) \sum_{w_1 \in V_1} \mu_1(w_1v_1) + d_{\overline{G_1^*}}(v_1) \sum_{w_2 \in V_2} \mu_2(w_2v_2) + d_{G_1^*}(v_1) \sum_{w_2 \in V_2} \mu_2(w_2v_2)$$

$$- 2(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$

$$= d_{G_1}(u_1) + d_{G_1}(v_1) + d_{G_2}(u_2) + d_{G_2}(v_2) + d_{\overline{G_2^*}}(u_2) d_{G_1}(u_1) + d_{\overline{G_1^*}}(u_1) d_{G_2}(u_2) + d_{G_1^*}(u_1) d_{G_2}(u_2)$$

$$+ d_{\overline{G_2^*}}(v_2) d_{G_1}(v_1) + d_{\overline{G_1^*}}(v_1) d_{G_2}(v_2) + d_{G_1^*}(v_1) d_{G_2}(v_2) - 2(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$

$$= d_{G_1}(u_1) + d_{G_1}(v_1) + d_{G_2}(u_2) + d_{G_2}(v_2) + d_{\overline{G_2^*}}(u_2) d_{G_1}(u_1) + d_{G_2}(u_2)(d_{\overline{G_1^*}}(u_1) + d_{G_1^*}(u_1))$$

$$+ d_{\overline{G_2^*}}(v_2) d_{G_1}(v_1) + d_{G_2}(v_2)(d_{\overline{G_1^*}}(v_1) + d_{G_1^*}(v_1)) - 2(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$

$$= d_{G_1}(u_1) + d_{G_1}(v_1) + d_{G_2}(u_2) + d_{G_2}(v_2) + d_{\overline{G_2^*}}(u_2) d_{G_1}(u_1) + d_{G_2}(u_2)(p_1 - 1)$$

$$+ d_{\overline{G_2^*}}(v_2) d_{G_1}(v_1) + d_{G_2}(v_2)(p_1 - 1) - 2(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$

$$\therefore d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1) + d_{G_1}(v_1) + d_{\overline{G_2^*}}(u_2) d_{G_1}(u_1) + d_{\overline{G_2^*}}(v_2) d_{G_1}(v_1)$$

$$+ p_1(d_{G_2}(u_2) + d_{G_2}(v_2)) - 2(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2)).$$

(iii). When $u_1v_1 \in E_1, u_2v_2 \notin E_2$,

$$(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2)) = \mu_1(u_1v_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2)$$

$$= \mu_1(u_1v_1).$$

$$\therefore d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1) + d_{G_1}(v_1) + d_{\overline{G_2^*}}(u_2) d_{G_1}(u_1) + d_{\overline{G_2^*}}(v_2) d_{G_1}(v_1)$$

$$+ p_1(d_{G_2}(u_2) + d_{G_2}(v_2)) - 2\mu_1(u_1v_1).$$

$$\therefore d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1v_1) + d_{\overline{G_2^*}}(u_2) d_{G_1}(u_1) + d_{\overline{G_2^*}}(v_2) d_{G_1}(v_1) + p_1(d_{G_2}(u_2) + d_{G_2}(v_2)).$$

(iv). When $u_1v_1 \notin E_1, u_2v_2 \in E_2$,

$(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2)) = \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \mu_2(u_2v_2)$

$= \mu_2(u_2v_2)$.

When $u_1v_1 \in E_1, u_2v_2 \in E_2$,

$(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2)) = \mu_1(u_1v_1) \wedge \mu_2(u_2v_2)$

$= \mu_2(u_2v_2)$.

$\therefore d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1) + d_{G_1}(v_1) + d_{\overline{G_2^*}}(u_2)d_{G_1}(u_1) + d_{\overline{G_2^*}}(v_2)d_{G_1}(v_1)$
$+ p_1(d_{G_2}(u_2) + d_{G_2}(v_2)) - 2\mu_2(u_2v_2)$.

$\therefore d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1) + d_{G_1}(v_1) + d_{G_2}(u_2v_2) + d_{\overline{G_2^*}}(u_2)d_{G_1}(u_1) + d_{\overline{G_2^*}}(v_2)d_{G_1}(v_1)$
$+ (p_1 - 1)(d_{G_2}(u_2) + d_{G_2}(v_2))$.

2. Proof is similar to the proof of (1).

Theorem 4.2:

Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs.

1. If $\sigma_1 \le \mu_2$ and σ_1 is a constant function with $\sigma_1(u) = c_1$ for all $u \in V_1$, then for any $(u_1,u_2)(v_1,v_2) \in E$,

(i). When $u_1 = v_1, u_2v_2 \in E_2$,

$d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(u_1,v_2)) = 2p_2 d_{G_1}(u_1) + c_1(d_{G_2^*}(u_2) + d_{G_2^*}(v_2))(d_{\overline{G_1^*}}(u_1) + 1) - 2c_1$,

(ii). When $u_2 = v_2, u_1v_1 \in E_1$,

$d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,u_2)) = d_{G_1}(u_1v_1) + c_1 d_{G_2^*}(u_2)(2 + d_{\overline{G_1^*}}(u_1) + d_{\overline{G_1^*}}(v_1))$
$+ (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1))$,

(iii). $d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1v_1) + (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1)) + c_1 d_{G_2^*}(u_2)(1 + d_{\overline{G_1^*}}(u_1))$
$+ c_1 d_{G_2^*}(v_2)(1 + d_{\overline{G_1^*}}(v_1))$, when $u_1v_1 \in E_1, u_2v_2 \notin E_2$ & $u_1v_1 \in E_1, u_2v_2 \in E_2$ and

(iv). $d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = p_2(d_{G_1}(u_1) + d_{G_1}(v_1)) + c_1 d_{G_2^*}(u_2)(1 + d_{\overline{G_1^*}}(u_1))$
$+ c_1 d_{G_2^*}(v_2)(1 + d_{\overline{G_1^*}}(v_1)) - 2c_1$, when $u_1v_1 \notin E_1, u_2v_2 \in E_2$.

2. If $\sigma_2 \le \mu_1$ and σ_2 is a constant function with $\sigma_2(u) = c_2$ for all $u \in V_2$, then for any $(u_1,u_2)(v_1,v_2) \in E$,

(i). When $u_1 = v_1, u_2v_2 \in E_2$,

$d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(u_1,v_2)) = d_{G_2}(u_2v_2) + c_2 d_{G_1^*}(u_1)(2 + d_{\overline{G_2^*}}(u_2) + d_{\overline{G_2^*}}(v_2))$
$+ (p_1 - 1)(d_{G_2}(u_2) + d_{G_2}(v_2))$,

(ii). When $u_2 = v_2, u_1v_1 \in E_1$,

$d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,u_2)) = 2p_1 d_{G_2}(u_2) + c_2(d_{G_1^*}(u_1) + d_{G_1^*}(v_1))(d_{\overline{G_2^*}}(u_2) + 1) - 2c_2$,

(iii). $d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = p_1(d_{G_2}(u_2) + d_{G_2}(v_2)) + c_2 d_{G_1^*}(u_1)(1 + d_{\overline{G_2^*}}(u_2))$
$+ c_2 d_{G_1^*}(v_1)(1 + d_{\overline{G_2^*}}(v_2)) - 2c_2$, when $u_1v_1 \in E_1, u_2v_2 \notin E_2$ and

(iv). $d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_2}(u_2v_2) + (p_1 - 1)(d_{G_2}(u_2) + d_{G_2}(v_2)) + c_2 d_{G_1^*}(u_1)(1 + d_{\overline{G_2^*}}(u_2))$
$+ c_2 d_{G_1^*}(v_1)(1 + d_{\overline{G_2^*}}(v_2))$, when $u_1v_1 \notin E_1, u_2v_2 \in E_2$ & $u_1v_1 \in E_1, u_2v_2 \in E_2$.

Proof:

1. We have $\sigma_1 \leq \mu_2$. Then by theorem 1.4 and theorem 1.5, $\sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$.

(i). From (4.1), when $u_1 = v_1, u_2v_2 \in E_2$,

$$d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(u_1,v_2)) = \sum_{u_2w_2 \in E_2, u_1=w_1} \sigma_1(u_1) \wedge \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1, u_2=w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2)$$
$$+ \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \sigma_1(u_1) \wedge \sigma_1(w_1) \wedge \mu_2(u_2w_2)$$
$$+ \sum_{u_1w_1 \in E_1, u_2w_2 \in E_2} \mu_1(u_1w_1) \wedge \mu_2(u_2w_2) + \sum_{w_2v_2 \in E_2, w_1=u_1} \sigma_1(u_1) \wedge \mu_2(w_2v_2) + \sum_{w_1u_1 \in E_1, w_2=v_2} \mu_1(w_1u_1) \wedge \sigma_2(v_2)$$
$$+ \sum_{w_1u_1 \in E_1, w_2v_2 \notin E_2} \mu_1(w_1u_1) \wedge \sigma_2(w_2) \wedge \sigma_2(v_2) + \sum_{w_1u_1 \notin E_1, w_2v_2 \in E_2} \sigma_1(w_1) \wedge \sigma_1(u_1) \wedge \mu_2(w_2v_2)$$
$$+ \sum_{w_1u_1 \in E_1, w_2v_2 \in E_2} \mu_1(w_1u_1) \wedge \mu_2(w_2v_2) - 2(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(u_1,v_2))$$
$$= \sum_{u_2w_2 \in E_2} \sigma_1(u_1) + \sum_{u_1w_1 \in E_1} \mu_1(u_1w_1) + \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \sigma_1(u_1) \wedge \sigma_1(w_1) + \sum_{u_1w_1 \in E_1, u_2w_2 \in E_2} \mu_1(u_1w_1)$$
$$+ \sum_{w_2v_2 \in E_2} \sigma_1(u_1) + \sum_{w_1u_1 \in E_1} \mu_1(w_1u_1) + \sum_{w_1u_1 \in E_1, w_2v_2 \notin E_2} \mu_1(w_1u_1) + \sum_{w_1u_1 \notin E_1, w_2v_2 \in E_2} \sigma_1(w_1) \wedge \sigma_1(u_1) + \sum_{w_1u_1 \in E_1, w_2v_2 \in E_2} \mu_1(w_1u_1)$$
$$-2(\sigma_1(u_1) \wedge \mu_2(u_2v_2))$$
$$= c_1 d_{G_2^*}(u_2) + d_{G_1}(u_1) + d_{\overline{G}_2^*}(u_2) \sum_{u_1w_1 \in E_1} \mu_1(u_1w_1) + d_{\overline{G}_1^*}(u_1) d_{G_2^*}(u_2) c_1 + d_{G_2^*}(u_2) \sum_{u_1w_1 \in E_1} \mu_1(u_1w_1)$$
$$+ c_1 d_{G_2^*}(v_2) + d_{G_1}(u_1) + d_{\overline{G}_2^*}(v_2) \sum_{w_1u_1 \in E_1} \mu_1(w_1u_1) + d_{\overline{G}_1^*}(u_1) d_{G_2^*}(v_2) c_1 + d_{G_2^*}(v_2) \sum_{w_1u_1 \in E_1} \mu_1(w_1u_1)$$
$$-2\sigma_1(u_1)$$
$$= c_1 d_{G_2^*}(u_2) + d_{G_1}(u_1) + d_{\overline{G}_2^*}(u_2) d_{G_1}(u_1) + d_{\overline{G}_1^*}(u_1) d_{G_2^*}(u_2) c_1 + d_{G_2^*}(u_2) d_{G_1}(u_1) + c_1 d_{G_2^*}(v_2)$$
$$+ d_{G_1}(u_1) + d_{\overline{G}_2^*}(v_2) d_{G_1}(u_1) + d_{\overline{G}_1^*}(u_1) d_{G_2^*}(v_2) c_1 + d_{G_2^*}(v_2) d_{G_1}(u_1) - 2c_1$$
$$= c_1(d_{G_2^*}(u_2) + d_{G_2^*}(v_2) - 2) + 2d_{G_1}(u_1) + d_{G_1}(u_1)(d_{G_2^*}(u_2) + d_{G_2^*}(v_2) + d_{\overline{G}_2^*}(u_2) + d_{\overline{G}_2^*}(v_2))$$
$$+ c_1 d_{\overline{G}_1^*}(u_1)(d_{G_2^*}(u_2) + d_{G_2^*}(v_2))$$
$$= c_1(d_{G_2^*}(u_2) + d_{G_2^*}(v_2))(1 + d_{\overline{G}_1^*}(u_1)) - 2c_1 + d_{G_1}(u_1)(p_2 - 1 + p_2 - 1 + 2)$$
$$\therefore d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(u_1,v_2)) = 2p_2 d_{G_1}(u_1) + c_1(d_{G_2^*}(u_2) + d_{G_2^*}(v_2))(d_{\overline{G}_1^*}(u_1) + 1) - 2c_1.$$

(ii). From (4.1), when $u_2 = v_2, u_1v_1 \in E_1$,

$$d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,u_2)) = \sum_{u_2w_2 \in E_2, u_1=w_1} \sigma_1(u_1) \wedge \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1, u_2=w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2)$$
$$+ \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \sigma_1(u_1) \wedge \sigma_1(w_1) \wedge \mu_2(u_2w_2)$$
$$+ \sum_{u_1w_1 \in E_1, u_2w_2 \in E_2} \mu_1(u_1w_1) \wedge \mu_2(u_2w_2) + \sum_{w_2u_2 \in E_2, w_1=u_1} \sigma_1(v_1) \wedge \mu_2(w_2u_2) + \sum_{w_1v_1 \in E_1, w_2=u_2} \mu_1(w_1v_1) \wedge \sigma_2(u_2)$$
$$+ \sum_{w_1v_1 \in E_1, w_2u_2 \notin E_2} \mu_1(w_1v_1) \wedge \sigma_2(w_2) \wedge \sigma_2(u_2) + \sum_{w_1v_1 \notin E_1, w_2u_2 \in E_2} \sigma_1(w_1) \wedge \sigma_1(v_1) \wedge \mu_2(w_2u_2)$$
$$+ \sum_{w_1v_1 \in E_1, w_2u_2 \in E_2} \mu_1(w_1v_1) \wedge \mu_2(w_2u_2) - 2(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,u_2))$$
$$= \sum_{u_2w_2 \in E_{21}} \sigma_1(u_1) + \sum_{u_1w_1 \in E_1} \mu_1(u_1w_1) + \sum_{u_1w_1 \in E_1, u_2w_2 \notin E_2} \mu_1(u_1w_1) + \sum_{u_1w_1 \notin E_1, u_2w_2 \in E_2} \sigma_1(u_1) \wedge \sigma_1(w_1) + \sum_{u_1w_1 \in E_1, w_2u_2 \in E_2} \mu_1(u_1w_1)$$

$+\sum_{w_2u_2\in E_2}\sigma_1(v_1)+\sum_{w_1v_1\in E_1}\mu_1(w_1v_1)+\sum_{w_1v_1\in E_1,w_2u_2\notin E_2}\mu_1(w_1v_1)+\sum_{w_1v_1\notin E_1,w_2u_2\in E_2}\sigma_1(w_1)\wedge\sigma_1(v_1)+\sum_{w_1v_1\in E_1,w_2u_2\in E_2}\mu_1(w_1v_1)$

$-2(\mu_1(u_1v_1)\wedge\sigma_2(u_2))$

$=c_1d_{G_2^*}(u_2)+\sum_{u_1w_1\in E_1}\mu_1(u_1w_1)+\sum_{w_1v_1\in E_1}\mu_1(w_1v_1)+d_{\overline{G}_2^*}(u_2)\sum_{u_1w_1\in E_1}\mu_1(u_1w_1)+\sum_{u_1w_1\notin E_1,u_2w_2\in E_2}c_1$

$+d_{G_2^*}(u_2)\sum_{u_1w_1\in E_1}\mu_1(u_1w_1)+c_1d_{G_2^*}(u_2)+d_{\overline{G}_2^*}(u_2)\sum_{w_1v_1\in E_1}\mu_1(w_1v_1)+\sum_{w_1v_1\notin E_1,w_2u_2\in E_2}c_1+d_{G_2^*}(u_2)\sum_{w_1v_1\in E_1}\mu_1(w_1v_1)$

$-2\mu_1(u_1v_1)$

$=2c_1d_{G_2^*}(u_2)+d_{G_1}(u_1v_1)+d_{\overline{G}_2^*}(u_2)d_{G_1}(u_1)+c_1d_{\overline{G}_1^*}(u_1)d_{G_2^*}(u_2)+d_{G_2^*}(u_2)d_{G_1}(u_1)$

$+d_{\overline{G}_2^*}(u_2)d_{G_1}(v_1)+c_1d_{\overline{G}_1^*}(v_1)d_{G_2^*}(u_2)+d_{G_2^*}(u_2)d_{G_1}(v_1)$

$=d_{G_1}(u_1v_1)+c_1d_{G_2^*}(u_2)(2+d_{\overline{G}_1^*}(u_1)+d_{\overline{G}_1^*}(v_1))+d_{G_1}(u_1)(d_{\overline{G}_2^*}(u_2)+d_{G_2^*}(u_2))$

$+d_{G_1}(v_1)(d_{\overline{G}_2^*}(u_2)+d_{G_2^*}(u_2))$

$\therefore d_{G_1\times_\gamma G_2}((u_1,u_2)(v_1,u_2))=d_{G_1}(u_1v_1)+c_1d_{G_2^*}(u_2)(2+d_{\overline{G}_1^*}(u_1)+d_{\overline{G}_1^*}(v_1))$

$\qquad\qquad +(p_2-1)(d_{G_1}(u_1)+d_{G_1}(v_1))$.

From (4.1), for any $(u_1,u_2)(v_1,v_2)\in E$,

$d_{G_1\times_\gamma G_2}((u_1,u_2)(v_1,v_2))=\sum_{u_2w_2\in E_2,u_1=w_1}\sigma_1(u_1)\wedge\mu_2(u_2w_2)+\sum_{u_1w_1\in E_1,u_2=w_2}\mu_1(u_1w_1)\wedge\sigma_2(u_2)$

$+\sum_{u_1w_1\in E_1,u_2w_2\notin E_2}\mu_1(u_1w_1)\wedge\sigma_2(u_2)\wedge\sigma_2(w_2)+\sum_{u_1w_1\notin E_1,u_2w_2\in E_2}\sigma_1(u_1)\wedge\sigma_1(w_1)\wedge\mu_2(u_2w_2)$

$+\sum_{u_1w_1\in E_1,u_2w_2\in E_2}\mu_1(u_1w_1)\wedge\mu_2(u_2w_2)+\sum_{w_2v_2\in E_2,w_1=u_1}\sigma_1(v_1)\wedge\mu_2(w_2v_2)+\sum_{w_1v_1\in E_1,w_2=v_2}\mu_1(w_1v_1)\wedge\sigma_2(v_2)$

$+\sum_{w_1v_1\in E_1,w_2v_2\notin E_2}\mu_1(w_1v_1)\wedge\sigma_2(w_2)\wedge\sigma_2(v_2)+\sum_{w_1v_1\notin E_1,w_2v_2\in E_2}\sigma_1(w_1)\wedge\sigma_1(v_1)\wedge\mu_2(w_2v_2)$

$+\sum_{w_1v_1\in E_1,w_2v_2\in E_2}\mu_1(w_1v_1)\wedge\mu_2(w_2v_2)-2(\mu_1\times_\gamma\mu_2)((u_1,u_2)(v_1,v_2))$

$=\sum_{u_2w_2\in E_2}\sigma_1(u_1)+\sum_{u_1w_1\in E_1}\mu_1(u_1w_1)+\sum_{u_1w_1\in E_1,u_2w_2\notin E_2}\mu_1(u_1w_1)+\sum_{u_1w_1\notin E_1,u_2w_2\in E_2}\sigma_1(u_1)\wedge\sigma_1(w_1)+\sum_{u_1w_1\in E_1,u_2w_2\in E_2}\mu_1(u_1w_1)$

$+\sum_{w_2v_2\in E_2}\sigma_1(v_1)+\sum_{w_1v_1\in E_1}\mu_1(w_1v_1)+\sum_{w_1v_1\in E_1,w_2v_2\notin E_2}\mu_1(w_1v_1)+\sum_{w_1v_1\notin E_1,w_2v_2\in E_2}\sigma_1(w_1)\wedge\sigma_1(v_1)+\sum_{w_1v_1\in E_1,w_2v_2\in E_2}\mu_1(w_1v_1)$

$-2(\mu_1\times_\gamma\mu_2)((u_1,u_2)(v_1,v_2))$

$=c_1d_{G_2^*}(u_2)+d_{G_1}(u_1)+d_{\overline{G}_2^*}(u_2)\sum_{u_1w_1\in E_1}\mu_1(u_1w_1)+\sum_{u_1w_1\notin E_1,u_2w_2\in E_2}c_1+d_{G_2^*}(u_2)\sum_{u_1w_1\in E_1}\mu_1(u_1w_1)+c_1d_{G_2^*}(v_2)$

$+d_{G_1}(v_1)+d_{\overline{G}_2^*}(v_2)\sum_{w_1v_1\in E_1}\mu_1(w_1v_1)+\sum_{w_1v_1\notin E_1,w_2v_2\in E_2}c_1+d_{G_2^*}(v_2)\sum_{w_1v_1\in E_1}\mu_1(w_1v_1)$

$-2(\mu_1\times_\gamma\mu_2)((u_1,u_2)(v_1,v_2))$

$=c_1(d_{G_2^*}(u_2)+d_{G_2^*}(v_2))+d_{G_1}(u_1)+d_{G_1}(v_1)+d_{\overline{G}_2^*}(u_2)d_{G_1}(u_1)+c_1d_{\overline{G}_1^*}(u_1)d_{G_2^*}(u_2)+d_{G_2^*}(u_2)d_{G_1}(u_1)$

$+d_{\overline{G}_2^*}(v_2)d_{G_1}(v_1)+c_1d_{\overline{G}_1^*}(v_1)d_{G_2^*}(v_2)+d_{G_2^*}(v_2)d_{G_1}(v_1)-2(\mu_1\times_\gamma\mu_2)((u_1,u_2)(v_1,v_2))$

$=d_{G_1}(u_1)+d_{G_1}(v_1)+c_1(d_{G_2^*}(u_2)+d_{G_2^*}(v_2))+d_{G_1}(u_1)(d_{\overline{G}_2^*}(u_2)+d_{G_2^*}(u_2))+c_1d_{\overline{G}_1^*}(u_1)d_{G_2^*}(u_2)$

$+d_{G_1}(v_1)(d_{\overline{G}_2^*}(v_2)+d_{G_2^*}(v_2))+c_1d_{\overline{G}_1^*}(v_1)d_{G_2^*}(v_2)-2(\mu_1\times_\gamma\mu_2)((u_1,u_2)(v_1,v_2))$

$=d_{G_1}(u_1)+d_{G_1}(v_1)+d_{G_1}(u_1)(p_2-1)+c_1d_{G_2^*}(u_2)(1+d_{\overline{G}_1^*}(u_1))+d_{G_1}(v_1)(p_2-1)$

$$+c_1 d_{G_2^*}(v_2)(1+d_{\overline{G}_1^*}(v_1)) - 2(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2))$$

$$\therefore d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = p_2(d_{G_1}(u_1)+d_{G_1}(v_1)) + c_1 d_{G_2^*}(u_2)(1+d_{\overline{G}_1^*}(u_1))$$
$$+c_1 d_{G_2^*}(v_2)(1+d_{\overline{G}_1^*}(v_1)) - 2(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2)).$$

(iii). When $u_1v_1 \in E_1, u_2v_2 \notin E_2$,

$$(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2)) = \mu_1(u_1v_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2)$$
$$= \mu_1(u_1v_1).$$

When $u_1v_1 \in E_1, u_2v_2 \in E_2$,

$$(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2)) = \mu_1(u_1v_1) \wedge \mu_2(u_2v_2)$$
$$= \mu_1(u_1v_1).$$

$$\therefore d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = p_2(d_{G_1}(u_1)+d_{G_1}(v_1)) + c_1 d_{G_2^*}(u_2)(1+d_{\overline{G}_1^*}(u_1))$$
$$+c_1 d_{G_2^*}(v_2)(1+d_{\overline{G}_1^*}(v_1)) - 2\mu_1(u_1v_1)$$

$$\therefore d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1v_1) + (p_2-1)(d_{G_1}(u_1)+d_{G_1}(v_1)) + c_1 d_{G_2^*}(u_2)(1+d_{\overline{G}_1^*}(u_1))$$
$$+c_1 d_{G_2^*}(v_2)(1+d_{\overline{G}_1^*}(v_1)).$$

(iv). When $u_1v_1 \notin E_1, u_2v_2 \in E_2$,

$$(\mu_1 \underset{\gamma}{\times} \mu_2)((u_1,u_2)(v_1,v_2)) = \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \mu_2(u_2v_2)$$
$$= \sigma_1(u_1) \wedge \sigma_1(v_1).$$

$$\therefore d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = p_2(d_{G_1}(u_1)+d_{G_1}(v_1)) + c_1 d_{G_2^*}(u_2)(1+d_{\overline{G}_1^*}(u_1))$$
$$+c_1 d_{G_2^*}(v_2)(1+d_{\overline{G}_1^*}(v_1)) - 2(\sigma_1(u_1) \wedge \sigma_1(v_1))$$

$$\therefore d_{G_1 \underset{\gamma}{\times} G_2}((u_1,u_2)(v_1,v_2)) = p_2(d_{G_1}(u_1)+d_{G_1}(v_1)) + c_1 d_{G_2^*}(u_2)(1+d_{\overline{G}_1^*}(u_1)) + c_1 d_{G_2^*}(v_2)(1+d_{\overline{G}_1^*}(v_1))$$
$$-2c_1.$$

2. Proof is similar to the proof of (1).

Remark 4.3:

When both σ = 1 and μ = 1, all the above formulae coincide with the corresponding formulae of crisp graphs.

V. Conclusion

In this paper, we have found the degree of edges in $G_1 \underset{\alpha}{\times} G_2, G_1 \underset{\beta}{\times} G_2$ and $G_1 \underset{\gamma}{\times} G_2$ in terms of the degree of vertices and edges in G_1 and G_2 and also in terms of the degree of vertices in G_1^* and G_2^* under some conditions. They will we more helpful especially when the graphs are very large. Also they will be useful in studying various conditions, properties of alpha product, beta product and gamma product of two fuzzy graphs.

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