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Π Generalized Semi Connectedness in Intuitionistic Fuzzy Topological Spaces

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Abstract: *The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy π generalized semi connectedness in intuitionistic fuzzy topological space. Some of their properties are explored.*
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I. Introduction

The concept of fuzzy set was introduced by Zadeh in his classical paper [10] in1965. Using the concept of fuzzy sets, Chang [2] introduced the concept of fuzzy topological space. In [1], Atanassov introduced the notion of intuitionistic fuzzy sets in 1986. Using the notion of intuitionistic fuzzy sets, Coker [3] defined the notion of intuitionistic fuzzy topological spaces in 1997. Turnali and Coke have introduced and investigated connectedness in intuitionistic fuzzy topological spaces in the year 2000. Later intuitionistic fuzzy rg-connectedness was introduced by Thakur and Rekka Chaturvedi in 2006. Recently many fuzzy topological concepts such as fuzzy connectedness have been generalized in intuitionistic fuzzy topological spaces. In this paper we have introduced intuitionistic fuzzy π generalized connectedness in fuzzy topological spaces. Also we have provided some characterizations of intuitionistic fuzzy π generalized semi connectedness.

II. Preliminaries

Definition 2.1: [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ where the functions $\mu_A(x): X \to [0, 1]$ and $\nu_A(x): X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x\in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}$. Then
(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in X\}$
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on a non empty X is a family τ of IFSs in X satisfying the following axioms:
(a) $0_\sim, 1_\sim \in \tau$
(b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i\in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [3] Let (X, τ) be an IFTS and A = ⟨ x, μ_A, ν_A ⟩ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by
int(A) = ∪ { G / G is an IFOS in X and G ⊆ A }
cl(A) = ∩ { K / K is an IFCS in X and A ⊆ K }.
Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$ [14].

Definition 2.5: An IFS A = {⟨ x, μ_A (x), ν_A(x) ⟩ / x ∈ X} in an IFTS (X, τ) is said to be an
(a) [4] intuitionistic fuzzy semi closed set (IFSCS in short) if int(cl (A)) ⊆ A
(b) [4] intuitionistic fuzzy α-closed set (IFαCS in short) if cl(int(cl(A))) ⊆ A
(c) [4] intuitionistic fuzzy pre-closed set (IFPCS in short) if cl(int(A)) ⊆ A
(d) [4] intuitionistic fuzzy regular closed set (IFRCS in short) if cl(int(A)) = A
(e) [9] intuitionistic fuzzy generalized closed set (IFGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS
(f) [5] intuitionistic fuzzy generalized semi closed set (IFπGSCS in short) if scl(A) ⊆ U , whenever A ⊆ U and U is an IFπOS .

Definition 2.6: [5] An IFS A is said to be an intuitionistic fuzzy π - generalized semi open set (IFπGSOS in short) in X if the complement A^c is an IFπGSCS in X.The family of all IFπGSCSs of an IFTS (X, τ) is denoted by IFπGSC(X).

Result 2.7:[5] Every IFCS, IFαCS, IFGCS, IFRCS, IFPCS, IFαGCS is an IFπGSCS but the converses need not be true in general.

Definition 2.8: [6] Let A be an IFS in an IFTS (X, τ). Then π-generalized Semi closure of A (πgscl(A) in short) and π-generalized Semi interior of A (πgsint(A) in short) are defined by
πgsint(A) = ∪ { G / G is an IFπGSOS in X and G ⊆ A }
πgscl(A) = ∩ { K / K is an IFπGSCS in X and A ⊆ K }.

Definition 2.10: [3] Let f be a mapping from an IFS X to an IFS Y. If B = {⟨y, μ_B(y), ν_B(y)⟩ / y∈Y}is an IFS in Y, then the pre-image of B under f denoted by f^{-1}(B), is the IFS in X defined by f^{-1}(B) = {⟨ x, f^{-1}(μ_B(x)), f^{-1} (ν_B(x)) ⟩ / x ∈ X}.
If A = {⟨ x, λ_A(x), ν_A(x) ⟩ / x ∈ X} is an IFS in X, then the image of A under f denoted by f(A) is the IFS in Y defined by f (A) = {⟨ y, f (λ_A(y)), f_(ν_A(y)) ⟩ / y ∈ Y} where f_(ν_A) = 1-f(1- ν_A).

Definition 2.11: [7] A mapping f: (X, τ) → (Y, σ) is called an Intuitionistic fuzzy π- generalized semi continuous mappings, (IFπGS continuous in short) if f^{-1}(B) is an IFπGSCS in (X, τ) for every IFCS B of (Y, σ).

Definition 2.12: [6] A mapping f: (X, τ) → (Y,σ) is called an intuitionistic fuzzy π generalized semi irresolute (IFπGS irresolute in short) if f^{-1}(B) is an IFπSGCS in (X, τ) for every IFπGSCS B of (Y, σ).

Definition 2.13: A mapping f: (X, τ) → (Y, σ) from an IFTS (X, τ) into an IFTS (Y,σ) is said to be an
(a) [8] intuitionistic fuzzy closed mapping (IFCM for short) if f(A) is an IFCS in Y for every IFCS A in X.
(b) [4] intuitionistic fuzzy semi closed mapping (IFSCM for short) if f(A) is an IFSCS in Y for every IFCS A in X.
(c) [4] intuitionistic fuzzy α-closed mapping (IFαCM for short) if f(A) is an IFαCS in Y for every IFCS A in X.

Definition 2.14: [5] An IFTS (X, τ) is said to be an intuitionistic fuzzy IF $\pi T_{1/2}$ space if every IFRWGCS in X is an IFCS in X.

Definition 2.15: [5] An IFTS (X, τ) is said to be an intuitionistic fuzzy IF $\pi g T_{1/2}$ space if every IFRWGCS in X is an IFPCS in X.

Definition 2.17:[9] An IFTS (X, τ) is said to be intuitionistic fuzzy C_5 - connected space if the only intuitionistic fuzzy sets which are both IFOS and IFCS are $0_\sim$ and $1_\sim$.

Definition 2.18:[9] An IFTS (X, τ) is said to be intuitionistic fuzzy GO-connected space if the only intuitionistic fuzzy sets which are both IFGOS and IFGCS are $0_\sim$ and $1_\sim$.

Definition 2.19:[9]An IFTS (X, τ) is an intuitionistic fuzzy C_5-connected between two IFS A and B if there is no IFOS E in (X, τ) such that $A \subseteq E$ and $E \bar{q} B$.

III. Intuitionistic fuzzy π generalized semi connected spaces

In this section, we have introduced intuitionistic fuzzy π generalized semi connected (IFπGS connected in short) space and studied some of its properties.

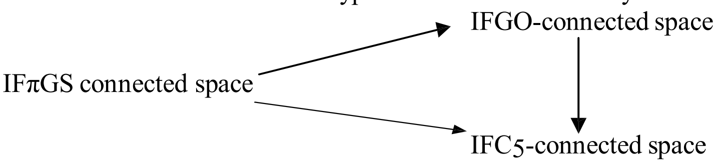
Definition 3.1: An IFTS (X, τ) is said to be an IFπGS connected space if the only intuitionistic fuzzy sets which are both IFπGSOS and IFπGSCS are $0_\sim$ and $1_\sim$

Theorem 3.2: Every IFπGS connected space is an intuitionistic fuzzy C_5-connected space but not conversely.
Proof: Let (X, τ) be an IFπGS connected space. Suppose (X, τ) is not an intuitionistic fuzzy C_5-connected space, then there exists a proper IFS A which is both intuitionistic fuzzy open and intuitionistic fuzzy closed in (X, τ). That is A is both IFπGSOS and IFπGSCS in (X, τ). This implies that (X, τ) is not an IFπGS connected space. Therefore we get a contradiction. Hence (X, τ) must be an intuitionistic fuzzy C_5-connected space.

Example 3.3: Let X = {a, b} and τ = {$0_\sim$, M, $1_\sim$} be an IFT on X, where M = ⟨x, (0.4, 0.3), (0.4, 0.5)⟩. Then (X, τ) is an IFC5-connected space but not an IFπGS connected space, since the IFS M in τ is both an IFπGSOS and an IFπGSCS in (X, τ).

Theorem 3.4: Every IFπGS connected space is an intuitionistic fuzzy GO-connected space but not conversely.
Proof: Let (X, τ) be an IFπGS connected space. Suppose (X, τ) is not an intuitionistic fuzzy GO-connected space, then there exists a proper IFS A which is both IFGOS and IFGCS in (X, τ). That is A is both IFπGSOS and IFπGSCS in (X, τ). This implies that (X, τ) is not an IFπGS connected space. That is we get a contradiction. Therefore (X, τ) must be an intuitionistic fuzzy GO-connected space.

Example 3.5: In Example 3.3, (X, □) is an IFGO-connected space but not an IFGSP connected space.
The relation between various types of intuitionistic fuzzy connectedness is given in the following diagram.



The reverse implications are not true in general in the above diagram.

Theorem 3.5: The IFTS (X, τ) is an IFπGS connected space if and only if there exists no non-zero IFπGSOS A and B in (X, τ) such that $A = B^c$.
Proof: Necessity: Let A and B be two IFπGSOS in (X, τ) such that $A \neq 0_\sim$ $B \neq 0_\sim$ and $A = B^c$. Therefore B^c is an IFπGSCS. Since $A \neq 0_\sim$, $B \neq 1_\sim$. This implies B is a proper IFS which is both IFπGSOS and IFπGSCS in (X, τ). Hence (X, τ) is not an IFπGS connected space. But it is a contradiction to our hypothesis. Thus there exists no non-zero IFπGSOS A and B in (X, τ) such that $A = B^c$.
Sufficiency: Let A be both IFπGSOS and IFπGSCS in (X, τ) such that $0_\sim \neq A \neq 1_\sim$. Now let $B = A^c$. Then B is an IFπGSOS and $B \neq 1_\sim$. This implies $B^c = A \neq 0_\sim$, which is a contradiction to our hypothesis. Therefore (X, τ) is an IFπGS connected space.

Theorem 3.6: Let (X, τ) be an $IF\pi_aT_{1/2}$ space, then the following are equivalent:
(i) (X, τ) is an IFπGS connected space
(ii) (X, τ) is an intuitionistic fuzzy GO-connected space
(iii) (X, τ) is an intuitionistic fuzzy C_5-connected space.
Proof: (i) ⇒ (ii): It is obvious from the Theorem 3.4.
(ii) ⇒ (iii): It is obvious.
(iii) ⇒ (i): Let (X, τ) be an intuitionistic fuzzy C_5-connected space. Suppose (X, τ) is not an IFπGS connected space, then there exists a proper IFS A in (X, τ) which is both IFπGSOS and IFπGSCS in (X, τ). But since (X, τ)

is an IF$\pi_a T_{1/2}$ space, A is both intuitionistic fuzzy open and intuitionistic fuzzy closed in (X, τ). This implies that (X, τ) is not an intuitionistic fuzzy C_5-connected, which is a contradiction to our hypothesis. Therefore (X, τ) must be an IFπGS connected space.

Theorem 3.7: If f : (X, τ) → (Y, σ) is an IFπGS continuous surjection and (X, τ) is an IFπGS connected space, then (Y, σ) is an intuitionistic fuzzy C_5 connected space.

Proof: Let (X, τ) be an IFπGS connected space. Suppose (Y, σ) is not an intuitionistic fuzzy C_5-connected space, then there exists a proper IFS A which is both intuitionistic fuzzy open and intuitionistic fuzzy closed in (Y, σ). Since f is an IFπGS continuous mapping, $f^{-1}(A)$ is both IFπGSOS and IFπGSCS in (X, τ). But it is a contradiction to our hypothesis. Hence (Y, σ) must be an intuitionistic fuzzy C_5-connected space.

Theorem 3.8: If f : (X, τ) → (Y, σ) is an IFπGS irresolute surjection and (X, τ) is an IFπGS connected space, then (Y, σ) is an IFπGS connected space.

Proof: Suppose (Y, σ) is not an IFπGS connected space, then there exists a proper IFS A such that A is both IFπGSOS and IFπGSCS in (Y, σ). Since f is an IFπGS irresolute mapping, $f^{-1}(A)$ is both IFπGSOS and IFπGSCS in (X, τ). But this is a contradiction to our hypothesis. Hence (Y, σ) must be an IFπGS connected space.

Definition 3.9: An IFTS (X, τ) is an intuitionistic fuzzy C_5-connected between two IFS A and B if there is no IFOS E in (X, τ) such that $A \subseteq E$ and $E\ \bar{q}\ B$.

Definition 3.10: An IFTS (X, τ) is an IFπGS connected between two IFS A and B if there is no IFπGSOS E in (X, τ) such that $A \subseteq E$ and $E\ \bar{q}\ B$.

Example 3.11: Let X = { a, b } and τ = { $0_\sim$, M, $1_\sim$ } be an IFT on X, where M = ⟨ x, (0.5, 0.3), (0.5, 0.1) ⟩. Then the IFTS (X, τ) is IFπGS connected between the IFS A= ⟨ x, (0.5, 0.4), (0.5, 0.3) ⟩ and B = ⟨ x, (0.5, 0.4), (0.5, 0.5) ⟩.

Theorem 3.12: If an IFTS (X, τ) is an IFπGS connected between two IFS A and B, then it is an intuitionistic fuzzy C_5-connected between A and B but the converse may not be true in general.

Proof: Suppose (X, τ) is not an intuitionistic fuzzy C_5-connected between A and B, then there exists an intuitionistic fuzzy open set E in (X, τ) such that $A \subseteq E$ and $E\ \bar{q}\ B$. Since every intuitionistic fuzzy open set is an IFπGSOS, there exists an IFπGSOS E in (X, τ) such that $A \subseteq E$ and $E\ \bar{q}\ B$. This implies (X, τ) is not an IFπGS connected between A and B. That is we get a contradiction to our hypothesis. Therefore the IFTS (X, τ) must be intuitionistic fuzzy C_5-connected between A and B.

Example 3.13: Let X = {a, b} and τ = { $0_\sim$, G, $1_\sim$ } be an IFT on X, where G= ⟨ x, (0.2, 0.2), (0.1, 0.2) ⟩. Then (X, τ) is an intuitionistic fuzzy C_5-connected between the IFS A = ⟨ x, (0.2, 0.3), (0.5, 0.5) ⟩ and B = ⟨ x, (0.4, 0.3), (0.4, 0.5) ⟩. But (X, τ) is not an IFπGS connected between A and B, since the IFS E = ⟨ x, (0.3, 0.3), (0.5, 0.4) ⟩ is an IFπGSOS such that $A \subseteq E$ and $E \subseteq B^c$.

Theorem 3.14: An IFTS (X, τ) is IFπGS connected between two IFSs A and B if and only if there is no IFπGSOS and IFπGSCS E in (X, τ) such that $A \subseteq E \subseteq B^c$.

Proof: Necessity: Let (X, τ) be IFπGS connected between A and B. Suppose that there exists an IFπGSOS and an IFπGSCS E in (X, τ) such that $A \subseteq E \subseteq B^c$, then $E\ \bar{q}\ B$ and $A \subseteq E$. This implies (X, τ) is not IFπGS connected between A and B, by Definition 3.9. A contradiction to our hypothesis. Therefore there exists no IFπGSOS and an IFπGSCS E in (X, τ) such that $A \subseteq E \subseteq B^c$.

Sufficiency: Suppose that (X, τ) is not IFπGS connected between A and B. Then there exists an IFπGSOS E in (X, τ) such that $A \subseteq E$ and $E\ \bar{q}\ B$. This implies that there exists an IF IFπGSOS E in (X, τ) such that $A \subseteq E \subseteq B^c$. But this is a contradiction to our hypothesis. Hence (X, τ) must be IFπGS connected between A and B.

Theorem 3.15: If an IFTS (X, τ) is IFπGS connected between A and B and $A \subseteq A_1$, $B \subseteq B_1$, then (X, τ) is an IFπGS connected between A_1 and B_1.

Proof: Suppose that (X, τ) is not an IFπGS connected between A_1 and B_1, then by Definition, there exists an IFπGSOS E in (X, τ) such that $A_1 \subseteq E$ and $E\ \bar{q}\ B_1$. This implies $E \subseteq B_1^c$ and $A_1 \subseteq E$. That is $A \subseteq A_1 \subseteq E$. Hence $A \subseteq E$. Since $E \subseteq B_1^c$, $B_1 \subseteq E^c$. That is $B \subseteq B_1 \subseteq E^c$ Hence $E \subseteq B^c$. Therefore (X, τ) is not an IFπGS connected between A and B. Hence we get a contradiction to our hypothesis. Thus X must be IFπGS connected between A_1 and B_1 .

Theorem 3.16: Let (X, τ) be an IFTS and A and B be IFS in (X, τ). If A q B, then X is an IFπGS connected between A and B.

Proof: Suppose (X, τ) is not IFπGS connected between A and B. Then there exists an IFπGSOS E in (X, τ) such that $A \subseteq E$ and $E \subseteq B^c$. This implies that $A \subseteq B^c$. That is A $\bar{q}$ B. But this is a contradiction to our hypothesis. Therefore X is must be an IFπGS connected between A and B.

Remark 3.17: The converse of the above theorem may not be true in general. This can be seen from the following example.

Example 3.18: In Example 3.14, (X, τ) is IFπGS connected between the IFSs A and B but since $\mu_A(x) < \nu_B(x)$, A $_q$ B is not possible.

Theorem 3.19: An IFTS (X, τ) is an IFπGS connected space if and only if there exists no non-zero IFπGSOS A and B in (X, τ) such that $B = A^c$, $B = (scl(A))^c$, $A = (scl(B))^c$.

Proof: Necessity: Assume that there exists IFS A and B such that $A \neq 0_\sim \neq B$, $B = A^c$, $B = (scl(A))^c$, $A = (scl(B))^c$. Since $(scl(A))^c$ and $(scl(B))^c$ are IFπGSOS in (X, τ), A and B are IFπGSOS in (X, τ). This implies (X, τ) is not an IFπGS connected space, which is a contradiction. Therefore there exists no non-zero IFπGSOS A and B in (X, τ) such that $B = A^c$,
$B = (scl(A))^c$, $A = (scl(B))^c$.

Sufficiency: Let A be both IFπGSOS and IFπGSCS in (X, τ) such that $1_\sim \neq A \neq 0_\sim$. Now by taking $B = A^c$, we obtain a contradiction to our hypothesis. Hence (X, τ) is an IFπGS connected space.

References

[1] Atanassov, K., **Intuitionistic fuzzy sets,** Fuzzy Sets and Systems, 20 (1986), 87-96.
[2] C.L.Chang, Fuzzy topological spaces, J.Math.Anal.Appl, 24(1968), 182-190.
[3] D.Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy sets and systems, 88(1997), 81-89.
[4] Joung Kon Jeon, Young Bae Jun and Jin Han Park, Intuitionistic fuzzy alpha continuity and intuitionistic fuzzy pre continuity, International journal of Mathematics and Mathematical Sciences, (2005), 3091-3101.
[5] S.Maragathavalli and K.Ramesh, Intuitionistic fuzzy π_ generalized semi closed sets, Advances in Theoretical and Applied Sciences, 1 (2012), 33-42.
[6] S.Maragathavalli and K.Ramesh, Intuitionistic fuzzy π- generalized semi irresolute mappings, International journal of mathematical Archive, 3 (2012), 1-7.
[7] S.Maragathavalli and K.Ramesh, Intuitionistic fuzzy π- generalized semi continuous mappings, International Journal of Computer Applications, 37 (2012), 30-34.
[8] Seok Jong Lee and Eun Pyo Lee, The category of intuitionistic fuzzy topological spaces, Bull. Korean Math. Soc, (2000), 63-76.
[9] S.S.Thakur and Rekha Chaturvedi, R.G-closed sets in intuitionistic fuzzy topological spaces, Universitatea Din Bacau Studii Si Cercertari Stiintifice, 6(2006), 257-272.
[10] L.A.Zadeh, Fuzzy sets, Information and control, 8(1965), 338-353.