

Mathematical Model of Dengue Disease Transmission Considering the incubation Period Both Intrinsic and Extrinsic

Rinancy Tumilaar¹, Paian Sianturi¹, Jaharuddin¹

¹(Department of Mathematics, Bogor Agricultural University, Indonesia)

Abstract: Dengue fever is an infectious disease in tropical regions, where its spread is transmitted by *Aedes aegypti* mosquito. Modeling the spread of dengue disease will facilitate in understanding the dynamics of the spread of disease in a population. There have been several mathematical models of the spread of dengue, in this paper will be modified a model that considering the incubation period. At its transmission in both humans and mosquitoes, viruses undergo an incubation period before the virus can move from mosquitoes to humans and vice versa humans to mosquitoes. In this paper will discuss the transmission of dengue in SIR model involving the incubation of virus in humans it called Intrinsic Incubation Period and model involving virus incubation period in humans and in mosquitoes. The virus incubation period in mosquitos it called Extrinsic Incubation Period. The fixed points were determined on this paper on both models, there were two fixed point namely free disease fixed point and endemic fixed point. Stability analysis performed on both models and numerical approach also performed. It was found that model with intrinsic incubation oscillates towards a stable value, and model with the effect of intrinsic and extrinsic incubation oscillates cyclically.

Keywords: dengue fever, incubation, mathematical model, SIR model, stability

I. Introduction

Dengue fever has been existing in Australia, Europe, Asia, South America and Africa since the 19th century. Dengue virus is transmitted by the bite of *Aedes aegypti* mosquito as the main factor. Modeling the transmission of dengue disease will facilitate in understanding the dynamics of the transmission of disease in a population. There have been several models that have been made by several researchers [1], [2], [3], [4].

In this study, will be assessed a model that refers to the study of Pongsumpun [1], a modified model related to the intrinsic incubation period, as well as a combination of intrinsic and extrinsic incubation period were considering in this paper.

Extrinsic incubation period is the period in which the start time of the entry of gametocytes to the mosquito's body to the stage where virus entered into the salivary glands, or in other words the period until the virus could be transmitted by mosquitoes.

Intrinsic incubation period is the time in which the virus is present in the human body until it is ready to transmit to mosquito.

II. Mathematical Model

The human population is divided into three classes, namely susceptible human (S_H), infected human (I_H), and recovered human (R_H). Susceptible humans are human that not immune and have not been infected. Infected humans are human that already infected and can transmit the virus to mosquito through its bite. Recovered humans were considered obtained immunity, thus no recovered human could get infected again.

Mosquitos populations divided into two classes, namely susceptible mosquitoes (S_V), and infected mosquitoes (I_V). Susceptible mosquitoes are susceptible to dengue fever. Infected mosquito is mosquito that could infect virus in to other individuals. It was assumed that human and mosquito population size constant so that the birth rate equals the death rate, average individual mosquito bites on humans per day is constant, and mosquitoes were never recovered after becoming infected.

The following are the parameters that exist in the model: N_T is total human population, N_V is total mosquito population, α is human birth rate, b is average of mosquito bites on humans, ζ_H is probability of dengue virus transmitted from mosquitoes to humans, ζ_H is probability of dengue virus transmitted from humans to mosquitoes, δ_H is a natural human death rate, r is rate of recovered of infected humans, D is rate of recruitment of mosquitoes, δ_V is natural mortality rate of mosquitoes.

In this work, the model developed by Pongsumpun [1] was used as the standard model as described in (1). Further, this model was modified also by Pongsumpun [1] by taking into account the effect of the so-called extrinsic incubation period as expressed in (2).

$$\frac{d}{dt} S^H = \alpha N_T - \frac{b \zeta_h}{N_T} S^H I^V - \delta_H S^H$$

$$\begin{aligned} \frac{d}{dt} I^H &= \frac{b\zeta_H}{N_T} S^H I^V - (\delta_H + r)I^H \\ \frac{d}{dt} R^H &= rI^H - \delta_h R^H \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{d}{dt} S^V &= D - \frac{b\zeta_V}{N_T} S^V I^H - \delta_V S^V \\ \frac{d}{dt} I^V &= \frac{b\zeta_V}{N_T} S^V I^H - \delta_V I^V \end{aligned}$$

and

$$\begin{aligned} \frac{d}{dt} S^H &= \alpha N_T - \frac{b\zeta_H}{N_T} S^H (1-c)I^V - \delta_H S^H \\ \frac{d}{dt} I^H &= \frac{b\zeta_H}{N_T} S^H (1-c)I^V - (\delta_h + r)I^H \\ \frac{d}{dt} R^H &= rI^H - \delta_H R^H \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{d}{dt} S^V &= D - \frac{b\zeta_V}{N_T} S^V I^H - \delta_V S^V \\ \frac{d}{dt} I^V &= \frac{b\zeta_V}{N_T} S^V I^H - \delta_V I^V \end{aligned}$$

2.1. Mathematical Model Considering Intrinsic Incubation

This model is a modification the model (1), considering the intrinsic incubation period. Let z is the percentage of infected humans but incapable of transmitting the disease. This is known as the intrinsic incubation. Thus, $(1-z)$ is the proportion of infected human that can transmit virus to mosquitoes. Therefore the equation (1) would result the following equation

$$\begin{aligned} \frac{d}{dt} S^H &= \alpha N_T - \frac{b\zeta_h}{N_T} S^H I^V - \delta_H S^H \\ \frac{d}{dt} I^H &= \frac{b\zeta_h}{N_T} S^H I^V - (\delta_H + r)I^H \\ \frac{d}{dt} R^H &= rI^H - \delta_H R^H \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{d}{dt} S^V &= D - \frac{b\zeta_V}{N_T} S^V (1-z)I^H - \delta_V S^V \\ \frac{d}{dt} I^V &= \frac{b\zeta_V}{N_T} S^V (1-z)I^H - \delta_V I^V \end{aligned}$$

given:

$$N_T = S^H + I^H + R^H \text{ and } N_V = S^V + I^V$$

To simplify the equation (3), it is assumed

$$S_H = \frac{S^H}{N_T}, I_H = \frac{I^H}{N_T}, R = \frac{R^H}{N_T}, S_V = \frac{S^V}{N_V}, I_V = \frac{I^V}{N_V}, \text{ and } N_V = \frac{D}{\delta_V} \quad (4)$$

and obtains

$$\begin{aligned} \frac{dS_H}{dt} &= \alpha - \gamma_H S_H I_V - \delta_H S_H \\ \frac{dI_H}{dt} &= \gamma_H S_H I_V - (\delta_H + r)I_H \end{aligned} \quad (5)$$

$$\frac{dI_V}{dt} = \gamma'_V (1 - I_V)I_H - \delta_V I_V$$

where $\gamma'_V = b\zeta'_V$ with $\zeta'_V = \zeta_V(1-z)$, and $\gamma_H = b\zeta_H n$ with $n = \frac{D/\delta_V}{N_T}$.

2.2. Mathematical Model Considering Intrinsic And Extrinsic Incubation

In this model both intrinsic and extrinsic incubation effect were considered. The system of differential equations obtained are:

$$\begin{aligned} \frac{d}{dt} S^H &= \alpha N_T - \frac{b\zeta_H}{N_T} (1-c)S^H I^V - \delta_H S^H \\ \frac{d}{dt} I^H &= \frac{b\zeta_H}{N_T} (1-c)S^H I^V - (\delta_H + r)I^H \\ \frac{d}{dt} R^H &= rI^H - \delta_H R^H \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{d}{dt}S^V &= D - \frac{b\zeta_V}{N_T}S^V(1-z)I^H - \delta_V S^V \\ \frac{d}{dt}I^V &= \frac{b\zeta_V}{N_T}S^V(1-z)I^H - \delta_V I^V \end{aligned}$$

The system equations in (6) were simplified by using equations (4), and results equations:

$$\begin{aligned} \frac{dS_H}{dt} &= \alpha - \gamma'_H S_H I_H - \delta_H S_H \\ \frac{dI_H}{dt} &= \gamma'_H S_H I_V - (\delta_H + r)I_H \\ \frac{dI_V}{dt} &= \gamma'_V (1 - I_V)I_H - \delta_V I_V \end{aligned} \tag{7}$$

where $\gamma'_V = b\zeta'_V$ with $\zeta'_V = \zeta_V(1-z)$, and $\gamma'_H = b\zeta'_H n$ with $n = \frac{D/\delta_V}{N_T}$.

III. Results

3.1. Analytical Results

The fixed point of the system of differential equations (5) and (7) were obtained by setting $\frac{dS_H}{dt} = 0$, $\frac{dI_H}{dt} = 0$ and $\frac{dI_V}{dt} = 0$, and produced two types of fixed point, Disease-free equilibrium (DFE) and the endemic fixed point.

3.1.1. The intrinsic Incubation Effect

The fixed points i.e, for the model with intrinsic incubation were denoted as $P^0(S_H, I_H, I_V) = P^0(1,0,0)$ and the endemic fixed point $P^*(S_H^*, I_H^*, I_V^*)$ with

$$\begin{aligned} S_H^* &= \frac{\delta_H b \zeta'_V + \delta_V (\delta_H + r)}{b^2 \zeta_H \zeta'_V n + \delta_H b \zeta'_V} \\ I_H^* &= \frac{\delta_H b^2 \zeta_H \zeta'_V n - \delta_H \delta_V (\delta_H + r)}{b^2 \zeta_H \zeta'_V n (\delta_H + r) + \delta_H b \zeta'_V (\delta_H + r)} \\ I_V^* &= \frac{\delta_H b^2 \zeta_H \zeta'_V n - \delta_H \delta_V (\delta_H + r)}{\delta_H b^2 \zeta_H \zeta'_V n + \delta_V b \zeta_H n (\delta_H + r)} \end{aligned} \tag{8}$$

to simplify it suppose $\zeta = \frac{b\zeta'_V}{\delta_V}$, $L = \frac{\delta_H + r}{\delta_H}$, $A^0 = \frac{b^2 \zeta_H \zeta'_V n}{\delta_V (\delta_H + r)}$, thus

$$S_H^* = \frac{L + \zeta}{\zeta + LA^0} I_H^* = \frac{A^0 - 1}{\zeta + LA^0} I_V^* = \frac{\zeta(A^0 - 1)}{A^0(\zeta + L)} \tag{9}$$

The fixed point $P^0(1,0,0)$ we obtain Jacobi matrix J_{P^0} which was determined using $\det(J_{P^0} - \lambda I) = 0$, and obtain

$$\lambda_1 = -\delta_H, \lambda_{2,3} = -q \pm \sqrt{q^2 - 4zk} \tag{10}$$

with

$$q = r + \delta_H + \delta_V, z = \delta_V(r + \delta_H), k = 1 - A^0, \gamma_H = b\zeta_H n, \gamma'_V = b\zeta'_V.$$

The eigenvalues λ_2 and λ_3 characterize the behavior of the system around the point P^0 . All eigenvalues will be negative if $A^0 < 1$, it means that the $P^0(1,0,0)$ will stable if $A^0 < 1$.

The fixed point $P^*(S_H^*, I_H^*, I_V^*)$ we obtain Jacobi matrix J_{P^*} which was determined using $\det(J_{P^*} - \lambda I) = 0$. The characteristic equation for J_{P^*} is

$$P(\lambda) = \lambda^3 + v_0 \lambda^2 + v_1 \lambda + v_2 \tag{11}$$

with

$$\begin{aligned} v_0 &= \delta_H \left(\frac{\zeta + LA^0}{\zeta + L} \right) + \delta_H L + \delta_V A^0 \left(\frac{\zeta + L}{\zeta + LA^0} \right) \\ v_1 &= \delta_H^2 L \left(\frac{\zeta + LA^0}{\zeta + L} \right) + \delta_V \delta_H A^0 + (A^0 - 1) \left(\frac{\delta_V \delta_H \zeta L}{\zeta + LA^0} \right) \\ v_2 &= \delta_V \delta_H^2 L (A^0 - 1) \end{aligned}$$

The eigenvalues of equation (11) is not easy to determine, therefore, the stability around the fixed point $P^*(S_H^*, I_H^*, I_V^*)$ will be investigated using the *Routh-Hurwitz criteria*.

Based on the *Routh-Hurwitz* criteria, the fixed point $P^*(S_H^*, I_H^*, I_V^*)$ will be stable if and only if the following requirements are met:

$$v_0 > 0, \text{ and } v_1 > 0, \text{ and } v_0 v_1 > v_2$$

Based on the condition above, if $A^0 > 1$ we obtain $v_0 > 0$, and $v_1 > 0$, and $v_0 v_1 > v_2$, Thus *Routh-Hurwitz* criteria are met. The fixed point $P^*(S_H^*, I_H^*, I_V^*)$ stable if $A^0 > 1$, in which further $\sqrt{A^0} = R_0$ was defined

as the basic reproduction number indicating the expectation number of infected human during the infection time [5].

3.1.2. The Intrinsic And Extrinsic Incubation Effects

The fixed points i.e, for the model with intrinsic and extrinsic incubation were denoted as $P^0(S_H, I_H, I_V) = P^0(1,0,0)$ and the endemic fixed point $P^*(S_H^*, I_H^*, I_V^*)$ with

$$S_H^* = \frac{\delta_H b \zeta'_V + \delta_V (\delta_H + r)}{b^2 \zeta'_H \zeta'_V n + \delta_H b \zeta'_V}$$

$$I_H^* = \frac{\delta_H b^2 \zeta'_H \zeta'_V n - \delta_H \delta_V (\delta_H + r)}{b^2 \zeta'_H \zeta'_V n (\delta_H + r) + \delta_H b \zeta'_V (\delta_H + r)} \tag{12}$$

$$I_V^* = \frac{\delta_H b^2 \zeta'_H \zeta'_V n - \delta_H \delta_V (\delta_H + r)}{\delta_H b^2 \zeta'_H \zeta'_V n + \delta_V b \zeta'_H n (\delta_H + r)}$$

to simplify it suppose $\zeta = \frac{b \zeta'_V}{\delta_V}$, $L = \frac{\delta_H + r}{\delta_H}$, $A^1 = \frac{b^2 \zeta'_H \zeta'_V n}{\delta_V (\delta_H + r)}$, thus

$$S_H^* = \frac{L + \zeta}{\zeta + L A^1} I_H^* = \frac{A^1 - 1}{\zeta + L A^1} I_V^* = \frac{\zeta (A^1 - 1)}{A^1 (\zeta + L)} \tag{13}$$

The fixed point $P^0(1,0,0)$ we obtain Jacobi matrix J_{P^0} which was determined using $\det(J_{P^0} - \lambda I) = 0$, and obtain

$$\lambda_1 = -\delta_H, \lambda_{2,3} = -q \pm \sqrt{q^2 - 4zk} \tag{14}$$

with

$$q = r + \delta_H + \delta_V, z = \delta_V (r + \delta_H), k = 1 - A^1, \gamma'_H = b \zeta'_H n, \gamma'_V = b \zeta'_V.$$

The eigenvalues λ_2 and λ_3 characterize the behavior of the system around the point P^0 . All eigenvalues will be negative if $A^1 < 1$, it means that the $P^0(1,0,0)$ will stable if $A^1 < 1$.

The fixed point $P^*(S_H^*, I_H^*, I_V^*)$ we obtain Jacobi matrix J_{P^*} which was determined using $\det(J_{P^*} - \lambda I) = 0$. The characteristic equation for J_{P^*} is

$$P(\lambda) = \lambda^3 + v_0 \lambda^2 + v_1 \lambda + v_2 \tag{15}$$

with

$$v_0 = \delta_H \left(\frac{\zeta + L A^1}{\zeta + L} \right) + \delta_H L + \delta_V A^1 \left(\frac{\zeta + L}{\zeta + L A^1} \right)$$

$$v_1 = \delta_H^2 L \left(\frac{\zeta + L A^1}{\zeta + L} \right) + \delta_V \delta_H A^1 + (A^1 - 1) \left(\frac{\delta_V \delta_H \zeta L}{\zeta + L A^1} \right)$$

$$v_2 = \delta_V \delta_H^2 L (A^1 - 1)$$

The eigenvalues of equation (15) is not easy to determine, therefore, the stability around the fixed point $P^*(S_H^*, I_H^*, I_V^*)$ will be investigated using the *Routh-Hurwitz criteria*.

Based on the *Routh-Hurwitz* criteria, the fixed point $P^*(S_H^*, I_H^*, I_V^*)$ will be stable if and only if the following requirements are met:

$$v_0 > 0, \text{ and } v_1 > 0, \text{ and } v_0 v_1 > v_2$$

Based on the condition above, if $A^1 > 1$ we obtain $v_0 > 0$, and $v_1 > 0$, and $v_0 v_1 > v_2$, Thus *Routh-Hurwitz* criteria are met. The fixed point $P^*(S_H^*, I_H^*, I_V^*)$ stable if $A^1 > 1$, in which further $\sqrt{A^1} = R_0$ was defined as the basic reproduction number indicating the expectation number of infected human during the infection time [5].

3.2. Numerical Results

Simulations performed because the system difficult to observe directly, from simulation could be learned things that could happen in population dynamics based on models. Simulation refers to stability analysis above. The value of the parameters:

$$\alpha_H = 0.0000391, \alpha_V = 0.071, \delta_H = 0.0000391, \delta_V = 0.071, \zeta_H = 0.5, \zeta_{vV} = 0.7, b = 0.6, r = 1/3, n = 10, z = 0.3.$$

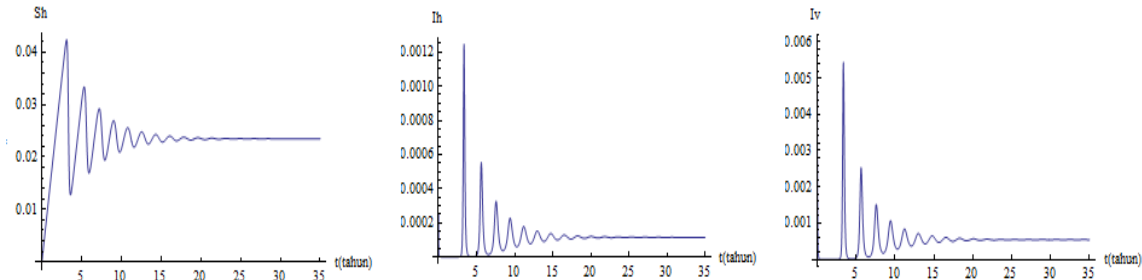


Figure 1 The proportion of susceptible humans (S_H), infected humans (I_H), and infected mosquitoes (I_V) model with intrinsic incubation.

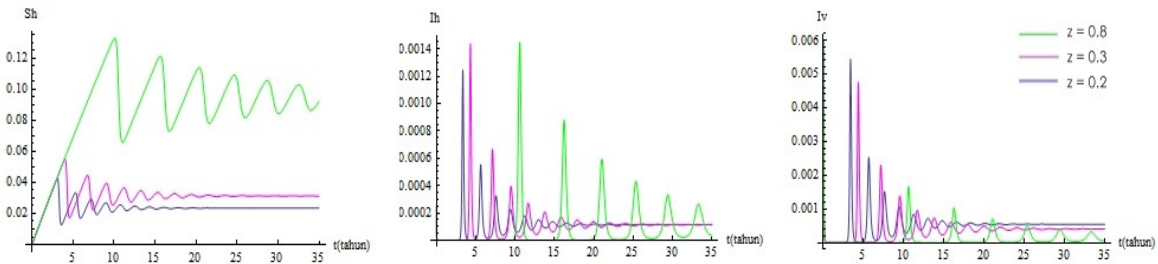


Figure 2 The proportion of susceptible humans (S_H), infected humans (I_H), and infected mosquitoes (I_V) model with intrinsic incubation, with a different value of z and other parameters remain.

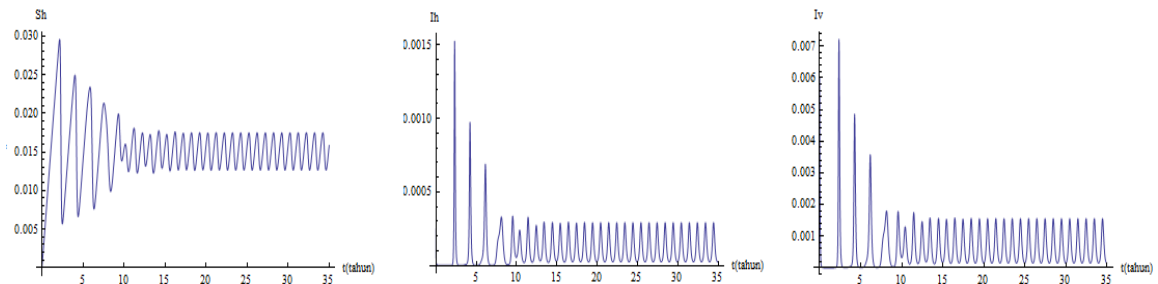


Figure 3 The proportion of susceptible humans (S_H), infected humans (I_H), and the proportion of infected mosquitoes (I_V) model with intrinsic and extrinsic incubation.

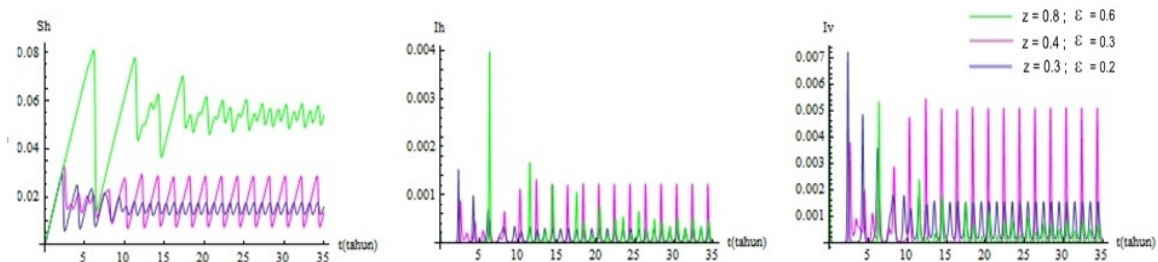


Figure 4 The proportion of susceptible humans (S_H), infected humans (I_H), and infected mosquitoes (I_V) model with intrinsic and extrinsic incubation with a different value of z and ϵ with values of other parameters remain.

IV. Conclusion

In general, the resulting model can indicate the presence in an endemic area for certain parameter values. We obtained two fixed points, namely the free disease fixed point and endemic fixed point to each model.

Analytical results show that the number of each subpopulation human and mosquito reaches a stable condition around the free disease fixed point with condition $R_0 < 1$ and stable around endemic fixed point with condition $R_0 > 1$.

Fig 1 and fig 3 shows that a model with intrinsic incubation oscillates towards a stable value, and model with the effect of extrinsic and intrinsic incubation period oscillates cyclically.

Fig 2 shows that the increase of the value of the proportion of people infected in the incubation period (z) caused an increase in the proportion of susceptible humans (S_H), decrease in proportion of infected humans (I_H) and infected mosquitoes (I_V), and time towards stable more slower.

Fig 4 shows that changes value of z and ε gives a different oscillation behavior in the proportion of susceptible humans (S_H), infected humans (I_H), and infected mosquitoes (I_V).

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