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Contra gp*- Continuous Functions

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Abstract: *In this paper, the authors introduce a new class of functions called contra gp*-continuous function in topological spaces. Some characterizations and several properties concerning contra gp*-continuous functions are obtained. Mathematics Subject Classification: 54 C 05, 54 C 08, 54 C10.*
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I. Introduction

In 1970, Dontchev introduced the notions of contra continuous function. A new class of function called contra b-continuous function introduced by Nasef. In 2009, A.A.Omari and M.S.M.Noorani have studied further properties of contra b-continuous functions. In this paper, we introduce the concept of contra gp*-continuous function via the notion of gp*-open set and study some of the applications of this function. We also introduce and study two new spaces called gp*-Hausdorff spaces, gp*-normal spaces and obtain some new results.

Throughout this paper (X, τ) and (Y, σ) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. Let $A \subseteq X$, the closure of A and interior of A will be denoted by cl (A) and int (A) respectively, union of all gp*-open sets X contained in A is called gp*-interior of A and it is denoted by gp*-int (A), the intersection of all gp*-closed sets of X containing A is called gp*-closure of A and it is denoted by gp*-cl(A).

II. Preliminaries.

Definition 2.1[8]: Let A subset A of a topological space (X, τ), is called a pre-open set if $A \subseteq$ Int(cl (A)).

Definition 2.2 [16]: Let A subset A of a topological space (X, τ), is called a generalized closed set (briefly g-closed) if cl (A) $\subseteq$ U whenever $A \subseteq U$ and U is open in X.

Definition 2.3 [10]: Let A subset A of a topological space (X, τ), is called a generalized pre- closed set (briefly gp- closed) if pcl (A) $\subseteq$ U whenever $A \subseteq U$ and U is open in X.

Definition 2.4 [7]: Let A subset A of a topological space (X, τ), is called a generalized pre-closed set (briefly pg-closed) if pcl(A) $\subseteq$ U whenever $A \subseteq U$ and U is pre-open in X.

Definition 2.5 [14]: Let A subset A of a topological space (X, τ), is called a generalized pre- closed set (briefly g*- closed) if cl (A) $\subseteq$ U whenever $A \subseteq U$ and U is g-open in X.

Definition 2.6 [18]: Let A subset A of a topological space (X, τ), is called a generalized pre- closed set (briefly g*p-closed) if pcl (A) $\subseteq$ U whenever $A \subseteq U$ and U is g-open in X.

Definition 2.7 [15]: Let A subset A of a topological space (X, τ), is called a generalized pre- closed set (briefly strongly g- closed) if cl (A) $\subseteq$ U whenever $A \subseteq U$ and U is g-open in X.

Definition 2.9 [17]: Let A subset A of a topological space (X, τ), is called a generalized pre- closed set (briefly g# closed) if cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is α g-open in X.

Definition 2.10 [4]: A subset A of a topological space (X, τ), is called gp^*-closed set if cl (A) $\subseteq$ U whenever $A \subseteq U$ and U is gp open in X.

Definition 2.2. A function f: $(X, \tau) \to (Y, \sigma)$ is called

(i) a contra continuous[1] if $f^{1}(V)$ is closed in (X, τ) for every open set V of (Y, σ).
(ii) a contra g*-continuous [14] if $f^{1}(V)$ is g*-closed in (X, τ) for every open set V of (Y, σ).
(iii) a contra pg-continuous [7] if $f^{1}(V)$ is pg-closed in (X, τ) for every open set V of (Y, σ).
(iv) a contra g*p-continuous [18] if $f^{1}(V)$ is g*p-closed in (X, τ) for every open set V of (Y, σ).
(v) a contra strongly g-continuous [15] if $f^{1}(V)$ is strongly g-closed in (X, τ) for every open set V of (Y, σ).
(vi) a contra g#-continuous [17] if $f^{1}(V)$ is g#-closed in (X, τ) for every open set V of (Y, σ).

III. Contra gp*Continuous Functions

In this section, we introduce contra gp*-continuous functions and investigate some of their properties.

Definition 3.1. A function f: $(X,\tau)\rightarrow(Y,\sigma)$ is called contra gp*-continuous if $f^{-1}(V)$ is gp*-closed in (X,τ) for every open set V in (Y,σ).

Example.3.2. Let X =Y={a,b,c}with $\tau=\{X,\varphi,\{a\},\{b\},\{a,b\}\}$ and $\sigma=\{Y,\varphi,\{a,b\}\}$. Define a function f: $(X,\tau)\rightarrow(Y,\sigma)$ by f(a) = b, f(b) = c, f(c) = a. Clearly f is contra gp*-continuous.

Definition3.3. [11] Let A be a subset of a space (X,τ).
(i) The set $\cap\{F\subset X: A\subset F$, F is gp*-closed$\}$ is called the gp*-closure of A and it is denoted by gp*-cl(A).
(ii) The set $\cup\{G\subset X: G\subset A$, G is gp*-open$\}$ is called the gp*-interior of A and it is denoted by gp*-int(A).

Lemma 3.4. For $x\in X$, $x\in$ gp*-cl (A) if and only if $U\cap A\neq\phi$ for every gp*-open set U containing x.

Proof.
Necessary part: Suppose there exists a gp*-open set U containing x such that $U\cap A=\varphi$. Since $A\subset X–U$, gp*-cl(A)$\subset X–U$. This implies $x\notin$ gp*-cl(A). This is a contradiction.
Sufficiency part: Suppose that $x\notin$ gp*-cl(A). Then $\exists$ a gp*-closed subset F containing A such that $x\notin F$. Then $x\in X$-F is gp*-open, (X-F) $\cap A=\varphi$. This is contradiction.

Lemma 3.5. The following properties hold for subsets A, B of a space X:
(i) $x\in$ ker(A) if and only if $A\cap F\neq\phi$ for any $F\in(X,x)$.
(ii) $A\subset$ ker(A) and A = ker(A) if A is open in X.
(iii) If $A\subset B$, then ker(A)$\subset$ker(B).

Theorem 3.6. Let f:$(X,\tau)\rightarrow(Y,\sigma)$ be a map. The following conditions are equivalent:
(i) f is contra gp*-continuous,
(ii) The inverse image of each closed in (Y,σ) is gp*-open in (X,τ),
(iii) For each $x\in X$ and each $F\in C(Y,f(x))$, there exists $U\in$ gp*-O(X), such that f(U) $\subset F$,
(iv) f(gp*-cl(X))$\subset$ker(f(A)), for every subset A of X,
(v) gp*(f^{-1}(B)) $\subset f^{-1}$(ker(B)), for every subset B of Y.

Proof: (i) $\Leftrightarrow$ (ii) and (ii)$\Rightarrow$(iii) are obvious.
(iii)$\Rightarrow$(ii): Let F be any closed set of Y and $x\in f^{-1}(F)$. Then $f(x)\in F$ and there exists $U_x\in$ gp*-O(X,x) such that $f(U_x)\subset F$. Hence we obtain $f^{-1}(F)=\cup\{U_x / x\in f^{-1}(F)\}\in$ gp*-O(X,x). Thus the inverse of each closed set in (Y,σ) is gp*-open in (X,τ).
(ii)$\Rightarrow$(iv). Let A be any subset of X. Suppose that $y\notin$ kerf(A)). By lemma there exists $F\in C(Y,y)$ such that f(A) $\cap F=\varphi$. Then, we have $A\cap f^{-1}(F)=\varphi$ and gp*-cl(A)$\cap f^{-1}(F)=\varphi$. Therefore, we obtain f(gp*-cl(A)) $\cap F=\varphi$ and $y\notin$ f(gp*-cl(A)). Hence we have f(gp*-cl(X))$\subset$ker(f(A)).
(iv) $\Rightarrow$(v): Let B be any subset of Y. By (iv) and Lemma, We have f(gp*-cl(f^{-1}(B))) $\subset$(ker(f(f^{-1}(B))) $\subset$ker(B) and gp*-cl(f^{-1}(B)) $\subset f^{-1}$(ker(B)).
(v) $\Rightarrow$(i): Let V be any open set of Y. By lemma We have gp*-cl(f^{-1}(V)) $\subset$ f^{-1}(ker(V)) = f^{-1}(V) and gp*-cl(f^{-1}(V)) = f^{-1}(V). It follows that f^{-1}(V) is gp*-closed in X. We have f is contra gp*-continuous.

Definition 3.7. A function f: $(X,\tau)\rightarrow(Y,\sigma)$ is called gp*-continuous if the pre image of every open set of Y is gp*-open in X.

Remark 3.8: The following two examples will show that the concept of gp*-continuity and contra gp*-continuity are independent from each other.

Example 3.9. Let X =Y={a,b,c}with $\tau=\{X,\varphi,\{a\},\{c\},\{a,c\}\}$ and $\sigma=\{Y,\varphi,\{b,c\}\}$. Define a function f: $(X,\tau)\rightarrow(Y,\sigma)$ by f(a) = a, f(b) = b, f(c) = c. Clearly f is contra gp*-continuous but f is not gp*-continuous. Because f^{-1}({b,c}) = {b,c} is not gp*-open in(X,τ) where {b,c} is open in (Y,σ).

Example 3.10. Let X =Y={a,b,c}with τ ={X,φ,{a},{a,b}} and σ ={Y,φ,{a,c}}. Define a function f: (X,τ)$\rightarrow$(Y, σ) by f(a) = c, f(b) = a, f(c) = b. Clearly f is gp*-continuous but f is not contra gp*-continuous. Because f^{-1}({a,c}) = {a,b} is not contra gp*-closed in(X,τ) where {a,c} is open in (Y,σ).

Theorem 3.11. If a function f: (X,τ) $\rightarrow$(Y,σ) is contra gp*-continuous and (Y,σ) is regular then f is gp*-continuous.

Proof: Let x be an arbitrary point of (X,τ) and V be an open set of (Y,σ) containing f(x). Since (Y,σ) is regular, there exists an open set W of (Y,σ) containing f(x) such that cl(W)$\subset$V. Since f is contra gp*-continuous, by theorem
There exists U$\in$gp*-O(X,x) such that f(U)$\subset$cl(W). Then f(U)$\subset$cl(W) $\subset$V. Hence f is gp*-continuous.

Theorem 3.12. Every contra g*-continuous function is contra gp*-continuous function.

Proof: Let V be an open set in (Y,σ). Since f is contra g*-continuous function, f^{-1}(V) is g*-closed in (X,τ). Every g*-closed set is gp*-closed. Hence f^{-1}(V) is gp*-closed in(X,τ). Thus f is contra gp*-continuous function.

Remark 3.13. The converse of theorem need not be true as shown in the following example.

Example 3.14. Let X =Y={a,b,c}with τ ={X,φ,{b},{a,b}} and σ ={Y,φ, {b,c}}. Define a function f: (X,τ)$\rightarrow$(Y, σ) by f(a) = b, f(b) = c, f(c) = a. Clearly f is contra gp*-continuous but f is not contra g*-continuous. Because f^{-1}({b,c}) = {a,b} is not g*-closed in(X,τ) where {b,c} is open in (Y,σ).

Theorem 3.15.
(i) Every contra pg-continuous function is contra gp*-continuous function.
(ii) Every contra g*p-continuous function is contra gp*-continuous function.
(iii)Every contra strongly g-continuous function is contra gp*-continuous function.
(iv) Every contra g#-continuous function is contra gp*-continuous function.

Remark 3.16. Converse of the above statements is not true as shown in the following example.

Example 3.17.
(i) Let X =Y={a,b,c}with τ ={X,φ,{a},{b},{a,b},{b,c}} and σ ={Y,φ,{b,c}}. Define a function f: (X,τ)$\rightarrow$(Y, σ) by f(a) = c, f(b) = a, f(c) = b. Clearly f is contra gp*-continuous but f is not contra pg-continuous. Because f^{-1}({b,c}) = {a,c} is not pg-closed in(X,τ) where {b,c} is open in (Y,σ).

(ii). Let X =Y={a,b,c}with τ ={X,φ,{a},{c},{a,c},{b,c}} and σ ={Y,φ,{a,b}}. Define a function f: (X,τ)$\rightarrow$(Y, σ) by f(a) = b, f(b) = c, f(c) = a. Clearly f is contra gp*-continuous but f is not contra g*p-continuous. Because f^{-1}({a,b}) = {a,c} is not g*p-closed in(X,τ) where {a,b} is open in (Y,σ).

(iii) Let X =Y={a,b,c}with τ ={X,φ,{a,b}} and σ ={Y,φ,{a}}. Define a function f: (X,τ)$\rightarrow$(Y, σ) by f(a) = c, f(b) = b, f(c) = a. Clearly f is contra gp*-continuous but f is not contra strongly g-continuous. Because f^{-1}({a}) = {c} is not strongly g -closed in(X,τ) where {a} is open in (Y,σ).

(iv) Let X =Y={a,b,c}with τ ={X,φ,{b}} and σ ={Y,φ,{a}}. Define a function f: (X,τ)$\rightarrow$(Y, σ) by f(a) = b, f(b) = c, f(c) = a. Clearly f is contra gp*-continuous but f is not contra g#-continuous. Because f^{-1}({a}) = {c}` is not g#-closed in(X,τ) where {a} is open in (Y,σ).

Remark 3.18 The concept of contra gp*-continuous and contra gp-continuous are independent as shown in the following examples.

Example 3.19. Let X =Y={a,b,c} with τ={X, φ,{a},{c},{a,c},{b,c}} and σ={Y, φ,{b,c}}. Define a function f: (X, τ) $\rightarrow$ (Y, σ) by f(a) = b, f(b) = a, f(c) = c. Clearly f is contra gp*-continuous but f is not contra gp-continuous. Because f^{-1}({b,c}) = {a,c} is not gp-closed in(X, τ) where {b,c} is open in (Y, σ).

Example 3.20 Let X =Y={a,b,c} with τ={X, φ,{c}} and σ={Y, φ,{a,b}}. Define a function f: (X, τ) $\rightarrow$ (Y, σ) by f(a) = b, f(b) = c, f(c) = a. Clearly f is contra gp-continuous but f is not contra gp*-continuous. Because f^{-1}({a,b}) = {a,c} is not gp*-closed in(X, τ) where {a,b} is open in (Y, σ).

Definition 3.21. A space (X, τ) is said to be (i) gp*-space if every gp*-open set of X is open in X, (ii) locally gp*-indiscrete if every gp*-open set of X is closed in X.

Theorem 3.22. If a function f : X $\rightarrow$ Y is contra gp*-continuous and X is gp*-space then f is contra continuous.

Proof: Let V $\in$ O(Y). Then f^{-1}(V) is gp*-closed in X. Since X is gp*-space, f^{-1}(V) is open in X. Hence f is contra continuous.

Theorem 3.23. Let X be locally gp*-indiscrete. If f: X $\rightarrow$ Y is contra gp*-continuous, then it is continuous.

Proof: Let V $\in$ O(Y). Then f^{-1}(V) is gp*-closed in X. Since X is locally gp*-indiscrete space, f^{-1}(V) is open in X. Hence f is continuous.

Definition 3.24. A function f : X $\rightarrow$ Y, the subset {(x,f(x)) : x $\in$ X} $\subset$ X $\times$ Y is called the graph of f and is denoted by G_f.

Definition 3.25. The graph G_f of a function f : X $\rightarrow$ Y is said to be contra gp*-closed if for each (x,y) $\in$ (X $\times$ Y) – G_f there exists U $\in$ gp*-O(X,y) and V $\in$ C(Y,y) such that (U $\times$ V) $\cap$ G_f .

Theorem 3.26. If a function f : X $\rightarrow$ Y is contra gp*-continuous and Y is Urysohn, then G_f is contra gp*-closed in the product space X $\times$ Y.

Proof: Let (x,y) $\in$ (X $\times$ Y) – G_f . Then y $\neq$ f(x) and there exist open sets H_1, H_2 such that f(x) $\in H_1$, y $\in H_2$ and cl(H_1) $\cap$ cl(H_2) = φ. From hypothesis, there exists V $\in$ gp*-O(X,x) such that f(V) $\subset$ cl(H_1). Therefore, we have f(V) $\cap$ cl(H_2) = φ. This shows that G_f is contra gp*-closed in the product space X $\times$ Y.

Theorem 3.27. If f: X $\rightarrow$ Y is gp*-continuous and Y is T_1, then Gf is contra gp*-closed in X $\times$ Y.

Proof. Let (x,y) $\in$ (X $\times$ Y) – G_f . Then y $\neq$ f(x) and there exist open set V of Y such that f(x) $\in$ V and y $\notin$ V. Since f is gp*-continuous, there exists U $\in$ (gp*-O(X,x) such that f(U) $\subset$ V. Therefore, we have f(U) $\cap$ (Y-V) = φ and (Y-V) $\in$ (gp*-C(Y,y). This shows that G_f is contra gp*-closed in X $\times$ Y.

Theorem 3.28. Let f : X $\rightarrow$ Y be a function and g : X $\rightarrow$ X $\times$ Y, the graph function of f, defined by g(x) = (x, f(x)) for every x $\in$ X. If g is contra gp*-continuous, then f is contra gp*-continuous.

Proof. Let U be an open set in Y, then X $\times$ U is an open set in X $\times$ Y. Since g is contra gp*-continuous. It follows that f^{-1}(U) = g^{-1}(X $\times$ U) is an gp*-closed in X. Hence f is gp*-continuous.

Theorem 3.29. If f : X $\rightarrow$ Y is a contra gp*-continuous function and g : Y $\rightarrow$ Z is a continuous function, then g $\circ$ f : X $\rightarrow$ Z is contra gp*-continuous.

Proof: Let V $\in$ O(Y). Then g^{-1}(V) is open in Y. Since f is contra gp*-continuous, f^{-1}(g^{-1}(V)) = $(g \circ f)^{-1}$ (V) is gp*-closed in X. Therefore, g $\circ$ f: X $\rightarrow$ Z is contra gp*-continuous.

Theorem 3.30. Let p: X $\times$ Y $\rightarrow$ Y be a projection. If A is gp*-closed subset pf X, then p^{-1}(A) = A $\times$ Y is gp*-closed subset of X $\times$ Y.

Proof: Let $A\times Y\subset U$ and U be a regular open set of $X\times Y$. Then U = $X\times Y$ for some regular open set of X. Since A is gp*-closed in X, bcl(A) and so bcl(A) $\times Y\subset V\times Y$ = U. Therefore $bcl(A\times Y)\subset U$. Hence $A\times Y$ is gp*-closed sub set of $X\times Y$.

IV. Applications.

Definition 4.1. A topological space (X, τ) is said to be gp*-Hausdorff space if for each pair of distinct points x and y in X there exists $U\in$ gp*-O(X,x) and $V\in$ gp*-O(X,y) such that $U\cap V=\varphi$.

Example 4.2. Let X ={a,b,c}with τ ={X, φ, {a}, {b}, {c}, {a,b}, {b,c}, {a,c}}. Let x and y be two distinct points of X, there exists an gp*-open neighbourhood of x and y respectively such that $\{x\}\cap\{y\}=\varphi$. Hence (X, τ) is gp*-Hausdorff space.

Theorem 4.3. If X is a topological space and for each pair of distinct points x_1 and x_2 in X, there exists a function f of X into Uryshon topological space Y such that $f(x_1)\neq f(x_2)$ and f is contra gp*-continuous at x_1 and x_2, then X is gp*-Hausdorff space.

Proof: Let x_1 and x_2 be any distinct points in X. By hypothesis, there is a Uryshon space Y and a function $f: X\rightarrow Y$ such that $f(x_1)\neq f(x_2)$ and f is contra gp*-continuous at x_1 and x_2. Let $y_i = f(x_i)$ for i = 1,2 then $y_1\neq y_2$. Since Y is Uryshon, there exists open sets U_{y1} and U_{y2} containing y_1 and y_2 respectively in Y such that $cl(U_{y1})\cap cl(U_{y2})=\varphi$. Since f is contra gp*-continuous at x_1 and x_2, there exists and gp*-open sets V_{x1} and V_{x2} containing x_1 and x_2 respectively in X such that $f(V_{xi})\subset cl(U_{yi})$ for i = 1,2. Hence we have $(V_{x1})\cap(V_{x2})=\varphi$. Therefore X is gp*-Hausdorff space.

Corollary 4.4. If f is contra gp*-continuous injection of a topological space X into a Uryshon space Y then Y is gp*-Hausdorff.

Proof: Let x_1 and x_2 be any distinct points in X. By hypothesis, f is contra gp*-continuous function of X into a Uryshon space Y such that $f(x_1)\neq f(x_2)$, because f is injective. Hence by theorem, X is gp*-Hausdorff.

Definition 4.5. A topological space (X, τ) is said to be gp*-normal if each pair of non-empty disjoint closed sets in (X, τ) can be separated by disjoint gp*-open sets in (X, τ).

Definition 4.6. A topological space (X, τ) is said to be ultra normal if each pair of non-empty disjoint closed sets in (X, τ) can be separated by disjoint clopen sets in (X, τ).

Theorem 4.7. If $f: X\rightarrow Y$ is a contra gp*-continuous function, closed, injection and Y is Ultra normal, then X is gp*-normal.

Proof: Let U and V be disjoint closed subsets of X. Since f is closed and injective, f(U) and f(V) are disjoint subsets of Y. Since Y is ultra normal, there exists disjoint closed sets A and B such that $f(U)\subset A$ and $f(V)\subset B$. Hence $U\subset f^{1}(A)$ and $V\subset f^{1}(B)$. Since f is contra gp*-continuous and injective, $f^{1}(A)$ and $f^{1}(B)$ are disjoint gp*-open sets in X. Hence X is gp*-normal.

Definition4.8. [13] A topological space X is said to be gp*-connected if X is not the union of two disjoint non-empty gp*-open sets of X.

Theorem 4.9. A contra gp*-continuous image of a gp*-connected space is connected.
Proof: Let $f: X\rightarrow Y$ is a contra gp*-continuous function of gp*-connected space X onto a topological space Y. If possible, let Y be disconnected. Let A and B form disconnectedness of Y. Then A and B are clopen and Y = $A\cup B$ where $A\cap B=\varphi$. Since f is contra gp*-continuous, X = $f^{1}(Y) = f^{1}(A\cup B) = f^{1}(A)\cup f^{1}(B)$ where $f^{1}(A)$ and $f^{1}(B)$ are non-empty gp*-open sets in X. Also $f^{1}(A)\cap f^{1}(B)=\varphi$. Hence X is non-gp*-connected which a contradiction is. Therefore Y is connected.
Theorem 4.10. Let X be gp*-connected and Y be T_1. If $f: X\rightarrow Y$ is a contra gp*-continuous, then f is constant.
Proof: Since Y is T_1 space v = $\{f^{1}(y): y\in Y\}$ is a disjoint gp*-open partition of X. If $|v|\geq 2$, then X is the union of two non empty gp*-open sets. Since X is gp*-connected, |v| = 1. Hence f is constant.

Theorem 4.11. If f: X$\rightarrow$Y is a contra gp*-continuous function from gp*-connected space X onto space Y, then Y is not a discrete space.
Proof: Suppose that Y is discrete. Let A be a proper non-empty open and closed subset of Y. Then $f^{-1}(A)$ is a proper non-empty gp*-clopen subset of X, which is a contradiction to the fact X is gp*-connected.

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