IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 10, Issue 4 Ver. IV (Jul-Aug. 2014), PP 102-107
www.iosrjournals.org

Effect of Darcy Dissipation on Melting From a Vertical Plate with Variable Temperature Embedded In Porous Medium

Umaru Mohammed[1], Hakimi Danladi [1] and Aliyu Ishaku Ma'ali[2]
[1]*Department of Mathematics and Statistics, Federal University of Technology, Minna, Nigeria.*
[2]*Department of Mathematics/computer science Ibrahihim Badamasi Babangida University Lapai,Niger State, Nigeria*

Abstract: *The effect of Darcy dissipation on melting from a vertical plate with variable temperature embedded in porous medium is numerically studied. The partial differential equations governing the problem under consideration have been transformed by a similarity transformation into a system of ordinary differential equation which is solved numerically by Runge-Kutta-Gill methods. Dimensionless velocity, Temperature and concentration profiles are presented graphically for various values of physical parameter. During the course of integration, it is found that Increasing the values of melting result into the decrease in local nusselt number.*
Keyword: *Liquid phase;Mixed convection; Melting effect; and porous medium*

I. Introduction

A situation where both the forced and free convection effects are of comparable order is called mixed or combined convection. The analysis of mixed convection boundary layer flow along a vertical plate embedded in a fluid saturated porous media has received considerable theoretical and practical interest. The phenomenon of mixed convection occurs in many technical and industrial problems such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low-velocity environment, solar collectors and so on. Several authors have studied the problem of mixed convection about different surface geometries. The analysis of convective transport in a porous medium with the inclusion of non-Darcian effects has also been a matter of study in recent years. The inertia effect is expected to be important at a higher flow rate and it can be accounted for through the addition of a velocity squared term in the momentum equation, which is known as the Forchheimer's extension of the

Darcy's law. A detailed review of convective heat transfer in Darcy and non-Darcy porous medium can be found in the book by Nield and Bejan [1]. The study of melting effect is considered by many previous authors. For example, Roberts [2] firstly presented ''shielding effect" to describe the melting phenomena of ice placed in a hot stream of air at a steady state. Later, from the point of view of boundary layer theory, film theory and penetration theory, Tien and Yen [3] studied the effect of melting on convective heat transfer between a melting body and surrounding fluid. Epstein and Cho [4] considered the laminar film condensation on a vertical melting surface for 1-D and 2-D systems based on nusselt's method to discuss the melting rate. They pointed out that as long as the melting solid is large compared with the thickness of thermal boundary layer, transient effects in the solid would be neglected. Sparrow et al. [5] studied the velocity and temperature fields, the heat transfer rate and the melting layer thickness by means of a finite-difference scheme in the melting region for natural convection. In problems dealing with porous media, the effects of melting, radiation and heat generation or absorption become important (Bakier [6], Gorla et al. [7], Cheng and Lin, [8], Tashtoush [9],Chamkha et al.[10] Kazmierczak et al. [11], Kazmierczak et al. [12] Chen et al. [13]). The problem of unsteady mixed convection boundary layer flow near the stagnation point on a heated vertical plate embedded in a fluid saturated porous medium with thermal radiation and variable viscosity was investigated by Hassanien and Al-arabi [14]. Murthy et al. [15] considered mixed convection flow of an absorbing fluid up a uniform non–Darcy porous medium supported by a semi-infinite ideally transparent vertical flat plate due to solar radiation. Chamkha et al. [16] presented a numerical study of coupled heat and mass transfer by boundary-layer free convection over a vertical flat plate embedded in a fluid-saturated porous medium in the presence of thermophoretic particle deposition and heat generation or absorption effects. Ali [17] discussed the effect of suction or injection on the free convection boundary layers induced by a heated vertical plate embedded in a saturated porous medium with an exponential decaying heat generation.

The present study focuses on the effect of Darcy dissipation on melting from a vertical plate with variable temperature embedded in porous medium. The governing system of non-linear partial differential equations is transformed into similarity non-linear ordinary differential equations which are solved numerically using shooting technique together with Runge-Kutta sixth-order integration scheme.

II. Mathematical Formulations

Consider the mixed convection heat transfer from a heated vertical plate embedded in a porous medium saturated with a Newtonian fluid. It is assumed that the plate constitutes the interface between the liquid phase and the solid phase during melting inside the porous matrix. The co-ordinate system and flow model are shown in the Figure **1**.The x coordinate is taken along the plate, the y –coordinate is measured normal to the plate, while the origin of the reference system is taken at the leading edge of the plate. The fluid flow is moderate and the permeability of the medium is assumed to be low so that the Forchheimer flow model is applicable and the boundary-drag effect is neglected. The plate is at a variable temperature T_m at which the material of the porous matrix melts. The liquid phase temperature is T ($>T_m$) and the temperature of the solid far from the interface is T_∞ ($<T_m$). The flow is steady, laminar and two dimensional. With the usual boundary layer and linear Boussinesq approximations, the governing equations, namely the equation of continuity, the momentum and the energy equations for the porous medium may be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial y}=\frac{-K\beta g}{\nu}\frac{\partial T}{\partial y} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\frac{u^2}{K} \tag{3}$$

In the above equations, K,α,ν,β *and* g are the permeability, equivalent thermal diffusivity, kinematic viscosity, thermal expansion coefficient and acceleration due to gravity respectively.
The above problem is solved subject to the following boundary conditions.

$$y=0,\quad T\to T_m=T_\infty+Ax^\lambda,\ k\frac{dT}{dy}=\rho\left[1+C_s\left(T_m-T_s\right)\right]v, \tag{4}$$

$$y\to\infty,\ T\to T_\infty,\ u\to U_\infty \tag{5}$$

Where A>0, we will designate as aiding flow when the buoyancy force has a component in the direction of free stream velocity i.e. $T=T_m=T_\infty+Ax^\lambda\theta(\eta)$,the first boundary condition on the melting surface simply stated that the temperature of the interface equals the melting temperature of the materials saturating the porous matrix. The second boundary condition at y=0 is a direct result of heat balance.
To seek similarity solution to equation (2) and (3) with boundary conditions (4) and (5), we introduce the following dimensionless variables

$$\eta=\sqrt{\frac{u_\infty}{\alpha}}\ yx^{\frac{\lambda-1}{2}}\quad \psi=\sqrt{\alpha u_\infty}\ x^{\frac{\lambda+1}{2}}f(\eta),\quad \theta(\eta)=\frac{T-T_m}{T_\infty-T_m} \tag{6}$$

Substituting equation (6) into equation (2) and (3), we obtain the following transformed governing equations:

$$f''+\frac{Gr}{\mathrm{Re}}\theta'=0 \tag{7}$$

$$\theta''+\frac{1}{2}(1+\lambda)f\theta'-\lambda f'\theta+\frac{Ec}{Da}f'^2=0 \tag{8}$$

The boundary conditions are

$$\eta=0,\theta=0,\ f(0)+2M\theta'(0)=0 \tag{9}$$

$$\eta\to\infty,\theta=1,\ f'=1 \tag{10}$$

Where $M = \dfrac{C_p\left(T_\infty - T_m\right)}{1 + C_s\left(T_m - T_s\right)}$ is the melting parameter, $\mathrm{Re} = \dfrac{U_\infty x}{\nu}$ is the Reynolds number, $Gr = \dfrac{Kg\beta_T\left(T_\infty - T_m\right)x}{\nu^2}$ is the Grashof number $Ec = \dfrac{u_0^2}{C_p\left(T_\infty - T_m\right)}$ is the Eckert number and $Da = \dfrac{K}{x^2}$ is the Darcy number

The ratio $\dfrac{Gr}{\mathrm{Re}}$ in equation (7) is a measure of the relative importance of free and force convection and is the controlling parameter for the present problem.

The heat transfer rate along the surface of the plate q_w can be computed from the Fourier heat conduction law.

$$q_w = -k\left.\frac{\partial T}{\partial y}\right|_{y=0} \tag{11}$$

The heat transfer results can be represented by the local Nusselt number Nu, which is defined as

$$Nu = \frac{hx}{k} = \frac{q_w x}{k\left(T_m - T_\infty\right)} \tag{12}$$

Where h denotes the local heat transfer coefficient and k represent the liquid phase thermal conductivity. Substituting equation (6) and (11) into equation (12) we obtain

$$\frac{Nu}{Ra^{\frac{1}{2}}} = \theta'(0). \tag{13}$$

III. Method of solution

The system of non-linear ordinary differential equation (7) - (8) together with the boundary conditions (9)-(10) is solved numerically using the Nuchtsheim-Swigerstshooting iteration technique together with sixth order Runge-Kutta integration scheme. In a shooting method, the missing (unspecified) initial condition at the initial point of the interval is assumed, and the differential equation is then integrated numerically as an initial value problem to the terminal point. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If a difference exists, another value of the missing initial condition must be assumed and the process is repeated. This process is continued until the agreement between the calculated and the given condition at the terminal point is within the specified degree of accuracy. For this type of iterative approach one naturally inquires whether or not there is a systematic way of finding each succeeding. The present results are validated by direct comparison with those obtained by Bakier [6] as shown in Table 1. It seen from this table that both results are in excellent agreement, which gives confidence in the numerical results obtained for the present problem.

IV. Discussion of Result

In order to test the accuracy of our results, numerical results are presented graphically in Figures. 2,3,4 and 5 and in Tables 1,2,3 and 4 for several set of values of the pertinent parameters such as melting parameter and Eckert number.

Figures 2 and 3 present the influence of melting parameter (M) on velocity and Temperature profiles respectively. It is obvious that increasing the melting parameter causes higher acceleration to the fluid flow which in turn, increases its motion and causes decrease in temperature. This is established by respective increases in the boundary layer thickness of velocity and temperature. Figures 4, 5 show the effect of Eckert number (Ec) on velocity and temperature profiles in the mixed convection flow. It is seen from the figures that the velocity curve decreases with the increase of Eckert number. However, with the increase of Eckert number, there is significant increase in heat generation due to fluid motion and this translates to temperature increase as shown in figure 4. Table 2 present the values of $f(0)$ and $\theta'(0)$ which are proportional respectively to the stream function and heat transfer rate from surface of the plate for various value of melting parameter and Eckert number. It is evident from the table that $f(0)$ and $\theta'(0)$ decrease on increasing melting parameter. On the other hand, the effect of Eckert number on $\theta'(0)$ is opposite, that is, Ec has tendency to enhance $\theta'(0)$. Table 3 shows the effect of buoyancy parameter and Eckert number on $f(0)$ and $\theta'(0)$ in the presence of melting parameter. It is observed from the table that as $\frac{Gr}{\text{Re}}$ increases, the stream function at wall decreases and the heat transfer rate increases. Also as the Eckert number increases, the stream function at wall decreases and heat transfer rate increases due to heating in the plate. In table 4 values of $f(0)$ and $\theta'(0)$ are listed for some values of λ and Ec. From table, it can be seen that as λ increases $f(0)$ increases and $\theta'(0)$ decreases. On the other hand $f(0)$ decreases and $\theta'(0)$ increases with increase in Eckert number.

Table1: Values of $\theta'(0), f(0)$ and $f'(0)$ for different values of buoyancy parameter and melting parameters, $Ec = 0$

		Bakair [4]			Present work		
M	Gr/Re	$\theta'(0)$	$f(0)$	$f'(0)$	$\theta'(0)$	$f(0)$	$f'(0)$
0.4	0.0	0.4570	-0.3656	1.0000	0.4570	-0.3656	1.0000
	1.4	0.6278	-0.5022	2.4000	0.6278	-0.5022	2.4000
	20	1.6865	-1.3492	21.000	1.6866	-1.3493	21.000
2.0	0.0	0.2799	-1.1197	1.0000	0.2743	-1.097	1.0000
	1.4	0.3823	-1.5291	2.4000	0.3808	-1.5232	2.4000
	3.0	0.4754	-1.9016	4.0000	0.4747	-1.8988	4.0000
	8.0	0.6902	-2.7611	9.0000	0.6902	-2.7607	9.0000
	10.0	0.7594	-3.0370	11.000	0.7593	-3.0375	11.000
	20.0	1.0383	-4.1530	21.000	1.0382	-4.1529	21.000

Table2: Values of $f(0)$ and $\theta'(0)$ for different values of Eckert number and melting parameters.

	Ec=0.1		Ec=0.2		Ec=0.3	
M	$f(0)$	$\theta'(0)$	$f(0)$	$\theta'(0)$	$f(0)$	$\theta'(0)$
0	**0**	2.3358	0	3.3128	0	4.0502
0.2	-0.8017	2.0042	-1.1246	2.8114	-1.3704	3.4760
0.4	-1.4251	1.7814	-1.9914	2.4893	-2.4258	3.0330
0.8	-2.3911	1,4944	-3.3387	2.0862	-4.0684	2.5428
1.0	-2.7896	1.3948	-3.8960	1.9480	-4.7507	2.3753

Table3: Values of $f(0)$ and $\theta'(0)$ for different values of Eckert number and buoyacy parameter.

	Ec=0.1		Ec=0.2		Ec=0.3	
$\frac{Gr}{\text{Re}}$	$f(0)$	$\theta'(0)$	$f(0)$	$\theta'(0)$	$f(0)$	$\theta'(0)$
0	-1.1683	1.1683	-1.7549	1.7549	-2.1999	2.1999
0.2	-1.2539	1.2539	-1.7963	1.7963	-2.1966	2.1966
0.4	-1.3535	1.3535	-1.9059	1.9059	-2.3200	2.3200
0.6	-1.4618	1.4618	-2.0456	2.0456	-2.4890	2.4890
0.8	-1.5761	1.5761	-2.2011	2.2011	-2.6797	2.6797

Table4: Values of $f(0)$ and $\theta'(0)$ for different values of Eckert number and constant parameter.

	Ec=0.1		Ec=0.2		Ec=0.3	
λ	$f(0)$	$\theta'(0)$	$f(0)$	$\theta'(0)$	$f(0)$	$\theta'(0)$
0.1	-1.7952	1.7952	-2.4809	2.4809	-3.0125	3.0125
0.2	-1.7430	1.7430	-2.4215	2.4215	-2.9452	2.9452
0.3	-1.6945	1.6945	-2.3661	2.3661	-2.8823	2.8823
0.4	-1.6494	1.6494	-2.3142	2.3142	-2.8234	2.8234
0.5	-1.6073	1.6073	-2.2656	2.2656	-2.7680	2.7680

V. Conclusion

In this study, the effect of Darcy dissipation on melting from a vertical plate with variable temperature embedded in porous medium are considered. The governing equations are derived using the boundary layer and boussineq approximation. These equations are transformed using a similarity transformation and then solved by six-order runge-kutta method. Graphical results for velocity and temperature as well as the nusselt number are presented and discuss for different parametric conditions. It is noted that the velocity and temperature as well as the heat transfer coefficients are significantly affected by the melting and Eckert number in the medium. The heat transfer coefficient is reducing with increasing value of the melting point. Compares with previously published work was performed, and the results were found to be in good agreement

Reference

[1]. Nield D A, Bejan A (2006) Convection in Porous Media, Springer-Verlag, New York.

[2]. Roberts L (1958). On the melting of a semi-infinite body of ice placed in a hot stream of air, J. Fluid Mech. 4: 505–528.

[3]. Tien C, Yen Y C (1965) The effect of melting on forced convection heat transfer, J. Appl. Meteorol. 523–527.

[4]. Epstein M, Cho DH (1976) Laminar film condensation on a vertical melting surface, ASMEJ. Heat Transfer 98: 108–113.

[5]. Sparrow E M, Patankar S V, Ramadhyani S (1977) Analysis of melting in the presence of natural convection in the melt region, ASME J. Heat Transfer 99: 520–526.

[6]. Bakier AY. (1997) Aiding and opposing mixed convection flow in melting from a vertical flat plate embedded in a porous medium, Transport in Porous Media. 29:127–139.

[7]. Gorla RSR, Mansour MA, Hassanien IA, Bakier AY (1999) Mixed convection effect on melting from a vertical plate, Transport in Porous

[8]. Cheng WT, Lin CH (2007), Melting effect on mixed convection heat transfer with aiding and opposing flows from the vertical plate in a liquid saturated porous medium, Int. J. Heat Mass Transfer 50: 3026– 3034.

[9]. Tashtoush B (2005) Magnetic and buoyancy effects on melting from a vertical plate embedded in saturated porous media, Energy Conversion and Management 46: 2566–2577.

[10]. Chamkha A J, Ahmed S E , Aloraier A S(2010) Melting and radiation effects on mixed convection from a vertical surface embedded in a non-Newtonian fluid saturated non-Darcy porous medium for aiding and opposing eternal flows International Journal of the Physical Sciences Vol. 5(7), pp. 1212-1224

[11]. Kazmierczak M, Poulikakos D, Pop I (1986) Melting from a flat plate embedded in a porous medium in the presence of steady natural convection, Numer. Heat Transfer 10: 571–581.

[12]. Kazmierczak M, Poulikakos D, Sadowski D (1987) Melting of a vertical plate in porous medium controlled by forced convection of a dissimilar fluid, Int. Comm. Heat Mass Transfer 14: 507–517.

[13]. Chen MM, Farhadieh R, Baker Jr L (1986) On free convection melting of a solid immersed in a hot dissimilar fluid, Int. J. Heat Mass Transfer 29: 1087–1093.

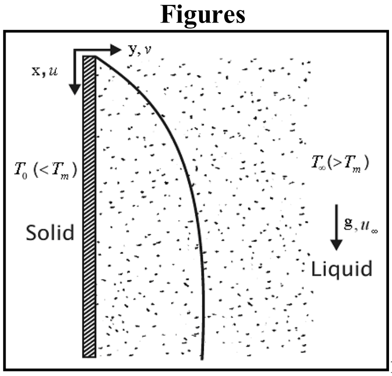
[14]. Hassanien IA, Al-arabi TH (2009) Non–Darcy unsteady mixed convection flow near the stagnation point on a heated vertical surface embedded in a porous medium with thermal radiation and variable viscosity, Commun. Nonlinear. Sci. Numer. Simulat. 14: 1366–1376.

[15]. Murthy PVSN, Mukherjee S, Kharagpur D, Srinivasacharya PVS, Krishna SSR (2004) Combined radiation and mixed convection from a vertical wall with suction/injection in a non-Darcy porous medium, Acta Mech. 168: 145–156.

[16]. Chamkha AJ, Al-Mudhaf AF, Pop I (2006) Effect of heat generation or absorption on thermophoretic free convection boundary layer from a vertical flat plate embedded in a porous medium, Int. Comm. Heat Mass Transfer 33: 1096–1102.

[17]. Ali M E (2007) The effect of lateral mass flux on the natural convection boundary layers induced by a heated vertical plate embedded in a saturated porous medium with internal heat generation, Int. J. Therm. Sci. 46: 157–163.

Figures



n

Figure 1 : The investigated physical model

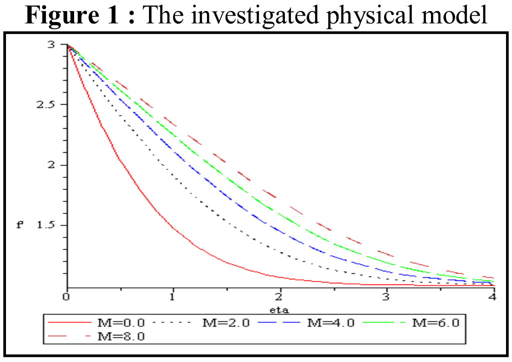


Figure 2. Velocity profile for different values of M, Gr/Re=1.0, Ec=0.1,λ=0.3 and Da=0.1

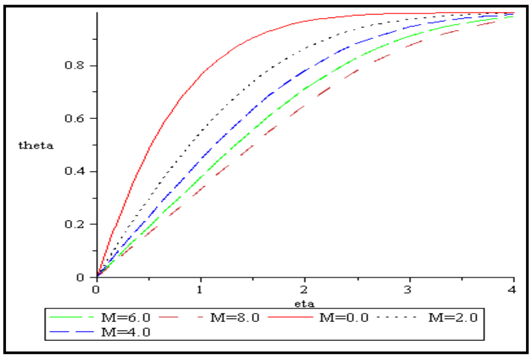


Figure 3. Temperature profile for different values of M Gr/Re=1.0, Ec=0.1,λ=0.3 and Da=0.1

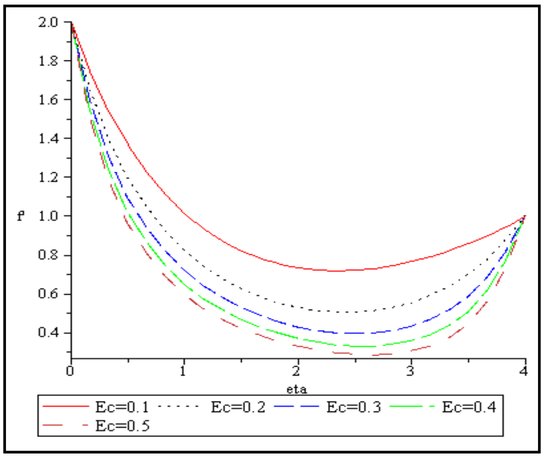


Figure 4. Velocity profile for different values of Ec, M=0.5, Gr/Re=1.0, λ=0.3and Da=0.1

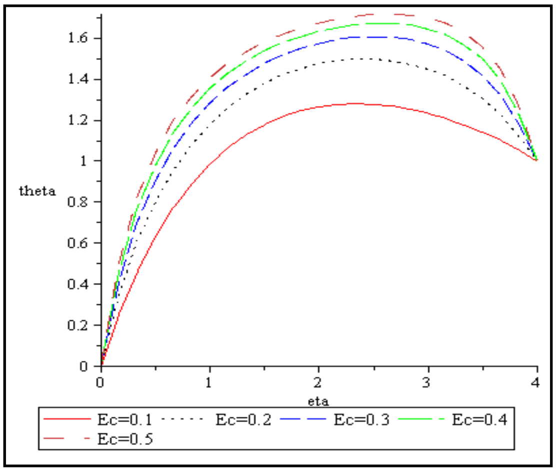


Figure 5. Temperature profile for different values of Ec, M=0.5, Gr/Re=1.0, λ=0.3and Da=0.1