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The Land Equivalent Ratio and Its Mutualistic Implications: Computational Approach

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Abstract: *In this paper, we develop the theoretical and computational per- spectives of tackling and calculating the concept of land equivalent ratio (LER). We apply this idea to discuss its application in a mutualistic interaction.*
Criteria for evaluating different intercropping situations are suggested also in this paper, and the land equivalent ration (LER) concept is considered for situations where intercropping has to be compared with growing each crop in pure stand. The need to use different standardizing sole crop yields in forming LERs is discussed, and a method of calculating an 'effective LER' is proposed to evaluate situations where the yield proportions achieved in intercropping are different from those that might be required by a farmer. The possible importance of effective LERs in indicating the proportions of crops likely to give biggest yield advantages is discussed.
Keywords: and phrases. LER, Mutualism, Numerical Simulation, ODE45, ODE23.

I. Introduction

What is known about the concept of the Land Equivalent Ratio? Within the research activities of crop science and crop protection, the benefits of growing two or more crops together or intercropping are enormous. Assessing these benefits invloves measuring productivity by using the concept of the Land Equivalent Ratio (LER) ([4], [5]). One of the means of understanding the dynamics of the concepts of biological interactions is through the process of mathematical modelling ([2], [3]).

Following our previous mathematical modelling of Lotka-Volterra-like competi- tion models ([2], [3]), we have selected the following parameters in the analysis of the problem we propose to study in this paper: the intraspecific coefficient values of 0.00165; the interspecific coefficient values of 0.0016 for the first variety of sorghum and 0.0015 for the second variety of sorghum; the daily intrinsic growth rate of

0.15 for the second variety of sorghum and the daily intrinsic growth rate of 0.16 for the first variety of sorghum. Without a detailed mathematical analysis that leads to the next vital background ecological information, using this set of model parameters provides us with four possible steady-state solutions namely the trivial steady-state (0, 0) where the two varieties of sorghum will go extinct followed by two other steady-state solutions (0, 90.49) and (96.52, 0) when either of the varieties will tend to survive at its carrying capacity. These two varieties of sorghum will coexist when the biomass of the first variety is 72.52 and the biomass of the second variety is 24.86. Since the inhibiting effect of the second variety on the growth of variety 1 is 0.96 [obtained by dividing the interspefic coefficient of 0.0016 of the first variety by its intraspecific coefficient of 0.00165] ([2]) is less than the ratio of the carrying capacity of the first variety to the carrying capacity of the second variety [that is dividing 96.52 by 90.49], it follows that the first variety of sorghum will survive under this simplifying tested formula. The second variety of sorghum will also survive because the inhibiting effect of the first variety on the growth of the second variety of sorghum is 0.9049 which is less than 0.9375 being the ratio of the carrying capacity of the second variety of sorghum to the carrying capacity of the first variety of sorghum.

This paper is organized into the following sections. Section 1 is focused in defin- ing the concept of the Land Equivalent Ratio and its practical application to agri- cultural data. Section 2 will discuss LER as it affects the biological interaction of mutualism ([3]). These calculations will be quantitatively discussed and analysed.

II. Numerical Calculation of the LER: Application to Mutualistic Interaction

In this section, we are interested in studying the effect of varying the daily intrinsic growth rate of a variety of sorghum on our estimated value of the Land Equivalent Ratio (LER). The goal is to investigate how this growth rate variation affects the incidence of each type of interaction following our recent mathematical modelling idea in the context of plant species interactions in a harsh environment ([3]).
By the assumption of the logistic growth of sorghum as a crop and invoking how the LER is calculated, we will set out to illustrate how we have carried out a systematic numerical calculation of the LER by using a powerful written MATLAB program which is based on the ODE45 simulation technique.

2.1. When the daily intrinsic growth rates are 0.04 and 0.06. If the daily intrinsic growth rates are 0.04 grams per day and 0.06 grams per day and using a complex interspecific competition model, we have used our MATLAB program to calculate the LERs for the first variety of sorghum hereby called species N_1 and also for the second variety of sorghum called species N_2 . The assumption is that these two varieties of sorghum are interacting and competing for a limited resource within the Northern Nigeria agricultural setting. Following [4] and [5], to calculate the partial LER of N_1 we have divided its intercrop yield within an appropriate unit by the value of its pure stand or monoculture yield. On the basis of the popular Lotka-Volterra model formulation or educational model in mathematical biology and our MATLAB simulation program, the following partial LER and total LER are calculated. These calculations are considered for a growing season of 90 days using our ODE45 MATLAB program. The unit of growth is grams.

Weekly Data	Sorghum Growth Data							
week number	S_1	S_{1i}	S_2	S_{2i}	S_1	S_{1i}	S_2	S_{2i}
1	1.00	1.00	2.00	2.00	1.00	1.00	2.00	2.00
2	1.35	2.58	5.55	5.49	1.56	2.58	5.56	5.49
3	1.96	6.45	14.48	14.11	2.58	6.45	14.55	14.11
4	3.24	15.06	32.65	31.31	4.86	15.06	33.08	31.31
5	6.74	30.60	58.02	54.56	11.37	30.60	59.86	54.56
6	17.24	50.40	82.19	73.73	31.60	50.40	88.83	73.73
7	46.35	66.86	107.46	84.07	84.52	66.86	129.86	84.07
8	107.93	76.35	152.34	88.40	182.76	76.35	210.80	88.40
9	204.22	80.70	236.66	90.00	326.46	80.70	345.45	90.00
10	333.46	82.50	359.18	90.60	501.63	82.50	514.43	90.60
11	479.85	83.20	499.90	90.80	668.33	83.20	676.52	90.80
12	614.12	83.47	629.33	90.87	793.40	83.47	797.06	90.87
13	714.85	83.57	727.04	90.90	870.65	83.57	871.58	90.90

Table 1. Calculation of Total LER; length of the growing season is 90 days; growth data of sorghum based on a 90 day growing sea- son; growth data S_1 represent the growth data for the first variety of sorghum in competition with sorghum variety S_2; growth data S_{1i} represent the growth data for variety S_1 growing in isolation of variety S_1; growth data S_2 represent the growth data for the sec- ond variety of sorghum in competition with sorghum variety S_1; growth data S_{2i} represent the growth data for variety S_2 growing in isolation of variety S_2

What do we learn from this Table 1? At the end of the 90 days when the intrinsic growth rate is 0.04, our partial LER for S_1 is 8.5537 (this estimated value was obtained by dividing 714.85 by 83.57 which are the growth data at the end of 90 days for variety S_1) which is followed by the partial LER for S_2 having an estimated value of 7.9986 (this estimated value was obtained by dividing 727.04 by 90.90 which are the growth data at the end of 90 days for variety S_2). By adding the values of these partial LERs for varieties S_1 and S_2 , we obtained our estimated total LER of 16.5523. This value of total LER shows the positive impact of growing sorghum variety S_1 with variety S_2 . Similarly, when the value of the daily intrinsic growth rate is 0.06, the total LER is 20.01 which indicates a positive impact of growing sorghum varieties together.

However, the interacting behaviour is called mutualistic interaction because the species growing together grow better than the isolated varieties.

By considering other variations of the daily intrinsic growth rate, we are inter- ested in finding how the qualitative behaviour of our LER is responding and which type of biological interaction is dominant.

In order to validate the appropriateness of using the notion of LER for the two varieties of sorghum, it would be useful to measure the size of the error between the LER of S_1 and the LER of S_2 when the daily intrinsic growth rate is 0.04. In this scenario, our estimated values of the p-norms of the difference between the LERs are 5.97 for the 1-norm, 1.94 for the 2-norm and 0.84 for the ∞-norm. In contrast, when the daily intrinsic growth rate is 0.06, our estimated values of the p-norms of the difference between the LERs are 6.12 for the 1-norm, 1.95 for the 2-norm and 0.83 for the ∞-norm.

2.2. When the daily intrinsic growth rates are 0.08 and 0.1. In this sub- section, we will consider the situation when the estimated values of our intrinsic growth rates are 0.08 and 0.1. Here, we will present a similar set of partial and total LER numerical calculations. Without delving into detailed explanation as we have done above, our next set of results are presented below.

Weekly Data	Sorghum Growth Data							
week number	S_1	S_{1i}	S_2	S_{2i}	S_1	S_{1i}	S_2	S_{2i}
1	1.00	1.00	2.00	2.00	1.00	1.00	2.00	2.00
2	1.80	2.58	5.60	5.49	2.10	2.58	5.60	5.49
3	3.40	6.45	14.60	14.11	4.40	6.45	14.70	14.11
4	7.30	15.06	33.70	31.31	10.70	15.06	34.50	31.31
5	18.80	30.60	62.70	54.56	30.40	30.60	67.20	54.56
6	54.90	50.40	100.30	73.73	89.20	50.40	119.40	73.73
7	140.10	66.86	167.80	84.07	214.00	66.86	227.00	84.07
8	283.70	76.35	298.90	88.40	414.50	76.35	418.30	88.40
9	481.40	80.70	489.20	90.00	662.40	80.70	658.20	90.00
10	689.50	82.50	690.40	90.60	878.00	82.50	867.60	90.60
11	850.90	83.20	848.00	90.80	1015.40	83.20	999.80	90.80
12	950.70	83.47	943.30	90.87	1085.70	83.47	1067.70	90.87
13	1002.50	83.57	993.40	90.90	1117.60	83.57	1099.00	90.90

Table 2. Calculation of Total LER; length of the growing season is 90 days; growth data of sorghum based on a 90 day growing sea- son; growth data S_1 represent the growth data for the first variety of sorghum in competition with sorghum variety S_2; growth data S_{1i} represent the growth data for variety S_1 growing in isolation of variety S_1; growth data S_2 represent the growth data for the sec- ond variety of sorghum in competition with sorghum variety S_1; growth data S_{2i} represent the growth data for variety S_2 growing in isolation of variety S_2

What do we learn from this Table 2? At the end of the 90 days when the intrinsic growth rate is 0.08, our partial LER for S_1 is 11.9954 whereas our partial LER for S_2 is 10.9296. By adding the values of these partial LERs for varieties S_1 and S_2, we obtained our estimated total LER of 22.925. This value of total LER shows the positive impact of growing sorghum variety S_1 with variety S_2. Similarly, when the value of the daily intrinsic growth rate is 0.1, the total LER is 25.4638 which indicates a positive impact of growing sorghum varieties together.

We will aim at measuring the size of the error between the LER of S_1 and the LER of S_2 when the daily intrinsic growth rates are 0.08 and 0.1. When the daily intrinsic growth rate is 0.08, our estimated values of the p-norms of the difference between the LERs are 6.89 for the 1-norm, 2.23 for the 2-norm and 1.07 for the ∞-norm. In contrast, when the daily intrinsic growth rate is 0.1, our estimated values of the p-norms of the difference between the LERs are 8.24 for the 1-norm, 2.78 for the 2-norm and 1.28 for the ∞-norm.

Henceforth, we will only present our other key results for other variations of the daily intrinsic growth rate for the first plant species. These results would concern the total LER for the two varieties of sorghum under a mutualistic interaction, the size of the error between the the LER of S_1 and the LER of S_2 when the daily intrinsic growth rate ranges from the value of 0.105 to 0.18 and the limiting biomasses between two interacting sorghum varieties and their isolated components. Our estimated calculations are presented as

Paramete	Data									
value	$LERS_1$	$LERS_2$	TLER	p1n	p2n	pmaxn	S_1	S_{1i}	S_2	S_{2i}
0.105	13.6976	12.3604	26.0580	8.702	2.960	1.337	1144.7	83.57	1123.5	90.90
0.110	14.0141	12.6259	26.6400	9.148	3.155	1.388	1171.2	83.57	1147.6	90.90
0.115	14.3343	12.8785	27.2128	9.70	3.36	1.46	1197.9	83.57	1170.60	90.90
0.120	14.6304	13.1438	27.7740	10.35	3.57	1.49	1222.7	83.57	1194.70	90.90
0.125	14.9351	13.3932	28.3283	10.99	3.80	1.54	1248.2	83.57	1217.40	90.90
0.130	15.2337	13.6426	28.8763	11.77	4.02	1.59	1273	83.57	1240	90.90
0.135	15.5316	13.8877	29.4193	12.78	4.30	1.64	1298	83.57	1262.30	90.90
0.140	15.8141	14.1420	29.9561	13.85	4.55	1.69	1321.60	83.57	1285.40	90.90
0.145	16.1212	14.3708	30.4920	15.00	4.85	1.75	1347.30	83.57	1306.20	90.90
0.150	16.4144	14.6086	31.023	16.195	5.158	1.834	1371.80	83.57	1327.90	90.90
0.155	16.6871	14.8608	31.5479	17.36	5.46	1.940	1394.6	83.57	1350.8	90.90
0.160	16.9769	15.0963	32.0732	18.60	5.80	2.06	1418.8	83.57	1372.20	90.90
0.165	17.2571	15.3361	32.5952	19.83	6.13	2.18	1442.40	83.57	1394.00	90.90
0.170	17.5491	15.5675	33.1166	21.07	6.47	2.34	1466.60	83.57	1415.00	90.90
0.175	17.8319	15.8042	33.6361	22.4307	6.85	2.50	1490.20	83.57	1436.50	90.90
0.180	18.1131	16.0411	34.1542	23.80	7.24	2.68	1513.70	83.57	1458.00	90.90

Table 3. Calculation of Total LER; length of the growing season is 90 days; growth data of sorghum based on a 90 day growing season; S_1 represents the limiting biomass for the first variety of sorghum in competition with sorghum variety S_2; S_{1i} represents the limiting biomass for the variety S_1 growing in isolation of va- riety S_1; S_2 represents the limiting biomass for the second variety of sorghum in competition with sorghum variety S_1; S_{2i} represents the limiting biomass for the variety S_2 growing in isolation of va- riety S_2

Here, the notations p1n represents $p-1$ norm, p2n represents $p-2$ norm and pmaxn represents the p maximum norm or the infinity norm.

III. Discussion of Results

We observe that as the daily intrinsic growth rate for species S_1 is slightly in- creased, the total LER and the size of the error between the LERs similarly in- crease. Within this interval space of increase in the value of the intrinsic growth rate, our numerical calculation estimates clearly show that the factor of mutualism is a dominant type of species interaction which we have not seen elsewhere.

IV. Concluding Remarks and Further Research

In this study, we have developed the theoretical and analytical perspectives of the concept of land equivalent ratio (LER) in which this idea has been systematically applied to demonstrate that mutualism is a dominant type of species interaction. The implication of this key result is that when two crop species are interacting mutualistically for a limited resource in the environment, the consistent positive value of the total LER clearly shows the positive advantage of growing one crop species with another crop species and that more land is required to sustain this beneficial interaction behaviour.

This present result is an extension of our first earlier study. Our numerical approach to tackling similar crop data is proposed to relate the idea of the (LER) to other types of species interactions such as commensalism and predation.

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