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Semi Global Dominating Set of Intuitionistic fuzzy graphs

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Abstract: *In this paper, new types of Intuitionistic fuzzy graphs are introduced and also some properties of the defined graphs are discussed. Also we defined the semi global Intuitionistic fuzzy dominating set and its number of Intuitionistic fuzzy graphs. Some results and bounds on semi global Intuitionistic fuzzy dominating number are derived, which is used in defence problems and bank transactions.*
Keywords: *Intuitionistic fuzzy graph, effective degree, semi complementary IFG, Semi-complete IFG, semi global Intuitionistic fuzzy dominating set.*
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I. Introduction:

Atanassov [1] introduced the concept of intuitionistic fuzzy (IF) relations and intuitionistic fuzzy graphs (IFGs). Research on the theory of intuitionistic fuzzy sets (IFSs) has been witnessing an exponential growth in Mathematics and its applications. R. Parvathy and M.G.Karunambigai's paper [4] introduced the concept of IFG and analyzed its components. Nagoor Gani, A and Sajitha Begum, S [3] defined degree, Order and Size in intuitionistic fuzzy graphs and extend the properties. The concept of Domination in fuzzy graphs is introduced by A. Somasundaram and S. Somasundaram [10] in the year 1998. Parvathi and Thamizhendhi[5] introduced the concepts of domination number in Intuitionistic fuzzy graphs. Domination is active subject in fuzzy graph and Intuitionistic fuzzy graphs, and has numerous applications to distributed computing, the web graph and adhoc networks. Study on domination concepts in Intuitionistic fuzzy graphs are more accurate than fuzzy graphs, which is useful in the traffic density and telecommunication systems. Global domination number of a graph is introduced by E. Sampathkumar [7] in 1989. Siva Rama Raju et. al. [9] analyzed the semi global domination in the crisp graph.

In this paper, we introduce new types of IFGs, semi complementary Intuitiontistic fuzzy graph and semi complete Intuitiontistic fuzzy graph which are useful in the defence problems and Bank transcations. Also we discussed the semi global Intuitionistic fuzzy domination set and its number in the IFG, which is useful to solve Transportation problems in more efficient way. Some bounds on semi global Intuitionistic fuzzy number are established.

II. Preliminary

Definition 2.1: An Intuitionistic fuzzy graph is of the form G = (V, E) where
(i) V={v1,v2,….,vn} such that μ_1: V→[0,1]and γ_1: V →[0,1] denote the degree of membership and non-membership of the element vi ЄV, respectively, and $0 \le \mu_1 (vi) + \gamma_1 (vi) \le 1$ for every vi ЄV, (i = 1,2, ……. n),
(ii) E ⊆ V x V where μ_2: V x V →[0,1] and γ_2: V x V →[0,1] are such that $\mu_2 (vi , vj) \le \min [\mu_1(vi), \mu_1(vj)]$ and $\gamma_2 (vi , vj) \le \max [\gamma_1(vi), \gamma_1(vj)]$ and $0 \le \mu_2 (vi, vj) + \gamma_2 (vi, vj) \le 1$ for every (vi ,vj) Є E, (i, j = 1,2, …,n)

Definition 2.2: An IFG H = < V', E' > is said to be an Intuitionistic fuzzy sub graph (IFSG) of the IFG, G = < V, E > if V' ⊆V and E' ⊆ E. In other words, if $\mu_{1i}' \le \mu_{1i}$; $\gamma_{1i}' \ge \gamma_{1i}$ and $\mu_{2ij}' \le \mu_{2ij}$; $\gamma_{2ij}' \ge \gamma_{2ij}$ for every i, j = 1,2………n.

Definition 2.3: Let G = (V,E) be a IFG. Then the cardinality of G is defined as

$$|G| = \left|\sum_{vi\in V} \frac{1+\mu 1\ (vi)-\gamma 1\ (vi)}{2} + \sum_{vi,vj\in E} \frac{1+\mu 2\ (vi,vj)-\gamma 2(vi,vj)}{2}\right|$$

Definition 2.4: The vertex cardinality of IFG G is defined by $|V| = \left|\sum_{vi\in V} \frac{1+\mu 1\ (vi)-\gamma 1\ (vi)}{2}\right| = p$ and The edge cardinality of IFG G is defined by $|E| = \left|\sum_{vi,vj\in E} \frac{1+\mu 2\ (vi,vj)-\ \gamma 2(vi,vj)}{2}\right| = q$.
The vertex cardinality of IFG is called the order of G and denoted by O(G). The edge cardinality of G is called the size of G, denoted by S(G).
Definition 2.5: An edge e = (x, y) of an IFG G = (V, E) is called an effective edge if
$\mu_2(x, y) = \mu_1(x) \wedge \mu_1 (y)$ and $\gamma_2(x, y) = \gamma_1(x) \vee \gamma_1(y)$.
Definition 2.6: An Intuitionistic fuzzy graph is complete if $\mu_{2ij} = \min (\mu_{1i}, \mu_{1j})$ and
$\gamma_{2ij} = \max (\gamma_{2i} ,\gamma_{2j})$ for all (vi , vj) Є V.

Definition 2.7: An Intuitionistic fuzzy graph G is said to be strong IFG if
$\mu_2(x, y) = \mu_1(x) \wedge \mu_1(y)$ and $\gamma_2(x, y) = \gamma_1(x) \vee \gamma_1(y)$ for all (vi , vj) $\in$ E. That is every edge is effective edge.
Definition 2.8 : The complement of an IFG G = (V, E) is denoted by $\bar{G} = (\bar{V}, \bar{E})$ and is defined as
i) $\bar{\mu}_1(v) = \mu_1(v)$ and $\bar{\gamma}_1(v) = \gamma_1(v)$
ii) $\bar{\mu}_2(u,v) = \mu_1(u) \wedge \mu_1(v) - \mu_2(u,v)$ and $\bar{\gamma}_2(u,v) = \gamma_1(u) \vee \gamma_1(v) - \gamma_2(u,v)$ for u,v in V
$\bar{G}$ also denoted by G^c.
Definition 2.9: Let G = (V,E) be an IFG. The neighbourhood of any vertex v is defined as
$N(v) = (N_\mu(v), N_\gamma(v))$, Where $N_\mu(v) = \{w \in V ;\ \mu_2(v.w) = \mu_1(v) \wedge \mu_1(w)\}$ and
$N_\gamma(v) = \{w \in V ;\ \gamma_2(v.w) = \gamma_1(v) \vee \gamma_1(w)\}$. $N[v] = N(v) \cup \{v\}$ is called the closed neighbourhood of v.
Definition 2.10: The neighbourhood degree of a vertex is defined as $d_N(v) = (d_{N\mu}(v), d_{N\gamma}(v))$ where $d_{N\mu}(v) = \sum_{w \in N(v)} \mu_1(w)$ and $d_{N\gamma}(v) = \sum_{w \in N(v)} \gamma_1(w)$.
The minimum neighbourhood degree is defined as $\delta_N(G) = (\delta_{N\mu}(v), \delta_{N\gamma}(v))$, where $\delta_{N\mu}(v) = \wedge \{ d_{N\mu}(v): v \in V\}$ and $\delta_{N\gamma}(v) = \wedge \{ d_{N\gamma}(v): v \in V\}$.
Definition 2.11: The effective degree of a vertex v in a IFG G = (V, E) is defined to be sum of the effective edges incident at v, and denoted by $d_E(v)$. The minimum effective degree of G is $\delta_E(G) = \wedge \{d_E(v)/v \in V\}$
Definition 2.12: Let G= (V, E) be an IFG. Let u, v $\in$ V, we say that u dominated v in G if there exist a strong arc between them. A subset D $\subseteq$V is said to be dominating set in G if for every v$\in$ V-D, there exist u in D such that u dominated v. The minimum scalar cardinality taken over all dominating set is called domination number and is denoted by $\gamma(G)$. The maximum scalar cardinality of a minimal domination set is called upper domination number and is denoted by the symbol $\Gamma(G)$.
Definition 2.13: Let G= (V, E) be an IFG. A subset D $\subseteq$V is said to be total dominating set in G if every vertex in V is dominated by a node in D. The minimum cardinality of all total dominating sets is called total Intuitionistic fuzzy domination number and denoted by $\gamma_t(G)$
Definition 2.14: An independent set of an Intuitionistic fuzzy graph G = (V, E) is a subset S of V such that no two vertices of S are adjacent in G.
Definition 2.15: A Bipartite IFG, G= (V,E) is said to be complete Bipartite IFG, if $\mu_2(v_i, v_j) = \mu_1(v_i) \wedge \mu_1(v_j)$ and $\gamma_2(v_i, v_j) = \gamma_1(v_i) \vee \gamma_1(v_j)$ for all $v_i \in V_1$ and $v_j \in V_2$. It is denoted by $K_{V1i\,,V2i}$.

III. Semi-Complementary IFG and Semi complete IFG

Definition 3.1: Let G = (V,E) be an IFG, then semi complementary IFG of G which is denoted by
$G^{sc} = (V^{sc}, E^{sc})$ defined as (i) $\mu_1^{sc}(v) = \mu_1(v)$ and $\gamma_1^{sc}(v) = \gamma_1(v)$ where $(\mu_1^{sc}(v), \gamma_1^{sc}(v)) \in V^{sc}$
(ii) E^{sc} = { uv $\notin$ E and $\exists$w such that uw and wv in E then $\mu_2^{sc}(u,v) = \mu_1(u) \wedge \mu_1(v)$, $\gamma_2^{sc}(u, v) = \gamma_1(u) \vee \gamma_1(v)$ }
Example: =

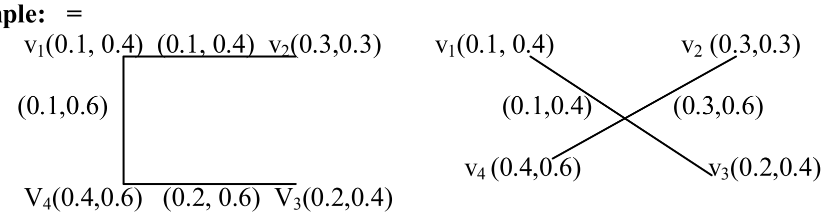

Fig- 1:Intuitionistic fuzzy graph **G^{sc}-Semi complementary IFG**

Observations:
(i) Let G be connected IFG with effective edges, but G^{sc} need not be connected IFG.
(ii) $(G^c)^c = G$ but $(G^{sc})^{sc} \neq G$
(iii) G^{sc} is spanning sub graph of G and $|E(G^c)| \geq |E(G^{sc})|$
(iv) Every edge (u, v) in G^{sc} is not neighbor in G.
(v) G be a complete IFG then $G^{sc} = G^c$ = null graph
(vi) G^{sc} having always effective edges only.

Definition 3.2: G = (V, E) be a connected IFG with effective edges which is said to be semi complete IFG, if every pair vertices have a common neighbor in G .

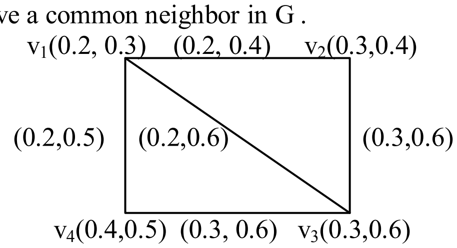

Fig-2:Semi complete IFG

Remark 3.3:

1. Every Complete IFG is semi complete IFG. But the converse is not true.
2. Every semi complete IFG has cycles.

Theorem 3.4: The necessary and sufficient condition for a connected IFG with effective edges to be semi complete IFG is any pair vertices lie on the same triangle or lie on two different triangles have a common vertex.

Proof: Since by the definition of Semi complete IFG, every pair of vertices have a common neighbor, that is any pair of vertices lie on the same triangle. If not, they lie on two different triangles have a common vertex. Otherwise we are not getting the common neighbor for some pair of vertices.

Example: In Fig-2, v_1, v_2 are lie on the same triangle $v_1v_2v_3$ and so on. But the pair of vertices v_2,v_4 are not lie on the same triangle but lie on two different triangles with common vertices v_1, v_3.

Theorem 3.5: Let G be the connected IFG with effective edges, then $G^c = G^{sc}$ if and only if between every pair of non-adjacent vertices there must be two effective edges.

Proof: Given G is connected IFG. Since G and G^{sc} have same vertex set we get $G^c = G^{sc}$ Implies uv $\in G^c$ if and only if uv also belongs to G^{sc}.

Similarly, let u and v are not adjacent in G then there must be two effective edges between them.

Theorem 3.6: Let G be semi complete IFG with effective edges, then $G^c = G^{sc}$.

Proof: Given G is semi complete IFG. Therefore between any pair of non adjacent vertices there must be two effective edges.

IF two vertices are adjacent in G then also there must be a path of two effective edges between them in G as it is a semi complete IFG.

i.e) $G^c = G^{sc}$.

Remark 3.7 : The converse of the above theorem is not true. i.e.) If $G^{sc} = G^c$ then G need not be semi complete IFG.

Example:

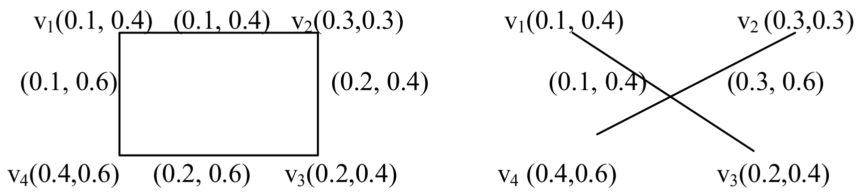

Fig -3: IFG – G and G^{sc}.

In the above IFG, $G^{sc} = G^c$ but G is not semi complete IFG.

Theorem 3.8 : G =(V,E) is IFG with effective edges and G^{sc} is also connected with effective edges then G is cyclic IFG.

Proof: Let e= uv ∈ E(G). which implies u, v are the vertices of G and G^{sc}.

Since G^{sc} is connected there is shortest uv path in G^{sc} . This induces a path P in G.

Now P ∪ {e} is a cycle in G. Thus G is cyclic IFG.

Remark 3.9 : The converse is need not true. That is G is cyclic G^{sc} need not connected.

Example: In Fig-3, The IFG G is cyclic but G^{sc} is not connected.

IV. Semi Global Intuitionistic Fuzzy Domination set of IFG

Definition 4.1: Let G = (V, E) be connected IFG. The set D ⊆V is said to be Semi global Intuitionistic fuzzy domination set (sgifd-set) of G if D is a Domimating set for both G and G^{sc} .

The minimum cardinality of all sgifd-sets of G is called Semi global Intuitionistic fuzzy domination number and is denoted by $\gamma_{sg}(G)$. The maximum cardinality of sgifd-sets is called upper semi global Intuitionistic fuzzy domination number and denoted by $\Gamma_{sg}(G)$

Example

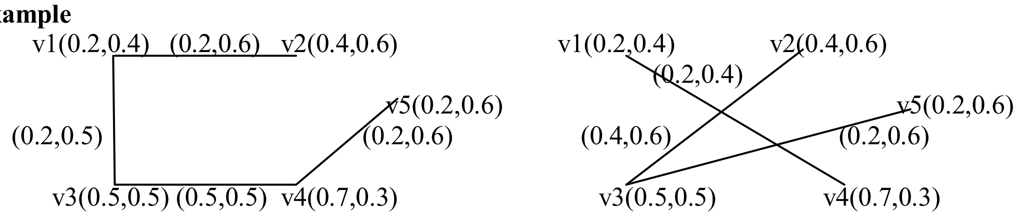

Fig – 4: IFG (G) with 5 vertices and G^{sc}.

Here The cardinalities of the vertices are |v1| = 0.4 , |v2| = 0.4 , |v3| = 0.5 , |v4| = 0.7 , |v5| = 0.3
The sgifd-sets are {v1,v2,v5}, {v1,v3,v4} and { v1, v3,v5} and their cardinalities are 1.1 , 1.6 and 1.2
Therefore, $\gamma_{sg}(G) = 1.1$ and $\Gamma_{sg}(G) = 1.6$

Proposition 4.2: The semi global Intuitionistic fuzzy dominating set is not singleton.
Proof: Since sgifd-set contain dominating set for both G and G^{sc} then at least two vertices are in the set.
i.e) The sgifd-set containing more than two vertices.

Proposition 4.3: Let G = (V, E) be complete IFG K_n, then $\gamma_{sg}(K_n) = p$.
Proof: Since the semi complementary IFG of Complete IFG is isolated vertices. i.e) sgifd-set contains all the vertices of G. Therefore $\gamma_{sg}(K_n) = p = O(G)$.
Proposition 4.4: Let G = (V, E) be complete Bipartite IFG $K_{V1i,V2i}$, Then $\gamma_{sg}(K_{V1i,V2i})$ = Min { | vi| + |vj | } Where $v_i \in V_1$ and $v_j \in V_2$.
Proof: The semi complement IFG of Complete Bipartite IFG is two partition sets V_1, V_2 and both are Complete IFG. Since G is complete Bipartite IFG, each $v_i \in V_1$ dominated all vertices all vertices of V_2 and vice versa. And that also dominating set in G^{sc}. we get, $\gamma_{sg}(K_{V1i,V2i})$ = Min { | v_i| + |v_j | } $v_i \in V_1$ and $v_j \in V_2$.

Definition 4.5: Let G = (V, E) be connected IFG. The set D ⊆V is said to be Global Intuitionistic fuzzy domination set (gifd-set) of G if D is a Domimating set for both G and G^c .
The minimum cardinality of all gifd-sets of G is called Global Intuitionistic fuzzy domination number and is denoted by $\gamma_g(G)$. The maximum cardinality of all gifd-sets of G is called Upper Global Intuitionistic fuzzy domination number and denoted by $\Gamma_g(G)$.

Example:

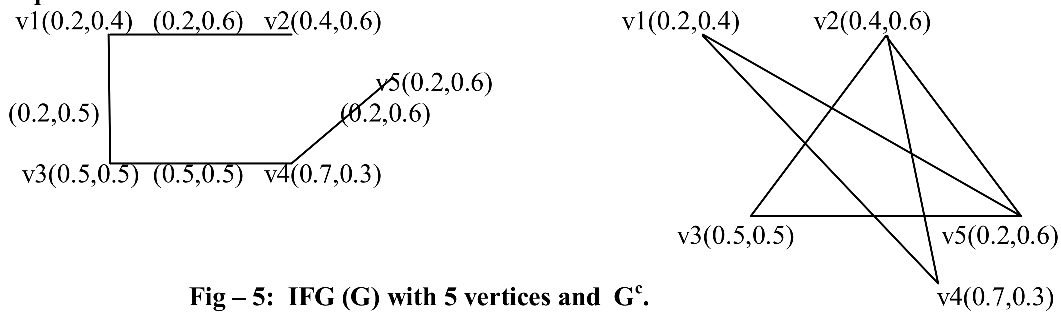

Fig – 5: IFG (G) with 5 vertices and G^c.

Here in G^c , $\mu_2^c(u, v) = \mu_1(u) \Lambda \mu_1(v)$ and $\gamma_2^c(u, v) = \gamma_1(u) V \gamma_1(v)$, for every u, v $\in V^c$.
In the above example, gifd-set is { v1, v5}. Therefore, γg(G) = 0.7 = Γg(G)

Remark: The sgifd-set is also gifd-set of G That is , $\gamma_g(G) \leq \gamma_{sg}(G)$.

Proposition 4.6: Let G be a semi complete IFG, D⊂V, Then D is sgifd-set in G if and only if D is a global Intuitionistic fuzzy domination set in G.
Example: In Fig-2, The sgifd-set is D = {v_1, v_2, v_3} and the same also gifd-set. That is $\gamma_{sg}(G) = 1.25= \gamma_g(G)$.

Theorem 4.7: Let G = (V, E) be connected IFG with effective edges, then Min{|V_i|+|V_j|} ≤ γsg(G) ≤ p, $i \neq j$ and for every v_i, $v_j \in V$.
Proof: We know that Semi global Intuitonistic fuzzy dominating set has at least two vertices. Let {vi, vj} are the vertices, then Min{|Vi|+|Vj|} = γsg(G)
If the set contains other than {vi, vj} then Min{|Vi|+|Vj|} < γsg(G), $i \neq j$
If the given G is complete IFG then sgifd-set contains all the vertices of the G, that is γsg(G)≤ O(G) = p
i.e.) We get, Min{|Vi|+|Vj|} ≤ $\gamma_{sg}(G)$ ≤ p.

Theorem 4.8: Let G be a semi complete IFG, Then γsg(G) ≥ Min { |vi| + |vj|+|vk|} , i≠j≠k
Proof: It is enough to prove sgifd-set containing at least three vertices.
Suppose sgifd-set contains less than three vertices
We know that sgifd-set not a singleton. i.e) sgifd-set contains at least two vertices
Let D = { v_1, v_2} be a sgifd-set in G.

Case 1: <D> is connected in G
Then v_1v_2 is an effective edge in G. By the definition of semi complete IFG, there is a v_3 in G such that <$v_1v_2v_3$> is triangle in G, i.e) D is not a Intuitonistic fuzzy domination set in G^{sc}.
Which is contradiction to D is a sgifd-set in G.

Case 2: <D> is disconnected in G. i.e.) There is no effective edge between v_1 and v_2 .
Since G is semi complete IFG, there is v_3 in G such that $v_1v_3$and v_3v_2 are the effective edges in G
Therefore, In G^{sc}, v_3 is not dominated by a vertex in D.
Which implies, D is not a sgifd-set in G
Which is contradiction to our assumption.
Therefore we get, $\gamma_{sg}(G) \geq$ Min { $|v_i| + |v_j|+|v_k|$} , $i \neq j \neq k$ for semi complete IFG.

Theorem 4.9: Let G = (V, E) be the IFG with effective edges . $\gamma_{sg}(G)$ = Min{|Vi|+|Vj|} $i \neq j$ if and only if there is an effective edge uv in G such that each vertex in V – {u, v} is adjacent to u or v but not both.
Proof: Suppose $\gamma_{sg}(G)$ = Min{|Vi|+|Vj|} $i\neq j$, We assume D = {u, v} be the sgifd-set in G
Let <D> is connected in G, then uv is an effective edge in G.
If any vertex w in V-{u, v} is adjacent to both u and v.
Which implies D is not a dominating set for G^{sc}. which is contradiction to our assumption. i.e) effective edge uv in G such that each vertex in V – {u, v} is adjacent to u or v but not both.
Conversely, each vertex in V – {u, v} is adjacent to u or v but not both, then γsg(G) = Min{|Vi|+|Vj|} $i\neq j$.

Theorem 4.10: The set $D\subset V$ is a sgifd-set in G if and only if each vertex in V-D lies on an effective edge whose end vertices are totally dominated by distinct vertices in D.
Proof: Let us assume D is a sgifd-set in G. Let $v_1 \in$ {V- D}. Then there exist distinct vertices v_2, v_3 in D such that v_1v_2 in E(G) and v_1v_3 in E(G^{sc}),
Since, the edge v_1v_3 is in E(G^{sc}), then there exist v_4 in V such that v_1v_4 and v_4v_3 are effective edges in G.
Case 1: Suppose $v_4 = v_2$, Then, v_1v_2 and v_2v_3 are effectives edges in G which implies v_1 lies on the edge v_1v_4 and v_1, v_4 are dominated by v_2 and v_3 respectively from D – {v_1}, D – { v_1,v_2}.
Case 2: Suppose $v_4 \neq v_2$, Then < v_2 v_1 v_4 v_3 > is a path in G which implies v_1 lies on the edge v_1v_4 and v_1, v_4 are dominated by v_2 and v_3 respectively from D – { v_1,v_4}
i.e) each vertex in V-D lies an effective edge whose end vertices are totally dominated by distinct vertices in D.
Conversely, assume $v_1 \in$ {V-D}. By our assumption there is an edge v_1v_2 in G such that v_1v_3, v_2v_4 are effective edges in G and {v_3,v_4} are in G ($v_3 \neq v_4$)
Now, If $v_3= v_2$ then <v_1 v_2 v_4> is a path in G and v_1v_2 is in G, but v_1v_4 is in G^{sc}.
And, If $v_3 \neq v_2$ then < v_3 v_1 v_2 v_4> is a path in G, which implies v_1v_3 is in G and v_1v_4 is in G^{sc}.
Hence , we have D is sgifd-set in G.

V. Conclusion

Here, new type of IFGs, Semi complementary Intuitiontistic fuzzy graph and Semi complete Intuitiontistic fuzzy graph are introduced. Some results on Semi complementary IFG and Semi complete IFG are derived. Also, the semi global Intuitionistic fuzzy domination set and its number of IFG are discussed, which is useful to solve Intuitionistic fuzzy Transportation problems in more efficient way. Some bounds on semi global Intuitionistic fuzzy domination number are established. Further some other domination parameters will be introduced in IFGs.

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