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Strongly α * Continuous Functions in Topolgical Spaces

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Abstract: *The Purpose Of This Paper Is To Introduce Strongly And Perfectly α *Continuous Maps And Basic Properties And Theorems Are Investigated. Also, We Introduced α * Open And Closed Maps And Their Properties Are Discussed.*
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I. Introduction

In 1960, Levine . N [3] introduced strong continuity in topological spaces. Beceren.Y [1] in 2000, introduced and studied on strongly α continuous functions. Also, in 1982 Malghan [5] introduced the generalized closed mappings Recently, S.Pious Missier and P. Anbarasi Rodrigo[8] have introduced the concept of α * -open sets and studied their properties. In this paper we introduce and investigate a new class of functions called strongly α * continuous functions. Also we studied about α * open and α * closed maps and their relations with various maps

II. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η) or X , Y , Z represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure and the interior of A respectively. The power set of X is denoted by P(X).

Definition 2.1: A function f: (X, τ) ⟶ (Y, σ) is called a **strongly continuous** [3]if $f^{-1}(O)$ is both open and closed in (X, τ) for each subset O in (Y, σ).
Definition 2.2: A function f: (X, τ) ⟶ (Y, σ) is called a α **– continuous** [4] if $f^{-1}(O)$ is a α **open** set [6] of (X, τ) for every open set O of (Y, σ).
Definition 2.3: A function f: (X, τ) ⟶ (Y, σ) is called a α *** continuous** [9]if $f^{-1}(O)$ is a α *open set of (X, τ) for every open set O of (Y, σ).
Definition 2.4: A function f: (X, τ) ⟶ (Y, σ) is called a **g – continuous** [10] if $f^{-1}(O)$ is a **g -open** set [2] of (X, τ) for every open set O of (Y, σ).
Definition 2.5: A function f: (X, τ) ⟶ (Y, σ) is called a **perfectly continuous** [7] if $f^{-1}(O)$ is both open and closed in (X, τ) for every open set O in (Y, σ).
Definition 2.6: A map f: (X, τ) ⟶ (Y, σ) is called a **g-closed** [5] if f(O) is g-closed in (Y, σ) for every closed set O in (X, τ).
Definition 2.7: A Topological space X is said to be α ***$T_{1/2}$ space** [9] if every α * open set of X is open in X.
Theorem 2.8[8]**:**
(i) Every open set is α *- open and every closed set is α *-closed set
(ii) Every α -open set is α *-open and every α -closed set is α *-closed.
(iii) Every g-open set is α *-open and every g-closed set is α *-closed.

III. Strongly α * Continuous Function

We introduce the following definition.
Definition 3.1: A function f: (X, τ) ⟶ (Y, σ) is called a strongly α * continuous if the inverse image of every α *open set in (Y, σ) is open in (X, τ).
Theorem 3.2: If a map f: X ⟶ Y from a topological spaces X into a topological spaces Y is strongly α * continuous then it is continuous .

Proof: Let O be a open set in Y. Since every open set is α *open, O is α *open in Y. Since f is strongly α *continuous , $f^{-1}(O)$ is open in X. Therefore f is continuous.

Remark 3.3: The following example supports that the converse of the above theorem is not true in general.

Example 3.4: Let X = Y = {a, b, c}, τ = {ϕ, {a}, X}, σ = {ϕ, {ab},Y}. Let g: (X, τ) ⟶(Y, σ) be defined by g (a) = g(b) = a, g(c) = b. Clearly, g is not strongly α *continuous since {a} is α * open set in Y but $g^{-1}(\{a\})$ = {a,b} is not a open set of X. However, g is continuous.

Theorem 3.5: A map f: X ⟶ Y from a topological spaces X into a topological spaces Y is strongly α * continuous if and only if the inverse image of every α * closed set in Y is closed in X.

Proof: Assume that f is strongly α * continuous. Let O be any α * closed set in Y. Then O^c is α *open in Y. Since f is strongly α * continuous, $f^{-1}(O^c)$ is open in X .But $f^{-1}(O^c)$= X/ $f^1(O)$ and so $f^{-1}(O)$ is closed in X.
Conversely, assume that the inverse image of every α * closed set in Y is closed in X . Then O^c is α *closed in Y. By assumption, $f^{-1}(O^c)$ is closed in X, but $f^{-1}(O^c)$= X/ $f^{1}(O)$ and so $f^{-1}(O)$ is open in X. Therefore, f is strongly α * continuous.

Theorem 3.6: If a map f: X ⟶ Y is strongly continuous then it is strongly α *continuous.

Proof: Assume that f is strongly continuous . Let O be any α *open set in Y. Since f is strongly continuous, $f^{-1}(O)$is open in X. Therefore, f is strongly α *continuous.

Remark 3.7: The converse of the above theorem need not be true.

Example 3.8: Let X = Y= {a, b, c}, τ = {ϕ,{a},{ab},{ac}, X} and σ = {ϕ, {a},{b},{ab},Y}. Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = a = f(b) , f(c) = c. clearly, f is strongly α *continuous. But $f^{-1}(\{a\})$ = {a,b} is open in X, but not closed in X. Therefore f is not strongly continuous.

Theorem 3.9: If a map f: X ⟶ Y is strongly α * continuous then it is α *continuous.

Proof: Let O be an open set in Y. By [8] O is α * open in Y. Since f is strongly α * continuous ⟹ $f^{-1}(O)$ is open in X. By [8] $f^{-1}(O)$ is α * open in X. Therefore, f is α * continuous.

Remark 3.10: The converse of the above theorem need not be true.

Example 3.11: : Let X = Y= {a, b, c, d}, τ = {ϕ, {a},{b},{ab}, X} and σ = {ϕ, {a},{ab},Y}, α *O(Y, σ)= P(X) . Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = f(d) = a, f(b) = b , f(c) = c. Clearly, f is α *continuous. But $f^{1}(\{a\})$ = {a,d} is not open in X. Therefore f is not strongly α *continuous.

Theorem 3.12: If a map f: X ⟶ Y is strongly α * continuous and a map g: Y ⟶ Z is α *continuous then g ∘ f: X ⟶ Z is continuous.

Proof: Let O be any open set in Z. Since g is α * continuous, $g^{-1}(O)$ is α * open in Y . Since f is strongly α * continuous $f^{-1}(g^{-1}(O))$ is open in X. But $(g \circ f)^{-1}(O) = f^{-1}(g^{-1}(O))$. Therefore, g ∘f is continuous.

Theorem 3.13: If a map f: X ⟶ Y is strongly α * continuous and a map g: Y ⟶ Z is α *irresolute , then g ∘ f: X ⟶ Z is strongly α * continuous.

Proof: Let O be any α *open set in Z. Since g is α * irresolute, $g^{-1}(O)$ is α * open in Y. Also, f is strongly α * continuous $f^{-1}(g^{-1}(O))$ is open in X. But $(g \circ f)^{-1}(O) = f^{-1}(g^{-1}(O))$ is open in X. Hence, g ∘ f: X ⟶ Z is strongly α * continuous.

Theorem 3.14: If a map f: X ⟶ Y is α * continuous and a map g: Y ⟶ Z is strongly α *continuous , then g ∘ f: X ⟶ Z is α * irresolute.

Proof: Let O be any α *open set in Z. Since g is strongly α * continuous, $g^{-1}(O)$ is open in Y. Also, f is α *continuous, $f^{1}(g^{-1}(O))$ is α *open in X. But $(g \circ f)^{-1}(O) = f^{-1}(g^{-1}(O))$. Hence, g ∘ f: X ⟶ Z is α * irresolute.

Theorem 3.15: Let X be any topological spaces and Y be a α *$T_{1/2}$ space and f: X ⟶ Y be a map. Then the following are equivalent

1) f is strongly α * continuous
2) f is continuous

Proof: (1) ⟹(2) Let O be any open set in Y. By thm [] O is α * open in Y. Then $f^{-1}(O)$ is open in X. Hence , f is continuous.

(2) ⟹(1) Let O be any α * open in (Y, σ). Since, (Y, σ) is a α *$T_{1/2}$ space, O is open in (Y, σ). Since, f is continuous. Then $f^{-1}(O)$ is open in (X, τ) . Hence, f is strongly α * continuous.

Theorem 3.16: Let f: (X, τ) ⟶ (Y, σ) be a map. Both (X, τ) and (Y, σ) are α *$T_{1/2}$ space. Then the following are equivalent.

1) f is α * irresolute
2) f is strongly α * continuous
3) f is continuous
4) f is α * continuous

Proof: The proof is obvious.

Theorem 3.17: The composition of two strongly α * continuous maps is strongly α * continuous.

Proof: Let O be a α * open set in (Z, η). Since, g is strongly α * continuous, we get $g^{-1}(O)$ is open in (Y, σ). By thm [8] $g^{-1}(O)$ is α *open in (Y, σ). As f is also strongly α * continuous, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is open in (X, τ). Hence, (g ∘ f) is strongly α * continuous.

Theorem 3.18: If f: (X, τ) ⟶ (Y, σ) and g: (Y, σ) ⟶ (Z, η) be any two maps .Then their composition g ∘ f : (X, τ) ⟶ (Z, η) is strongly α * continuous if g is strongly α * continuous and f is continuous.

Proof: Let O be a α * open in (Z, η) . Since, g is strongly α * continuous, $g^{-1}(O)$ is open in (Y, σ) . Since f is continuous , $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is open in (X, τ). Hence, (g ∘ f) is strongly α * continuous.

IV. Perfectly α * Continuous Function

Definition 4.1: A map f: (X, τ) ⟶ (Y, σ) is said to be perfectly α * continuous if the inverse image of every α *open set in (Y, σ) is both open and closed in (X, τ) .

Theorem 4.2: If a map f: (X, τ) ⟶ (Y, σ) from a topological space (X, τ) into a topological space (Y, σ) is perfectly α * continuous then it is strongly α * continuous.

Proof: Assume that f is perfectly α * continuous. Let O be any α *open set in (Y, σ) . Since, f is perfectly α * continuous, $f^{-1}(O)$ is open in (X, τ) . Therefore, f is strongly α * continuous.

Remark 4.3: The converse of the above theorem need not be true.

Example 4.4: Let X = Y= {a, b, c,d}, τ = {ϕ,{a},{b},{ab},{bc},{abc}, X} and σ = {ϕ, {a},{b},{c},{ab},{ac},{bc},{abc},Y}. Let f: (X, τ) ⟶(Y, σ) be defined by f(b) = a = f(c) , f(a) = c , f(d) = d. clearly, f is strongly α *continuous. But $f^{-1}(\{a\}) = \{b,c\}$ is open in X, but not closed in X. Therefore f is not perfectly α * continuous.

Theorem 4.5: If a map f: (X, τ) ⟶ (Y, σ) from a topological space (X, τ) into a topological space (Y, σ) is perfectly α * continuous then it is perfectly continuous .

Proof: Let O be an open set in Y. By thm [8] O is an α *open set in (Y, σ). Since f is perfectly α * continuous, $f^{-1}(O)$ is both open and closed in (X, τ) . Therefore, f is perfectly continuous.

Remark 4.6 : The converse of the above theorem need not be true.

Example 4.7: Let X = Y= {a, b, c}, τ = {ϕ,{a},{bc}, X} and σ = {ϕ, {a},Y}. Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = a , f(b) = b , f(c) = c. clearly, f is perfectly continuous. But the inverse image of α *open set in (Y, σ) [$f^{-1}(\{ac\}) = \{ac\}$] is not open and closed in X. Therefore f is not perfectly α * continuous.

Theorem 4.8: A map f: (X, τ) ⟶ (Y, σ) from a topological space (X, τ) into a topological space (Y, σ) is perfectly α * continuous if and only if $f^{-1}(O)$ is both open and closed in (X, τ) for every α *closed set O in (Y, σ).

Proof: Let O be any α *closed set in (Y, σ).Then O^c is α *open in (Y, σ). Since, f is perfectly α * continuous , $f^{-1}(O^c)$ is both open and closed in (X, τ) . But $f^{-1}(O^c) = X/ f^{-1}(O)$ and so $f^{-1}(O)$ is both open and closed in (X, τ) .

Conversely, assume that the inverse image of every α *closed set in (Y, σ) is both open and closed in (X, τ).Let O be any α *open in (Y, σ).Then O^c is α *closed in (Y, σ).By assumption $f^{-1}(O^c)$ is both open and closed in (X, τ) . But $f^{-1}(O^c) = X/ f^{-1}(O)$ and so $f^{-1}(O)$ is both open and closed in (X, τ).Therefore, f is perfectly α * continuous.

Theorem 4.9: Let (X, τ) be a discrete topological space and (Y, σ) be any topological space. Let f: (X, τ) ⟶(Y, σ) be a map, then the following statements are true.

1) f is strongly α * continuous
2) f is perfectly α * continuous

Proof: (1) ⟹(2) Let O be any α * open set in (Y, σ) . By hypothesis, $f^{-1}(O)$ is open in (X, τ). Since (X, τ) is a discrete space, $f^{-1}(O)$ is closed in (X, τ). $f^{-1}(O)$ is both open and closed in (X, τ). Hence, f is perfectly α * continuous.

(2) ⟹(1) Let O be any α * open set in (Y, σ). Then, $f^{-1}(O)$ is both open and closed in (X, τ). Hence, f is strongly α * continuous.

Theorem 4.10: If f: (X, τ) ⟶ (Y, σ) and g: (Y, σ) ⟶ (Z, η) are perfectly α * continuous , then their composition g ∘ f : (X, τ) ⟶ (Z, η) is also perfectly α * continuous .

Proof: Let O be a α * open set in (Z, η) . Since, g is perfectly α * continuous. We get that $g^{-1}(O)$ is open and closed in (Y, σ). By thm [8] $g^{-1}(O)$ is α * open in (Y, σ). Since f is perfectly α * continuous, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is both open and closed in (X, τ) . Hence, g ∘ f is perfectly α * continuous.

Theorem 4.11: If f: (X, τ) ⟶ (Y, σ) and g: (Y, σ) ⟶ (Z, η) be any two maps . Then their composition is strongly α * continuous if g is perfectly α * continuous and f is continuous.
Proof: Let O be any α * open set in (Z, η). Then, $g^{-1}(O)$ is open and closed in (Y, σ). Since, f is continuous. $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is open in (X, τ) . Hence, g ∘ f is strongly α * continuous.
Theorem 4.12: If a map f: (X, τ) ⟶ (Y, σ) is perfectly α * continuous and a map g: (Y, σ) ⟶ (Z, η) is strongly α * continuous then the composition g ∘ f : (X, τ) ⟶ (Z, η) is perfectly α * continuous.
Proof: Let O be any α * open set in (Z, η). Then, $g^{-1}(O)$ is open in (Y, σ).By thm [8]$g^{-1}(O)$ is α *open in (Y, σ).By hypothesis, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is both open and closed in (X, τ) . Therefore, g ∘ f is perfectly α * continuous.

V. α * Open maps and α * Closed maps

Definition 5.1: A map f: (X, τ) ⟶ (Y, σ) is called a α * open if image of each open set in X is α * open in Y.
Definition 5.2: A map f: (X, τ) ⟶ (Y, σ) is called a α * closed if image of each closed set in X is α *closed in Y
Theorem 5.3: Every closed map is α *closed map.
Proof: The proof follows from the definitions and fact that every closed set is α *closed.
Remark 5.4: The converse of the above theorem need not be true.
Example 5.5: Let X = Y= {a, b, c}, τ = {ϕ,{a},{ab},{ac}, X} and σ = {ϕ, {a},{b},{ab},Y}. Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = a , f(b) = b, f(c) = c. clearly, f is α *closed but not closed as the image of closed set {b} in X is {b} which is not closed set in Y.
Theorem 5.6: Every g-closed map is α *closed.
Proof: Let O be a closed set in X. Since f is g-closed map, f(O) is g-closed in Y. By [8] f(O) is α *closed in Y. Therefore, f is α *closed map.
Remark 5.7: The converse of the above theorem need not be true.
Example 5.8: Let X = Y= {a, b, c}, τ = {ϕ,{a},{ab},{ac}, X} and σ = {ϕ, {a},{b},{ab},Y} , GC(Y, σ) = {ϕ,{c},{ac},{bc},Y} . Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = a , f(b) = b, f(c) = c . clearly, f is α *closed but not g - closed as the image of closed set {b} in X is {b} which is not g - closed set in Y.

Theorem 5.9: Every α - closed map is α *closed.
Proof: The proof follows from the definition and by Thm [8] .
Remark 5.10: The converse of the above theorem need not be true.
Example 5.11: Let X = Y= {a, b, c, d }, τ = {ϕ,{a},{b},{c},{ab},{ac},{bc},{abc}, X} and σ = {ϕ, {a},{ab},{abc},Y} , α *C(Y, σ) = {ϕ,{b},{c},{d},{ad},{bc},{bd},{cd},{acd},{bcd}, {abd},Y}, α C(Y, σ) = {ϕ,{b},{c},{d},{bc},{bd},{cd},{bcd},Y} . Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = c , f(b) = b, f(c) = a , f(d) = d clearly, f is α *closed but not α - closed as the image of closed set {cd} in X is {ad} which is not α - closed set in Y.
Remark 5.12: The composition of two α * closed maps need not be α * closed in general as shown in the following example.
Example 5.13: Consider X = Y = Z = {a, b, c, d }, τ = {ϕ,{a},{b},{c},{ab},{ac},{bc},{abc}, X} and σ = {ϕ, {a},{ab},{abc},Y}, η ={ϕ,{a},{b},{ab},{abc},Z}, α *C(Y σ) = {ϕ,{b},{c},{d},{ad},{bc},{bd},{cd},{acd},{bcd}, {abd},Y}, α *C (Z, η) = {ϕ , {c},{d},{ad},{bd},{cd},{abd},{acd},{bcd}}. Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = a , f(b) = b, f(c) = c , f(d) = d. Clearly, f is α *closed. Consider the map g: Y ⟶ Z defined g(a) = a, g(b) = b, g(c) = d , g(d) = c, clearly g is α *closed. But g ∘f : X ⟶ Z is not a α *closed, g ∘f ({ad}) = g (f{ad}) = g (ad) = ac which is not a α *closed in Z.
Theorem 5.14: A map f: (X, τ) ⟶ (Y, σ) is α * closed if and only if α * cl (f(A)) ⊆ f(cl(A)) for each set A in X.
Proof: Suppose that f is a α * closed map. Since for each set A in X, cl(A) is closed set in X, then f(cl(A)) is a α * closed set in Y. Since, f(A) ⊆ f(cl(A)), then α * cl (f(A)) ⊆ f(cl(A))
Conversely, suppose A is a closed set in X. Since α * cl (f(A)) is the smallest α * closed set containing f(A), then f(A) ⊆ α * cl (f(A)) ⊆ f(cl(A)) = f(A). Thus, f(A) = α * cl (f(A)). Hence, f (A) is a α * closed set in Y. Therefore, f is a α * closed map.
Theorem 5.15: If f: (X, τ) ⟶ (Y, σ) is closed map and g: (Y, σ) ⟶ (Z, η) is α * closed, then the composition g ∘f : X ⟶ Z is α * closed map.

Proof: Let O be any closed set in X. Since f is closed map. f(O) is closed set in Y. Since, g is α * closed map, g(f(O)) is α * closed in Z which implies g ∘f ({O}) = g(f{O}) is α * closed and hence , g ∘f is α * closed.

Remark 5.16: If f: (X, τ) ⟶ (Y, σ) is α *closed map and g: (Y, σ) ⟶ (Z, η) is closed, then the composition g ∘f : X ⟶ Z is not α * closed map as shown in the following example.

Example 5.17: Consider X = Y = Z = {a, b, c, d }, τ = {ϕ,{a},{abc}, X} and σ = {ϕ, {ab},{abc},Y}, η ={ϕ,{a},{ab},{abc},Z}, α *C(Y, σ)= {ϕ , {a} , {b} , {c} , {d} , {ac} , {ad} , {bc} , {bd} , {cd} , {acd} ,{bcd}, {abd},Y}, α *C (Z, η) = {ϕ ,{b}, {c},{d},{bc},{ad},{bd},{cd},{abd},{acd},{bcd}}. Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = d , f(b) = c, f(c) = b , f(d) = a. Clearly, f is α *closed. Consider the map g: Y ⟶ Z defined g(a) = a, g(b) = b, g(c) = c , g(d) = d, clearly g is closed. But g ∘f : X ⟶ Z is not a α *closed, g ∘f ({d}) = g (f{d}) = g (a) = a which is not a α *closed in Z.

Theorem 5.18: Let (X, τ) , (Z, η) be topological spaces and (Y, σ) be topological spaces where every α *closed subset is closed. Then the composition g ∘f : (X, τ) ⟶ (Z, η) of the α *closed , f: (X, τ) ⟶ (Y, σ) and g: (Y, σ) ⟶ (Z, η) is α *closed.

Proof: Let O be a closed set in X . Since, f is α *closed, f(O) is α *closed in Y. By hypothesis, f(O) is closed . Since g is α *closed, g(f{O}) is α *closed in Z and g(f{O}) = g ∘f (O). Therefore, g ∘f is α *closed.

Theorem 5.19: If f: (X, τ) ⟶ (Y, σ) is g - closed map and g: (Y, σ) ⟶ (Z, η) is α *closed and (Y, σ) is $T_{1/2}$ spaces . Then the composition g ∘f : (X, τ) ⟶ (Z, η) is α *closed map.

Proof: Let O be a closed set in (X, τ). Since f is g – closed , f(O) is g – closed in (Y, σ) and g is α *closed which implies g(f(O)) is α *closed in Z and g(f(O)) = g ∘f (O). Therefore, g ∘f is α *closed.

Theorem 5.20: Let f: (X, τ) ⟶ (Y, σ) and g: (Y, σ) ⟶ (Z, η) be two mappings such that their composition g ∘f : (X, τ) ⟶ (Z, η) be α *closed mapping. Then the following statements are true.

1. If f is continuous and surjective, then g is α *closed.
2. If g is α * - irresolute and injective, then f is α *closed.
3. If f is g – continuous , surjective and (X, τ) is a $T_{1/2}$ spaces, then g is α *closed.
4. If g is strongly α * continuous and injective, then f is α *closed.

Proof: 1. Let O be a closed set in (Y, σ) . Since , f is continuous, $f^1(O)$ is closed in (X, τ) . Since, g ∘f is α *closed which implies g ∘f ($f^1(O)$) is α *closed in (Z, η). That is g(O) is α *closed in (Z, η), since f is surjective. Therefore, g is α *closed.

2. Let O be a closed set in (X, τ). Since g ∘f is α *closed, g ∘f (O) is α *closed in (Z, η), Since g is α * - irresolute, g^{-1}(g ∘f (O)) is α *closed in (Y, σ). That is f(O) is α *closed in (Y, σ). Since f is injective. Therefore, f is α *closed.

3. Let O be a closed set of (Y, σ). Since, f is g- continuous , $f^{-1}(O)$ is g – closed in (X, τ) and (X, τ) is a $T_{1/2}$ spaces, $f^{-1}(O)$ is closed in (X, τ) . Since , g ∘f is α *closed which implies , g ∘f (f^{-1} (O)) is α *closed in (Z, η). That is g(O) is α *closed in (Z, η) , since f is surjective. Therefore, g is α *closed.

4. Let O be a closed set of (X, τ).Since, g ∘ f is α *closed which implies , g ∘f (O) is α *closed in (Z, η). Since, g is strongly α * continuous, g^{-1}(g ∘f (O)) is closed in (Y, σ).That is f(O) is closed in (Y, σ). Since g is injective, f is α *closed.

Theorem 5.21: A map f: (X, τ) ⟶ (Y, σ) is α * open if and only if f(int(A)) ⊆ α * int (f(A)) for each set A in X.

Proof: Suppose that f is a α * open map. Since int (A) ⊆A, then f(int (A)) ⊆ f(A). By hypothesis, f(int (A)) is a α * open and α * int (f(A)) is the largest α * open set contained in f(A).Hence f(int(A)) ⊆ α * int (f(A)).

Conversely, suppose A is an open set in X. Then f(int(A)) ⊆ α * int (f(A)) . Since int (A) = A, then f(A) ⊆ α * int (f(A)). Therefore, f(A) is a α * open set in (Y, σ) and f is α * open map.

Theorem 5.22: Let (X, τ) , (Y, σ) and (Z , η) be three topologies spaces f: (X, τ) ⟶ (Y, σ) and g: (Y, σ) ⟶ (Z , η) be two maps. Then

1. If (g ∘ f) is α * open and f is continuous, then g is α * open.
2. If (g ∘ f) is open and g is α *continuous, then f is α * open map.

Proof:

1. Let A be an open set in Y .Then, $f^{-1}(A)$ is an open set in X. Since (g ∘ f) is α * open map, then (g ∘ f) ($f^{-1}(A)$) = g(f(($f^{-1}(A)$)) = g(A) is α * open set in Z. Therefore, g is a α * open map.

2. Let A be an open set in X. Then, g(f(A)) is an open set in Z. Therefore, $g^{-1}(g(f(A))) = f(A)$ is a α * open set in Y . Hence, f is a α * open map.

Theorem 5.23: Let f: (X, τ) ⟶ (Y, σ) be a bijective map. Then the following are equivalent:

(1) f is a α * open map.

(2) f is a α * closed map.

(3) f^{-1} is a α * continuous map.

Proof:

(1) ⇒ (2) Suppose O is a closed set in X. Then X\O is an open set in X and by (1) f(X\O) is a α * open in Y. Since, f is bijective, then f(X\O) = Y\ f(O). Hence, f(O) is a α * closed in Y. Therefore, f is a α * closed map.

(2) ⇒ (3) Let f is a α * closed map and O be closed set in X. Since, f is bijective then $(f^{-1})^{-1}(O) = f(O)$ which is a α * closed set in Y. Therefore, f is a α * continuous map.

(3) ⇒ (1) Let O be an open set in X. Since, f^{-1} is a α * continuous map then $(f^{-1})^{-1}(O) = f(O)$ is a α * open set in Y. Hence, f is α * open map.

Theorem 5.24: A map f: (X, τ) ⟶ (Y, σ) is α * open if and only if for any subset O of (Y, σ) and any closed set of (X, τ) containing $f^{-1}(O)$, there exists a α * closed set A of (Y, σ) containing O such that $f^{-1}(A) \subset F$

Proof: Suppose f is α * open. Let $O \subset Y$ and F be a closed set of (X, τ) such that $f^{-1}(O) \subset F$. Now X- F is an open set in (X, τ). Since f is α * open map, f (X – F) is α * open set in (Y, σ). Then, A = Y - f (X – F) is a α * closed set in (Y, σ). Note that $f^{-1}(O) \subset F$ implies $O \subset A$ and $f^{-1}(A) = X - f^{-1}(X - F) \subset X - (X - F) = F$. That is , $f^{-1}(A) \subset F$.

Conversely, let B be an open set of (X, τ) . Then, $f^{-1}((f(B))^c) \subset B^c$ and B^c is a closed set in (X, τ). By hypothesis, there exists a α * closed set A of (Y, σ) such that $(f(B))^c \subset A$ and $f^{-1}(A) \subset B^c$ and so $B \subset (f^{-1}(A))^c$. Hence, $A^c \subset f(B) \subset f((f^{-1}(A)))^c$ which implies $f(B) = A^c$. Since, A^c is a α * open. f (B) is α * open in (Y, σ) and therefore f is α * open map.

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