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One Interesting Family Of 3-Tuple with Property $D(k^2+4)$

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Abstract: *This paper concerns with the study of constructing a special family of 3-tuples (a,b,c) such that the product of any two elements of the set added with k-times their sum increased by* k^2+4 *is a Perfect square.*
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I. Introduction

The Problem of constructing the sets with property that product of any two of its distinct elements is one less than a square has a very long history and such sets were studied by Diophantus [1]. A set of positive integers $\{a_1, a_2 ......a_m\}$ is said to have the property D(n), $n \in Z - \{0\}$, if $a_i a_j + n$, a perfect square for all $1 \le i < j \le m$ and such a set is called a Diophantine m-tuples with property D(n). Many mathematicians considered the Construction of different formulations of Diophantine triples with property D(n) for any arbitrary integer n and also, for any linear polynomials in n. In this context, one may refer [2-19] for an extensive review of various problem on Diophantine triples. In [20-22], special mention is provided because it differs from the earlier one. This paper aims at constructing an interesting of 3-tuples different from the earlier one. The interesting triple is constructed where the product of any two elements of the set added with their sum multiplied by k and increased by k^2+4 is a Perfect square.

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II. Method Of Analysis

Let $a = x-2, b = x+2$ such that

$$ab + k(a+b) + (k^2+4) = (x+k)^2$$

Let c be any non zero integer such that

$$ac + k(a+c) + (k^2+4) = p^2$$
$$bc + k(b+c) + (k^2+4) = q^2 \qquad (1)$$

Using some algebra,

$$(b+k)p^2 - (a+k)q^2 = 4(b-a) \qquad (2)$$

Introducing the linear transformations

$$p = X + (a+k)T$$
$$q = X + (b+k)T \qquad (3)$$

We have $X^2 = [x^2 - 4 + 2kx + k^2]T^2 + 4$ (4)

which is in the form of a Pell equation.

Let $T_0 = 1, X_0 = x+k$ be the initial solutions of (4).

From (3), $p = 2x + 2k - 2$

From (1), $c = 4x + 3k$

Hence $(x-2, x+2, 4x+3k)$ is the interesting 3-tuple satisfying the required property. Repeating the above process, one can generate many 3-tuples satisfying the required property. For illustration, a few generated triples are given below.

$$(x-2, x+2, 4x+3k),\ (x+2, 4x+3k, 9x+8k+6)$$

$$(4x+3k, 9x+8k+6, 25x+24k+10),\ (9x+8k+6, 25x+24k+10, 64x+63k+32)$$

Some numerical examples are presented below:

k	Triples (a,b,c)	property
3	(3,7,29)	D(13)
5	(12,55,136)	D(29)
7	(65,161,453)	D(53)
10	(140,400,1046)	D(104)

Remark:

Replacing x and k by a Gaussian integer and irrational numbers respectively in each of the above triples, it is noted that each resulting triple is a Gaussian triple and irrational triple satisfying the required property.

III. Conclusion

To conclude, one may search for triples consisting of polygonal and centered polygonal numbers with suitable property.

x	k	Triples (a, b, c)	property
2+i	1	(i,4+i,11+4i), (4+i,11+4i,32+9i), (11+4i,32+9i,84+25i), (32+9i,84+25i,223+64i)	D(5)
5+3i	1+i	(3+3i,7+3i,23+15i), (7+3i,23+15i,59+35i), (23+15i,59+35i,159+99i), (59+35i,159+99i,415+255i)	D(2i+4)
3+i	$1+i\sqrt{5}$	$(1+i, 5+i, 15+i(4+3\sqrt{5}))$, $(5+i, 15+i(4+3\sqrt{5}), 41+i(9+8\sqrt{5}))$, $(15+i(4+3\sqrt{5}), 41+i(9+8\sqrt{5}), 159+i(25+24\sqrt{5}))$, $(41+i(9+8\sqrt{5}), 159+i(25+24\sqrt{5}), 287+i(64+63\sqrt{5}))$	D$(i2\sqrt{5})$
$1+\sqrt{3}$	$\sqrt{3}$	$(\sqrt{3}-1, 3+\sqrt{3}, 4+7\sqrt{3})$, $(3+\sqrt{3}, 4+7\sqrt{3}, 15+17\sqrt{3})$ $(4+7\sqrt{3}, 15+17\sqrt{3}, 35+49\sqrt{3})$, $(15+17\sqrt{3}, 35+49\sqrt{3}, 96+127\sqrt{3})$	D(7)

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