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Study of $(T, \xi; \eta)$-invex function of second kind and Equivalence between GNVIP and DGNVIP

Bibhakar Kodamasingh
Department of Mathematics Institute of Technical Education and Research Siksha O Anusandhan University Bhubaneswar,Odisha,751030,India.

Abstract: *In this paper, we introduce the concept of $(T, \xi; \eta)$-invex function of second kind .Further,we establish the equivalence between the problems $(GNVIP)$ and $(DGNVIP)$ using $(T, \xi; \eta)$-invex function of second kind.*
Key Words and Phrases. *Nonlinear variational ineqality problems, Invex set,$\eta-$invexset,$\eta-$preinvexfunction,$(T, \xi; \eta)$-invex function and $\eta-$hemicontinuous,η-monotone function.*

I. Introduction

It has been shown that a wide class of linear and nonlinear problems arising in various branches of mathematics and engineering sciences can be studied in the unified and general framework of variational inequalities. The variational inequality problem(VIP) is to:
find $x_0 \in K$ such that

$$\langle T(x_0), x - x_0 \rangle \geq 0. \quad \text{for all } x \in K. \qquad (1.1)$$

For generalization of VIP,Chipot has studied the nonlinear variational inequality problem with two operators in Banach spaces as follows. Let K be a nonempty subset of a Banach space X with dualspace X^*. Let $T : K \to X^*$ and $\xi \in X^*$ be two mappings. The nonlinear variational inequality problem(NVIP) is to:
find $u \in K$ such that

$$\langle T(u), x - u\rangle \geq \langle \xi, x - u\rangle \quad \text{for all } x \in K. \qquad (1.2)$$

Let X be any reflexive real Banach space and $K \subset X$. Let $L(X, R^n) \equiv R^n$ be the set of linear continuous functionals from X to R^n. Let

$$\eta : K \times K \to X$$

be a vector valued function. Let
and
$T : K \to L(X, R^n) \equiv R^n$
$\xi : K \to L(X, R^n) \equiv R^n$
be any nonlinear maps. Let $F : K \to R^n$ be any map. The pair $\langle \mathbf{f}, z\rangle$ denote the value of $\mathbf{f} \in L(X, R^n) \equiv R^n$ at $z \in X$.

The Generalized Nonlinear Variational Inequality Problem $(GNVIP)$ is to:
find $x_0 \in K$ such that

$$\langle T(x_0), \eta(x, x_0) \rangle \geq \langle \xi(x_0), \eta(x, x_0) \rangle \quad \text{for all } x \in K, \qquad (GNVIP)$$

and the Dual Generalized Nonlinear Variational Inequality Problem $(DGNVIP)$ is to:
find $x_0 \in K$ such that

$$\langle T(x), \eta(x, x_0) \rangle \geq \langle \xi(x), \eta(x, x_0)\rangle \text{ for all } x \in K. \qquad (DGNVIP)$$

Definition 1.1. [1] Let K be a nonempty subset of R^n. Let

$$\eta : K \times K \to \mathsf{R}^n$$

be a vector valued mapping. K is said to be η-invex set if for all $x, u \in K$ and for all $t \in [0, 1]$, we have

Definition 1.2. The mapping
$u + t\eta(x, u) \in K$.

$T : X \to L(X, Y)$

is said to be η-hemicontinuous on K if for any sequence $\{u_n\}$ converging to x_0 along a line, the sequence $\{T(u_n)\}$ weakly converging to $T(x_0)$, i.e., the map

$$\lambda \to T(u + \lambda\eta(x, u))$$

of $[0, 1]$ into Y is continuous, when Y is endowed with its weak topology.

Definition 1.3. [10] Let X be topological vector space and K be a nonempty subset of X. Let (Y, P) be an ordered topological vector space equipped with the closed convex pointed cone such that $\mathrm{int}P \neq \varphi$ where $\mathrm{int}P$ denotes the interior of P. Let $L(X, Y)$ be the set of all continuous linear functionals from X to Y. Let the pair $\langle f, x\rangle$ denote the value of $f \in L(X, Y)$ at $x \in X$. Let

and be any two maps. Let be any vector valued function. Then

$$F : K \to Y,$$

$$T : K \to L(X, Y)$$

$$\eta : K \times K \to X$$

(a) F is T-η-invex on K if for all $x, u \in K$, we have

$F(x) - F(u) - \langle T(u), \eta(x, u)\rangle \not\in -\mathrm{int}P,$

(b) F is not T-η-invex on K if for all $x, u \in K$, we have

$-F(x) + F(u) + \langle T(u), \eta(x, u)\rangle \not\in -\mathrm{int}P,$

alternatively;

$F(x) - F(u) - \langle T(u), \eta(x, u)\rangle \not\in \mathrm{int}P.$

Definition 1.4. [10] The mapping $T : K \to L(X, Y)$ is said to be η-monotone if there exist a vector function $\eta : K \times K \to X$ such that

$$\langle T(u), \eta(x, u)\rangle + \langle T(x), \eta(u, x)\rangle \not\in \mathrm{int}P \quad \text{for all } x, u \in K.$$

As an extension work of Minty's Lemma, the equivalence theorems of two different nonlin- ear variational inequalities associated with two different operators is studied in the following theorem.

Theorem 1.5. **([7], p.6)** Let K be a nonempty, closed and convex subset of a Banach space X and T a monotone operator from K into X^*, the dual of X which is continuous on finite dimensional subspaces of X. Then for $\xi \in X^*$, $x \in K$,

$$x_0 \in K,\ \langle T(x_0), x - x_0\rangle \geq \langle \xi, x - x_0\rangle$$

if and only if

$$x_0 \in K,\ \langle T(x), x - x_0\rangle \geq \langle \xi, x - x_0\rangle .$$

The following result characterizes the equivalence of different generalized nonlinear vari- ational inequality problems in a η-invex set.

Theorem 1.6. Let K be a nonempty subset of a reflexive real Banach space X and let

$$T : K \to L(X, R^n) \equiv R^n$$

And be any maps. Let
$\xi : K \to L(X, R^n) \equiv R^n$

$$\eta : K \times K \to X$$

be any vector valued function. Assume that:

(i) K is η-invex cone in X,

(ii) $(T - \xi)$ is generalized η-monotone on K, (iii) T and ξ are η-hemicontinuous on K. Then the following are equivalent:
(A) find $u \in K$ such that $\langle T(u), \eta(x, u)\rangle \geq \langle \xi(u), \eta(x, u)\rangle$ for all $x \in K$,

(B) find $u \in K$ such that $\langle T(x), \eta(x, u)\rangle \geq \langle \xi(x), \eta(x, u)\rangle$ for all $x \in K$.
Note 1.7 $Y=R^n$ and $P=R^n_+$ then the term"$y \not\in - \text{int}P$ "will be replaced by
$Y \geq 0$, "$y \not\in \text{int}P$ "will be $Y \leq 0$ and "$y \not\in (- \text{int}P \cup \text{int}P$ ")will be $Y=0$.
The existence of η-monotone function is reflected in the following result.

Theorem1.8[10] Let X be a topological vector space and (Y,P) be an order topological vector space equipped with closed convex pointed cone P such that $\text{int}P \neq \varphi$. Let $L(X,Y)$ be the set of continuous linear functionals from X to Y and

$$\eta : K \times K \to X$$

be a vector-valued function where K is any subset of X. Let

$$T : K \to L(X, Y)$$

be an operator. Let the function
$F : K \to Y$
be T-η-invex in K. Then T is η-monotone.

Behera and Das [10] have studied the generalized vector complementarity problem (GVCP)
and explored their results as follows.

Theorem 1.9. ([10], Theorem 3.4, p. 6) Let K be a nonempty compact cone in a topological vector space X. Let (Y,P) be an ordered topological vector space equipped with a convex pointed cone P with $\text{int}P \neq \varphi$ Let $L(X, Y)$ be the set of continuous linear
functionals from X to Y and let be continuous mappings. Let
$\eta : K \times K \to X$

$T : K \to L(X, Y)$

be an arbitrary continuous map and let K be η-invex. Let the following conditions hold: (a) $\langle T(x), \eta(x, x)\rangle =_P 0$ for all $x \in K$,
(b) for each $u \in K$, the set

$$B(u) = \{x \in K : \langle T(u), \eta(x, u)\rangle \in -\operatorname{int}P\} \text{ is an } \eta\text{-invex set},$$

(c) for at least one $x \in K$, the set

$$\{u \in K : \langle T(u), \eta(x, u)\rangle \notin -\operatorname{int}P\} \text{ is compact},$$

(d) η satisfies condition C_0.

Then there exist $x_0 \in K$ such that
$\langle T(x_0), \eta(x, x_0)\rangle \notin (-\operatorname{int}P \cup \operatorname{int}P)$ for all $x \in K$.

2 $(T,\xi;\eta)$-invex function and $(T,\xi;\eta)$-incave function

To study the theory of generalized nonlinear variational inequality problem GNVIP , dual generalized nonlinear variational inequality problem DGNVIP , generalized nonlinear F - variational inequality problem GNVIP-F and the dual generalized nonlinear F -variational inequality problem DGNVIP-F. we introduce the concept of $(T,\xi;\eta)$-invex functions in R^n as follows.

Let K be a nonempty subset of a reflexive real Banach space X and let

$$T : K \to L(X, R^n) \equiv R^n$$

And $\xi : K \to L(X, R^n) \equiv R^n$

be any maps. We define the general form of $(T,\xi;\eta)$-invexity of F as follows.

Definition 2.1. The mapping $F : K \to R^n$ is said to be

(a) $(T,\xi;\eta)$-invex function of first kind on K, if there exist a vector function

$$\eta : K \times K \to X$$

such that

$$F(x) - F(u) - \langle \xi(u), \eta(x, u)\rangle \geq \langle T(u), \eta(x, u)\rangle \quad \text{for all } x, u \in K,$$

(b) $(T,\xi;\eta)$-invex function of first kind at point $x_0 \in K$ if

$$F(x) - F(x_0) - \langle \xi(u), \eta(x, x_0)\rangle \geq \langle T(x_0), \eta(x, x_0)\rangle \quad \text{for all } x \in K.$$

Definition 2.2. The mapping $F : K \to R^n$ is said to be

(a) $(T,\xi;\eta)$-invex function of second kind on K, if there exist a vector function

$$\eta : K \times K \to X$$

such that

$$F(x) - F(u) + \langle \xi(u), \eta(x, u)\rangle \geq \langle T(u), \eta(x, u)\rangle \quad \text{for all } x, u \in K,$$

(b) $(T,\xi;\eta)$-invex function of second kind at point $x_0 \in K$ if

$$F(x) - F(x_0) + \langle \xi, \eta(x, x_0)\rangle \geq \langle T(x_0), \eta(x, x_0)\rangle \quad \text{for all } x \in K.$$

Remark 2.3. Let $y = R^n$, $P = R^n_+$ and $\xi \equiv 0$, then the definition of $(T, \xi; \eta)$-invex function coincides with the definition of T-η-invex function.

Note 2.4. The mapping $F : K \to R^n$ is $(T, \xi; \eta)$-invex function of second kind on K, equivalently saying F is $(T, -\xi; \eta)$-invex function of first kind on K.

Definition 2.5. The mapping $F : K \to R^n$ is said to be

(a) $(T, \xi; \eta)$-incave function of first kind on K, if there exist a vector function

$$\eta : K \times K \to X$$

such that

$$F(x) - F(u) - \langle \xi(u), \eta(x, u) \rangle \le \langle T(u), \eta(x, u) \rangle \quad \text{for all } x, u \in K,$$

(b) $(T, \xi; \eta)$-incave function of first kind at point $x_0 \in K$ if

$$F(x) - F(x_0) - \langle \xi(u), \eta(x, x_0) \rangle \le \langle T(x_0), \eta(x, x_0) \rangle \quad \text{for all } x \in K.$$

Definition 2.6. The mapping $F : K \to R^n$ is said to be

(a) $(T, \xi; \eta)$-incave function of second kind on K, if there exist a vector function

$$\eta : K \times K \to X$$

such that

$$F(x) - F(u) + \langle \xi(u), \eta(x, u) \rangle \le \langle T(u), \eta(x, u) \rangle \quad \text{for all } x, u \in K,$$

(b) $(T, \xi; \eta)$-incave function of second kind at point $x_0 \in K$ if

$$F(x) - F(x_0) + \langle \xi(u), \eta(x, x_0) \rangle \le \langle T(x_0), \eta(x, x_0) \rangle \quad \text{for all } x \in K.$$

Example 2.7. Let X be a reflexive real Banach space. Let $K \subset X$ and

$$\eta : K \times K \to X$$

be a vector-valued function. Let

$$p : K \times K \times [0, 1] \to K$$

be any continuous map defined by the rule:

$$p(x, u, t) = \begin{cases} u & \text{if } t = 0; \\ u + t\eta(x, u) & \text{if } 0 < t < 1; \\ x & \text{if } t = 1 \end{cases}$$

for all $x, u \in K$, and $t \in [0, 1]$ where $p(x, u, t)$ represents the path joining the two points $x, u \in K$. Clearly K is a η-invex set. Let

$$F : K \to R^n$$

be a twice differentiable function on K such that for each $x \in K$,

$$\langle \nabla F^2(x), y\rangle \geq 0$$

for all $y \in X$ and

$$\langle \nabla F^2(x), y_2\rangle \leq \langle \nabla F^2(x), y_1\rangle$$

for $y_1 < y_2$ in X, i.e., $\nabla F^2(x)$ is weakly decreasing on X. Using Taylor's series expansion, we obtain

$$\begin{aligned} F(x) &= F(u + \eta(x, u)) \\ &= F(u) + \langle \nabla F(u), \eta(x,u)\rangle + \\ &\langle \nabla^2 F(u), \frac{\eta\hat{}2\,(x,u)}{2}\rangle + R_n \end{aligned}$$

where R_n is remainder term. Since $\nabla F^2(u)$ is weakly decreasing on X, we have for each $u \in K$,

$$\langle \nabla F^2(u), z_1\rangle \geq \langle \nabla F^2(u), z_2\rangle$$

for $z_1 < z_2$ in X. So from the above expression, we get

$$F(x) - F(u) - \langle \nabla F(u), \eta(x, u)\rangle \geq \langle \nabla F^2(u), \eta^2(x, u)\rangle + R_n$$

for all $x, u \in K$. Defining $T = \nabla F$, and $\xi = \nabla^2 F$ by the rule

$$\langle T(u), \eta(x, u)\rangle = \langle \nabla F(u), \eta(x, u)\rangle,$$

$$\langle \xi(u), \eta(x, u)\rangle = \langle \nabla F^2(u), \eta^2(x, u)\rangle$$

for all $x, u \in K$, we get

$$F(x) - F(u) - \langle T(u), \eta(x, u)\rangle \geq \langle \xi(u), \eta(x, u)\rangle + R_n$$

for all $x, u \in K$. Neglecting the remainder term, we get

$$F(x) - F(u) - \langle T(u), \eta(x, u)\rangle \geq \langle \xi(u), \eta(x, u)\rangle$$

for all $x, u \in K$. Thus

$$F(x) - F(u) - \langle \xi(u), \eta(x, u)\rangle \geq \langle T(u), \eta(x, u)\rangle \qquad (2.1)$$

for all $x, u \in K$, i.e., F is $(T, \xi; \eta)$-invex function of first kind on K. Again for each $x \in K$, $\langle \nabla F^2(x), y\rangle \geq 0$ for all $y \in X$, so

$\langle \xi(u), \eta(x, u)\rangle = \langle \nabla F^2(u), \eta^2(x, u)\rangle \geq 0$ for all $x, u \in K$. From (2.1), we get
$F(x) - F(u) - \langle hT(u), \eta(x, u)\rangle = \langle \xi(u), \eta(x, u) \geq 0$

for all $x, u \in K$ which together with the above expression gives

$$F(x) - F(u) - \langle T(u), \eta(x, u)\rangle + \rangle \xi(u), \eta(x, u)\rangle \geq 0,$$

i.e.,
$$F(x) - F(u) + \langle \xi(u), \eta(x, u)\rangle \geq \langle T(u), \eta(x, u)\rangle$$

for all $x, u \in K$. Thus F is $(T, \xi; \eta)$-invex function of second kind on K.

In the following theorem, we study the concept that presence of $(T, \xi; \eta)$-invex function of second kind on any set K imply η-monotoneness of $(T - \xi)$.

Theorem 2.8. Let K be a nonempty subset of a reflexive real Banach space X and let

$$T : K \to L(X, R^n) \equiv R^n$$

And be any maps. Let

$\xi : K \to L(X, R^n) \equiv R^n$

$$\eta : K \times K \to X$$

be any vector valued function. $(T - \xi)$ is η-monotone on K if the mapping

$$F : K \to R^n$$

is $(T, \xi; \eta)$-invex function of second kind on K.

Proof. Since F is $(T, \xi; \eta)$-invex function of second kind on K, we get

$F(x) - F(u) + \langle \xi(u), \eta(x, u) \rangle \geq \langle T(u), \eta(x, u) \rangle$

for all $x, u \in K$. Interchanging x and u, we get

$F(u) - F(x) + \langle \xi(x), \eta(u, x) \rangle \geq \langle T(x), \eta(u, x) \rangle$

for all $x, u \in K$. Adding the equations, we get

$\langle \xi(x), \eta(u, x) \rangle + \langle \xi(u), \eta(x, u) \rangle \geq \langle T(x), \eta(u, x) \rangle + \langle T(u), \eta(x, u) \rangle$

for all $x, u \in K$, i.e.,

$- \langle \xi(x), \eta(u, x) \rangle - \langle \xi(u), \eta(x, u) \rangle \leq - \langle T(x), \eta(u, x) \rangle - \langle T(u), \eta(x, u) \rangle$

for all $x, u \in K$. Thus

$\langle (T - \xi)(u), \eta(x, u) \rangle + \langle (T - \xi)(x), \eta(u, x) \rangle \leq 0$

for all $x, u \in K$. So $(T - \xi)$ is η-monotone on K. This completes the proof of the theorem. □

Theorem 2.9. Let K be a nonempty subset of a reflexive real Banach space X and let

$$T : K \to L(X, R^n) \equiv R^n$$

and

be any maps. Let

$\xi : K \to L(X, R^n) \equiv R^n$

$$\eta : K \times K \to X$$

be any vector valued function. The mapping

$$F : K \to R^n$$

is $(T, \xi; \eta)$-invex function of second kind on K. Let η be antisymmetric on K. Then $(T - \xi)$ is generalized η-monotone on K.

Proof. Since all the conditions of Theorem 2.8 are satisfied, $(T - \xi)$ is η-monotone on K, i.e.,

$$\langle (T - \xi)(u), \eta(x, u) \rangle + \langle (T - \xi)(x), \eta(u, x) \rangle \leq 0$$

for all $x, u \in K$. Since η is antisymmetric on K, we have

$$\eta(x, u) + \eta(u, x) = 0$$

for all $x, u \in K$,
i.e.,
for all $x, u \in K$ .Since and we have and $\eta(u, x) = -\eta(x, u)$
$T(u) \in L(X, R^n) \equiv R^n$

$\xi(u) \in L(X, R^n) \equiv R^n$,

$\langle T(u), kz\rangle = k\langle T(u), z\rangle$

$\langle \xi(u), kz\rangle = k\langle \xi(u), z\rangle$
for all $z \in X$ and any nonzero scalar k. From the above inequality, we get

$\langle (T-\xi)(u), \eta(x, u)\rangle + \langle (T-\xi)(x), -\eta(x, u)\rangle \le 0$

for all $x, u \in K$, i.e.,
for all $x, u \in K$. Thus for all $x, u \in K$, i.e.,

$\langle (T-\xi)(u), \eta(x, u)\rangle - \langle (T-\xi)(x), \eta(x, u)\rangle \le 0$

$\langle (T-\xi)(x), \eta(x, u)\rangle - \langle (T-\xi)(u), \eta(x,u)\rangle \ge 0$

$\langle (T-\xi)(x) - (T-\xi)(u), \eta(x, u)\rangle \ge 0$

for all $x, u \in K$. Hence $(T-\xi)$ is generalized η-monotone on K. This completes the proof of the theorem.
□

II. Equivalence Theorems

In this section, we establish the equivalence between the problems (GNVIP) and (DGNVIP) using $(T,\xi;\eta)$-invex function of second kind.

Theorem 3.1. Let K be a nonempty subset of a reflexive real Banach space X and let

$$T : K \to L(X, R^n) \equiv R^n$$

And $\xi : K \to L(X, R^n) \equiv R^n$
be any maps. Let

be any vector valued function. Let

$$\eta : K \times K \to X$$
$$F : K \to R^n$$

be $(T,\xi;\eta)$-invex function of second kind on K. Assume that:

(i) K is η-invex cone in X,

(ii) T and ξ are η-hemicontinuous on K,

(iii) η is antisymmetric on K. Then the following are equivalent:
(A) find $u \in K$ such that $\langle T(u), \eta(x, u)i \ge \langle \xi(u), \eta(x, u)\rangle$ for all $x \in K$,

(B) find $u \in K$ such that $\langle T(x), \eta(x, u)\rangle \ge \langle \xi(x), \eta(x, u)\rangle$ for all $x \in K$.
Proof. By Theorem 2.8, F is $(T,\xi;\eta)$-invex function of second kind on K, imply $(T-\xi)$ is generalized η-monotone on K. Again we have

(i) K is η-invex cone in X,

(ii) T and ξ are η-hemicontinuous on K, (iii) η is antisymmetric on K.
Since all the conditions of Theorem 1.6 are satisfied, the problems
(A) find $u \in K$ such that $\langle T(u), \eta(x, u) \rangle \geq \langle \xi(u), \eta(x, u) \rangle$ for all $x \in K$, (B) find $u \in K$ such that $\langle T(x), \eta(x, u) \rangle \geq \langle \xi(x), \eta(x, u) \rangle$ for all $x \in K$ are equivalent. This completes the proof of the theorem. □

Proposition 3.2. Let K be a nonempty subset of a reflexive real Banach space X and let

$$T : K \to L(X, R^n) \equiv R^n$$

And be any maps. Let

$\xi : K \to L(X, R^n) \equiv R^n$

$$\eta : K \times K \to X$$

be any vector valued function. Let

$F : K \to R^n$

be $(T, \xi; \eta)$-invex function of second kind on K. If $x_0 \in K$ solves the problem GVIP given by:

"find $x_0 \in K$ such that

$\langle \xi(x_0), \eta(x, x_0) \rangle \leq 0$ for all $x \in K$."

Then F is $(T, \xi; \eta)$-invex function of first kind at $x_0 \in K$.

Proof. Since $x_0 \in K$ solves the problem (GVIP), we have

$$\langle \xi(x_0), \eta(x, x_0) \rangle \leq 0$$

for all $x \in K$. F is $(T, \xi; \eta)$-invex function of second kind on K, i.e.,

$$F(x) - F(u) + \langle \xi(u), \eta(x, u) \rangle \geq \langle T(u), \eta(x, u) \rangle$$

for all $x, u \in K$. At $u = x_0$, we get

$$F(x) - F(x_0) + \langle \xi(x_0), \eta(x, x_0) \rangle \geq \langle T(x_0), \eta(x, x_0) \rangle$$

for all $x \in K$, i.e.,

$F(x) - F(x_0) \geq \langle T(x0), \eta(x, x_0) \rangle - \langle \xi(x_0), \eta(x, x_0) \rangle$
$\geq \langle T(x_0), \eta(x, x_0) \rangle$

for all $x \in K$ which together with the first equation gives

$F(x) - F(x_0) - \langle \xi(x_0), \eta(x, x_0) \rangle \geq \langle T(x_0), \eta(x, x_0) \rangle$

for all $x \in K$. Hence F is $(T, \xi; \eta)$-invex function of first kind at $x_0 \in K$. This completes the proof of the theorem. □

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