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Manpower Systems with Recruitment Period Depending On Erlang Departure Times during Busy Period

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Abstract: *A System in which employees leave the organization during busy period and new employees are recruited after the busy period is treated. After the busy period, recruitment starts. In Model-A busy period has general distribution and the inter departure time of employees has Erlang Phase two distribution and in Model-B General Departure time and Erlang operation time is also treated. Joint Laplace stieltjes Transforms of busy period and recruitment time and their means are obtained.*
Key Words: *Manpower system, Loss of Manpower and Recruitment, Joint transform, Erlang Phase two*

I. Introduction

It is very common in business world to face the availability and shortage of Manpower periods. These two periods are liable to be dependent on each other due to various reasons such as larger number of employees leaving the organization, unavailability of suitable persons in the job market, stringent recruitment policies and such similar reasons. There are many reasons for an employee to leave the organization. It may be for higher salary or to join family or for higher education etc… If more persons leave, then the business would be severely affected during busy period. As loss and shortcomings are inevitable and fund management is to be done during busy period and in the recruitment period, one may have to speed up recruitments using different strategies in order to start business early. The duration of busy period and the duration to recruit employees are random and they occur alternately in a business organization. When a busy period is long, one may like to speed up recruitment so as to start the next busy period early. A systematic approach to manpower system has been made as early as 1947 by Vajda [8] and others. For clear understanding of manpower planning one can refer to Bartholomew [1], Grinold and Marshall [2] and Vajda [9]. Lesson [3] have given methods to compute wastages (Resignation, dismissal and death) and promotion intensities which produce the proportions corresponding to some desired planning proposals. Stochastic models are designed for wastage and promotion in manpower system by Vassiliou [10]. Subramanian V [6] has made an attempt to provide optimal policy for recruitment training, promotion, and wastages in manpower planning models with special provisions such as time bound promotions, cost of training and voluntary retirement scheme. For other manpower models one may refer Setlhare K [5]. For three characteristics system in manpower models one may refer to Mohan C and Ramanarayanan R [4]. Manpower system with Recruitment Period Depending on Attrition during the busy period by K.Usha and R.Ramanarayanan [7]. In this Paper we consider two models. In model A the busy period of operation of an organization has general distribution and the inter–departure time of employees has Erlang distribution phase 2 and in model B the busy period of operation has exponential distribution and the inter-departure time of employees has general distribution. After the busy period, recruitments of employees start to fill up the vacancies due to employees leaving the organization during the busy period. Two distinct distributions for the recruitment times depending on the length of busy period is within or exceeding a random threshold length is considered. The joint transform is obtained to derive the mean number of employees left and recruitment time . Numerical examples are also presented for illustration.

1.1 Model A: General Busy Period and Erlang departure

The main assumptions of the model are given below

1. The busy period T of an organization is a random variable with cumulative distribution function (Cdf) F(x) and probability density function (pdf) f(x)
2. The inter-departure times of employees are independent random variables with Erlang distribution with rate λ. Let N be the number of employees left during the operation time T phase 2 and the recruitment time S has general distribution with Cdf $G_1(y)$ and pdf $g_1(y)$ when the operation time T is less than the threshold time U and it has Cdf $G_2(y)$ and pdf $g_2(x)$ when T is more than the threshold U to speed up recruitment.
3. Threshold U has exponential distribution with parameter μ.We may derive the joint distribution of T, N, and S as follows.

The joint probability density function of *T* and *S* and probability function of *N* is

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}P\left(T\le x,\ N=n,\ S\le y\right)\right)=f\left(x,n,y\right)$$

$$=f\left(x\right)\begin{Bmatrix} e^{-\lambda x}\left(\frac{(\lambda x)^{2n}}{(2n)!}+\frac{\left(\lambda x\right)^{2n+1}}{\left(2n+1\right)!}\right)e^{-\mu x}g_{1}\left(y\right)+ \\ e^{-\lambda x}\left(\frac{(\lambda x)^{2n}}{(2n)!}+\frac{\left(\lambda x\right)^{2n+1}}{\left(2n+1\right)!}\right)\left(1-e^{-\mu x}\right)g_{2}\left(y\right)\end{Bmatrix} \qquad ....(1)$$

n=1,2,3, . . .

The first term of equation (1) is the part of the pdf that the busy period is *x*, the recruitment time is *y*, the number of employees left is *n* and the busy period duration is within the threshold time. The second term of equation (1) is the part of the pdf that the busy period is *x*, the recruitment time is *y*, the number of employees left is *n* and the busy period duration exceeds the threshold time.Since the inter departure times of employees Erlang distribution with phase2 and rate λ exactly 2n or 2n+1 exponential times are completed during the busy period (0,x) is considered for writing equation(1).
Let us define the joint Laplace transform cum generating function as follows.

$$E\left[e^{-tT}e^{-sS}\ \theta^{N}\right]=\int_{0}^{\infty}\int_{0}^{\infty}\sum_{n=0}^{\infty}f\left(x,n,y\right)e^{-tx}.e^{-sy}\ \theta^{n}\ dx\,dy$$

$$=\int_{0}^{\infty}\int_{0}^{\infty}\sum_{n=0}^{\infty}e^{-tx}e^{-sy}\begin{bmatrix} f\left(x\right)e^{-\lambda x}\left(\frac{(\lambda x)^{2n}}{(2n)!}+\frac{\left(\lambda x\right)^{2n+1}}{\left(2n+1\right)!}\right)e^{-\mu x}g_{1}\left(y\right)+ \\ f\left(x\right)e^{-\lambda x}\left(\frac{(\lambda x)^{2n}}{(2n)!}+\frac{\left(\lambda x\right)^{2n+1}}{\left(2n+1\right)!}\right)\left(1-e^{-\mu x}\right)g_{2}\left(y\right)\end{bmatrix}\theta^{n}\,dxdy,$$

$$0\le\theta\le1,\ \text{and}\ T,\ S\ge0$$

On simplification, we obtain

$$=\frac{1}{2}\left[g^{*}{}_{1}\left(s\right)-g_{2}^{*}\left(s\right)\right]\left[\left(\frac{\sqrt{\theta}+1}{\sqrt{\theta}}\right)f^{*}\left(\lambda+t+\mu-\lambda\theta^{1/2}\right)+\left(\frac{\sqrt{\theta}-1}{\sqrt{\theta}}\right)f^{*}\left(\lambda+t+\mu+\lambda\theta^{1/2}\right)\right]+$$

$$\frac{1}{2}g_{2}^{*}\left(s\right)\left[\left(\frac{\sqrt{\theta}+1}{\sqrt{\theta}}\right)f^{*}\left(\lambda+t-\lambda\theta^{1/2}\right)+\left(\frac{\sqrt{\theta}-1}{\sqrt{\theta}}\right)f^{*}\left(\lambda+t+\lambda\theta^{1/2}\right)\right] \quad ......(2)$$

Here * indicates Laplace transform.
Put θ=1 in equation (2), we get

$$E\left[e^{-sS}e^{-tT}\right]=\left[g_{1}^{*}(s)-g_{2}^{*}(s)\right]f^{*}(t+\mu)+g_{2}^{*}\left(s\right)f^{*}\left(t\right) \quad ....(3)$$

Using differentiation of equation (3), we get

$$E(T) = -\left[f^{*'}(0) \right]$$

Assume that f(.) is exponential density with parameter δ .

$$E(T) = \frac{1}{\delta} \quad \text{................(4)}$$

Using differentiation of equation (3), we get

$$E(S) = \left[E(S_1) - E(S_2) \right] f^*(\mu) + E(S_2)$$

Assume that f (.) is exponential density with parameter δ

$$E(S) = \left[E(S_1) - E(S_2) \right] \left(\frac{\delta}{(\delta + \mu)} \right) + \mathrm{E}(S_2) \qquad \text{.................(5)}$$

Put s= 0 and t=0 in equation (2), we get

$$E\left[\theta^N\right] = \frac{1}{2}\left[\left(\frac{\sqrt{\theta}+1}{\sqrt{\theta}} \right) f^*\left(\lambda - \theta^{1/2}\lambda\right) + \left(\frac{\sqrt{\theta}-1}{\sqrt{\theta}} \right) f^*(\lambda + \theta^{1/2}\lambda) \right] \qquad \text{.............(6)}$$

Using differentiation of equation(6), we get

$$E\left[N\right] = \left(\frac{2\lambda^2}{\delta\left(\delta + 2\lambda\right)} \right) \qquad \text{...........(7)}$$

1.2 Model B: General Departure Time and Erlang Operation Time

The Main assumptions of the model are given below

1. The operation time T is a random variable with Erlang distribution function with parameter λ and phase
2. The inter-departure times of employees are independent random variables with Cdf $F(x)$ and pdf $f(x)$. Let N be the number of employees left during the operation time T.
3. The recruitment time S has general distribution with Cdf $G_1(Y)$ and pdf $g_1(y)$ when the operation time T is less than the threshold time U and it has Cdf $G_2(Y)$ and pdf $g_2(y)$.When T is more than the thresholed U to speed up recruitment.
4. The threshold has exponential distribution with parameter μ. We may derive the joint distribution of T, N, and S as follows.

The joint probability density function of T and S and probability function of N is

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y} P\left(T \le x,\ \ N = n,\ \ S \le y\right) \right) = f\left(x, n, y\right)$$

$$= \lambda\left(\lambda x\right)e^{-\lambda x}\left[F_n\left(x\right) - F_{n+1}\left(x\right)\right]e^{-\mu x}g_1\left(y\right) + \lambda\left(\lambda x\right)e^{-\lambda x}\left[F_n\left(x\right) - F_{n+1}\left(x\right)\right]\left(1 - e^{-\mu x}\right)g_2\left(y\right), \quad n = 1, 2, 3, \ldots \qquad \text{- (8)}$$

The first term of equation (8) is part of the pdf that the operation time is x, the recruitment time is y, the number of employees left is n and the operation duration is within the threshold time. The second term of equation (8) is part of the pdf that the operation time is x, the recruitment time is y, the number of employees left is n and the operation duration exceeds the threshold time where $F_k(x)$ is the k fold Cdf convolution and is the probability that the time for k departure of employees is less than x, for k = 1,2, 3,…

Let us define the joint Laplace transform cum generating function as follows,

$$E\left[e^{-tT}e^{-sS}\theta^N\right] = \int_0^\infty\int_0^\infty\sum_0^\infty f\left(x, n, y\right)e^{-tx}e^{-sy}\theta^n dxdy \ , 0 \le \theta \le 1, \text{ and } t, s \ \ge 0.$$

$$= \int_0^\infty \int_0^\infty \sum_0^\infty \Big[\lambda(\lambda x) e^{-\lambda x} \left(F_n(x) - F_{n+1}(x) \right) e^{-\mu x} \; g_1(y) + \\ \lambda(\lambda x) e^{-\lambda x} \left(F_n(x) - F_{n+1}(x) \right) \left(1 - e^{-\mu x} \right) g_2(y) \Big] e^{-tx} e^{-sy} \theta^n dxdy$$

On simplification, we obtain

$$= \left[g_1^*(s) - g_2^*(s) \right] \left\{ \left(\frac{\lambda}{t+\lambda+\mu} \right)^2 \frac{\left(1 - f^*(t+\lambda+\mu)\right)}{1-\theta f^*(t+\lambda+\mu)} - \frac{\lambda^2}{(t+\lambda+\mu)} \frac{d}{dT} \left(\frac{1-f(T)}{1-\theta . f(T)} \right) \right\}$$

$$+ g_2^*(s) \left\{ \left(\frac{\lambda}{\lambda+t} \right)^2 \frac{\left(1 - f^*(t+\lambda)\right)}{1-\theta \; f^*(t+\lambda)} - \left(\frac{\lambda^2}{t+\lambda} \right) \frac{d}{dT^{'}} \left[\frac{1-f^*(T^{'})}{1-\theta . f^*(T^{'})} \right] \right\} \quad ....(9)$$

$$Here \; T = (t+\lambda+\mu) \; and \; T^{'} = (t+\lambda)$$

Put θ = 1 in equation (9), we get

$$E\left[e^{-tT} e^{-sS} \right] = \left[g_1^*(s) - g_2^*(s) \right] \frac{\lambda^2}{(t+\lambda+\mu)^2} + g_2^*(s) \frac{\lambda^2}{(\lambda+t)^2} \quad ....(10)$$

Put s=0 in equation (10), we get

$$E\left[e^{-tT} \right] = g_2^*(0) \left(\frac{\lambda}{\lambda + t} \right)^2 \quad ....(11)$$

Using differentiation of equation (11), we get

$$E[T] = \left(\frac{2}{\lambda} \right) \quad ....(12)$$

Put t =0 in equation (10), we get

$$E\left[e^{-sS} \right] = \left(g_1^*(s) - g_2^*(s) \right) \left(\frac{\lambda}{\lambda + \mu} \right)^2 + g_2^*(s) \quad ....(13)$$

Using differentiation of equation (13), we get

$$E[S] = \left(E[S_1] - E[S_2] \right) \left(\frac{\lambda}{\lambda + \mu} \right)^2 + E(S_2) \quad ....(14)$$

Put s = 0 and t = 0 in equation (9), we get

$$E\left[\theta^N \right] = \left(\frac{1-f^*(\lambda)}{1-\theta f^*(\lambda)} \right) - \lambda \frac{d}{d\lambda} \left(\frac{1-f^*(\lambda)}{1-\theta f^*(\lambda)} \right) \quad ....(15)$$

Assume that f (.) is exponential density with parameter δ

Using differentiation of equation(15) , we get

$$E[N] = \delta E[T] \quad ....(16)$$

1.3 Numerical Illustration

1.3.1 Model-A (General Busy Period and Erlang Departure)

We consider the case of busy period, the inter-departure time of employees, recruitment time S_1 and recruitment time S_2 have Erlang distribution with rate parameter values respectively δ, λ, $E[S_1]$ and $E[S_2]$.

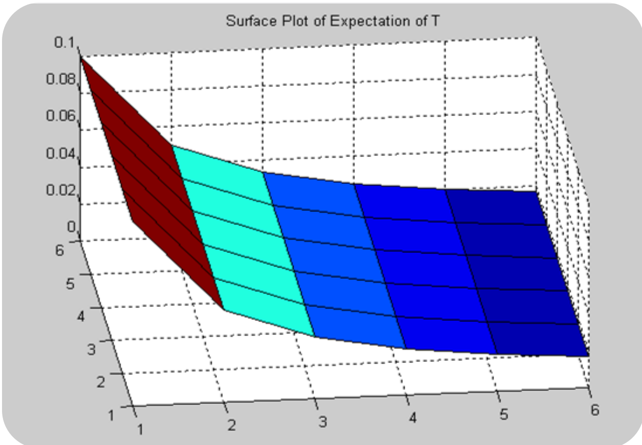
We fix the value of $\lambda=2$, $E[S_1]=2$ and $E[S_2]=1$

We vary the busy period parameter δ= 10,20,30,40,50,60 and the threshold rate parameter μ=5,10,15,20,25,30 in the equations (5),(6) and (7), we get

Table of E[T]

E[T]						
$\frac{\mu \rightarrow}{\delta}$	5	10	15	20	25	30
10	.1000	.0500	.0333	.0250	.0200	.0167
20	.1000	.0500	.0333	.0250	.0200	.0167
30	.1000	.0500	.0333	.0250	.0200	.0167
40	.1000	.0500	.0333	.0250	.0200	.0167
50	.1000	.0500	.0333	.0250	.0200	.0167
60	.1000	.0500	.0333	.0250	.0200	.0167

3D-Graph of E[T]

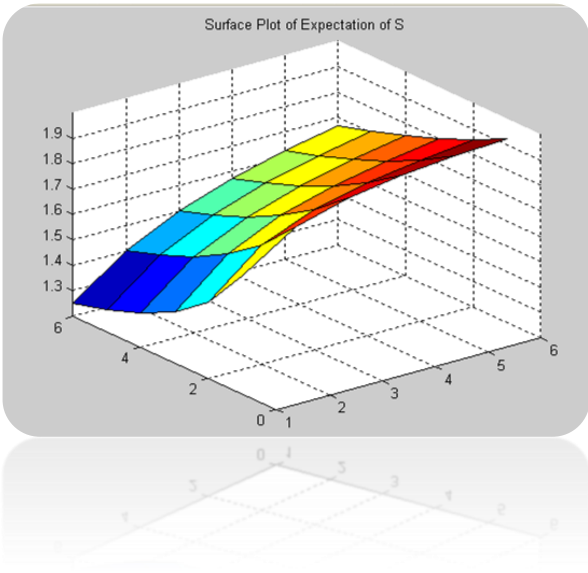


As the value of μ increases, E[T] decreases and δ increases, E[T] remains the same.

Table of E[S]

E[S]						
$\frac{\mu \rightarrow}{\delta}$	5	10	15	20	25	30
10	1.6667	1.8000	1.8571	1.8889	1.9091	1.9231
20	1.5000	1.6667	1.7500	1.8000	1.8333	1.8571
30	1.4000	1.5714	1.6667	1.7273	1.7692	1.8000
40	1.3333	1.5000	1.6000	1.6667	1.7143	1.7500
50	1.2857	1.4444	1.5455	1.6154	1.6667	1.7059
60	1.2500	1.4000	1.5000	1.5714	1.6250	1.6667

3D-Graph of E[S]

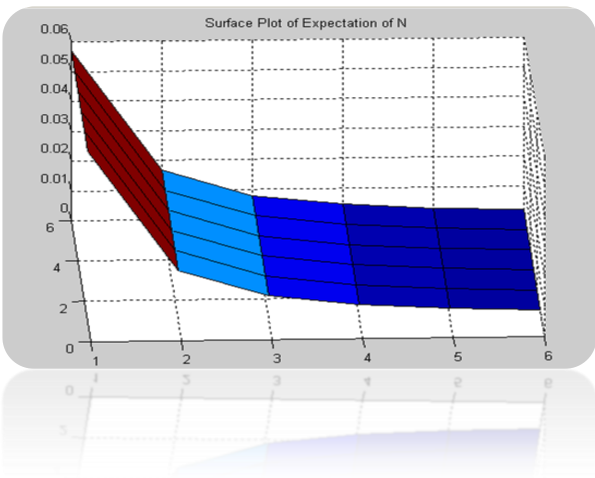


As the value of μ increases, E[S] increases and the value of δ increases, E[S] decreases.

Table of E[N]

E[N]						
$\frac{\mu \rightarrow}{\delta}$	5	10	15	20	25	30
10	0.0571	0.0167	0.0078	0.0045	0.0030	0.0021
20	0.0571	0.0167	0.0078	0.0045	0.0030	0.0021
30	0.0571	0.0167	0.0078	0.0045	0.0030	0.0021
40	0.0571	0.0167	0.0078	0.0045	0.0030	0.0021
50	0.0571	0.0167	0.0078	0.0045	0.0030	0.0021
60	0.0571	0.0167	0.0078	0.0045	0.0030	0.0021

3D-Graph of E[N]



As the value of μ increases, E[N] decreases and δ increases, E[N] remains the same.

1.3.2 Model-B (General Departure Time and Erlang Operation Time)

We consider the case of busy period, the inter-departure time of employees, recruitment time S_1 and recruitment time S_2 have Erlang distribution with rate parameter values respectively δ, λ, $E[S_1]$ and $E[S_2]$.

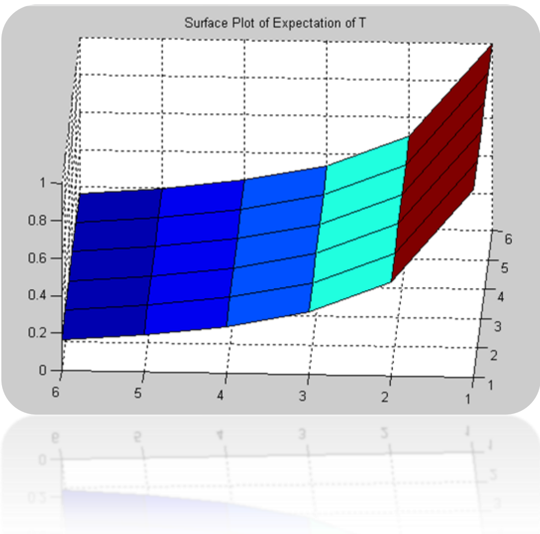
We fix the value of $E[S_1]=2$ and $E[S_2]=1$ and for the different values of λ=2,4,6,8,10,12.

We vary the busy period parameter δ= 10,20,30,40,50,60 and the threshold rate parameter μ=5,10,15,20,25,30 in the equations (12),(14) and (15), we get

Table of E[T]

E[T]						
$\frac{\mu \rightarrow}{\lambda}$	5	10	15	20	25	30
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
6	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
8	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500
10	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
12	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667

3D-Graph of E[T]



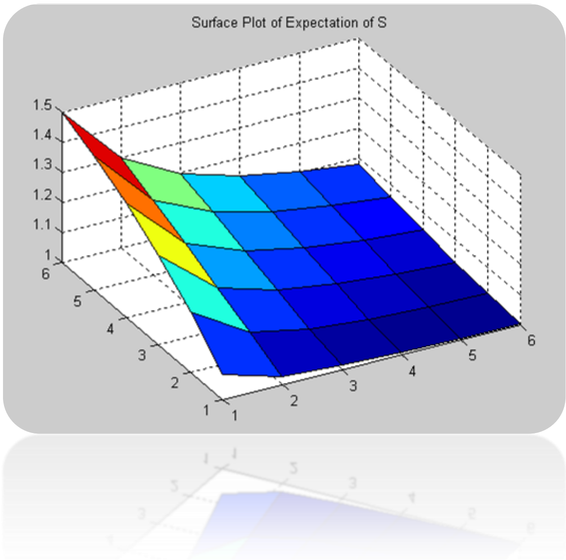
As the value of μ increases, E[T] remains the same and the value of λ increases, E[T] decreases.

E[S]						
$\frac{\mu \rightarrow}{\lambda}$	5	10	15	20	25	30
2	1.0816	1.0278	1.0138	1.0083	1.0055	1.0039
4	1.1975	1.0816	1.0443	1.0278	1.0190	1.0138

Table of E[S]

6	1.2975	1.1406	1.0816	1.0533	1.0375	1.0278
8	1.3787	1.1975	1.1210	1.0816	1.0588	1.0443
10	1.4444	1.2500	1.1600	1.1111	1.0816	1.0625
12	1.4983	1.2975	1.1975	1.1406	1.1052	1.0816

3D-Graph of E[S]

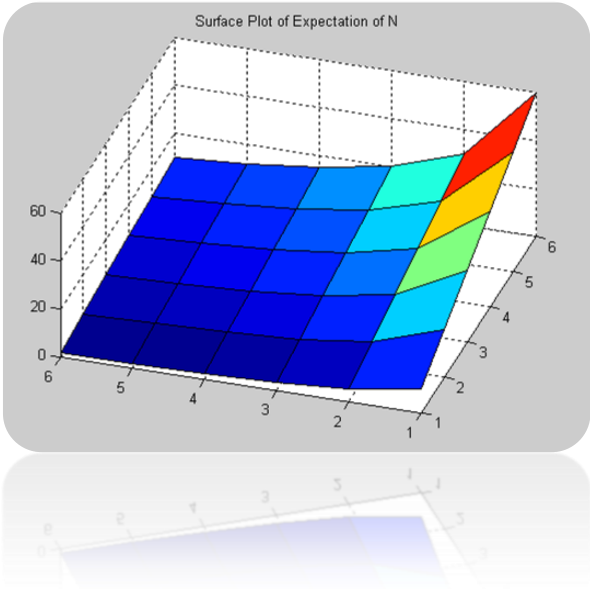


As the value of λ increases, E[S] increases and the value of μ increases, E[S] decreases.But the diagonal values of E[S] remains unchanged.

Table of E[N]

E[N]						
$\mu \rightarrow$ / λ	5	10	15	20	25	30
2	10.0000	20.0000	30.0000	40.0000	50.0000	60.0000
4	5.0000	10.0000	15.0000	20.0000	25.0000	30.0000
6	3.3333	6.6667	10.0000	13.3333	16.6667	20.0000
8	2.5000	5.0000	7.5000	10.0000	12.5000	15.0000
10	2.0000	4.0000	6.0000	8.0000	10.0000	12.0000
12	1.6667	3.3333	5.0000	6.6667	8.3333	10.0000

3D-Graph of E[N]



As the value of λ increases, E[N] increases and the value of μ increases, E[N] decreases.But the diagonal values of E[N] remains unchanged.

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