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A Non-Stationary Transition Probabilities for a Reservoir elevation of Hydro Electric Power Dam

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Abstract: *This paper demonstrates the application of Markov chain model with non-stationary transition probabilities to study data of the Reservoir elevation of Shiroro dam in the dry and raining seasons. The result indicates an optimal of 40% and 49% transition probabilities at equilibrium for dry season and wet season respectively. The result confirms the reality on the ground that higher reservoir elevation is obtained more during the raining season. Conversely, the lower reservoir elevation is experienced largely in the dry season. The variation of the reservoir elevation directly affects the hydro electric power generation and the availability of the other dam resources. Markov chain model could be used as a predictive device for studying reservoir elevation of Shiroro dam. These predictions might be used for the management of the dam resources.*
Keywords: *Markov chain, non-stationary, transition, probability, dam, reservoir, elevation.*

I. Introduction

Stochastic models are influential and widely used tools in reservoir management. Policy makers and administrators in the industry take into consideration quantitative predictions on decision-making policies. Markov chain models have been applied to many areas of Dam related problems recently reported in [1] was a study that used a first and second order Markov chain models to dry and wet periods of annual stream flow series to produce a stochastic structure of hydrological droughts. They found that the second – order Markov (MC2) model in general gives results that are in better agreement with simulation results as compared with the first order (MC1) model. In [2] was considered a case of two connected dams, a capture and supply dam with a regular demand of one unit from the supply dam and random inputs into the capture dam. Modeling the system as a Markov chain she derived a transition probability matrix with a general block structure to solve for the invariant state probability vector of the water stored in the dams. Contained in [3] is the computed reservoir water release policy for Shiroro hydro-electric Dam using a probabilistic dynamic programming model. In the simulation, he used the State variable as the reservoir storage volume and uncertain nature of the inflow process is accounted for in the model by considering different possible inflow volume and their inflow probabilities. In [4] is reported a three state Markov model to study the reservoir elevation of Shiroro dam in continuous time. It is observed that the assumption of stationary transition probabilities for the model is not very appropriate, since it does not take into consideration the seasonal variation in Nigeria. It is therefore important to consider in this paper the same model in discrete time with non-stationary transition probabilities. The two models could be used side by side to make prediction of the reservoir elevation of the dam.

Shiroro dam was built primary for the production of Hydro electric power (H E P). The water level (Reservoir Elevation) of dam rises and falls seasonally every year unpredictably. Sometimes the dam overflows and the run ways are opened when the dam is filled to capacity. Some other time the water level is so low so that some or all the turbines are shut down due to shortage of water. In view of these uncertainties, it is an attempt in this paper to formulate a Markov chain model that could be used to make prediction of the reservoir elevation quantitatively.

II. Formulation Of Markov Chain Model

Consider a random variable X that is indexed by time parameter n, such a process is referred as stochastic process X_n, n = 0,1,2,… , suppose that the stochastic process $\{X_n, n = 0, 1, 2, \ldots\}$ takes on a finite or countable number of possible values such that:

$$p\{X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, \ldots, X_1 = i_1, X_0 = i_0\} = P_{ij} \quad (1)$$

for all states i_0, $i_1, \ldots, i_{n-1}, i, j$ and all $n \geq 0$. Such a stochastic process is known as a *Markov chain,[5]*. Since probabilities are nonnegative and since the process must make a transition into some states, we have that.

$$P_{ij} \geq 0, \quad i, j, \geq 0; \quad \sum_{j=0}^{\infty} P_{ij} = 1, \quad i = 0,1,\ldots..$$

It has been observed that the reservoir elevation of the Shiroro dam that can tolerate turbine operation has the highest and lowest reservoir elevation of 382.50m and 358.25m respectively. Any short fall below

358.25m in reservoir elevation will amount to shutdown of some or all turbine engines resulting to load shedding. Similarly, if the water level rises above 382.50m, it incidentally calls for opening of windows for overflows which eventually affects the downstream farming and ecological activities.

Let us consider reservoir elevation of Shiroro dam in a month to be random variable X. Suppose that the random variable is collected for several months to constitute a stochastic process X_n of first order dependence presented in (1). It is assumed that this process is modeled by a three state Markov chain thus:
State 1: Elevation below 367.50m. Low reservoir elevation.
State 2: Elevation between 367.50m. – 376.25m. Middle reservoir elevation.
State 3: Elevation above 376.25m. Upper reservoir elevation.
The possible transitions between the states is described by the following transition diagram.

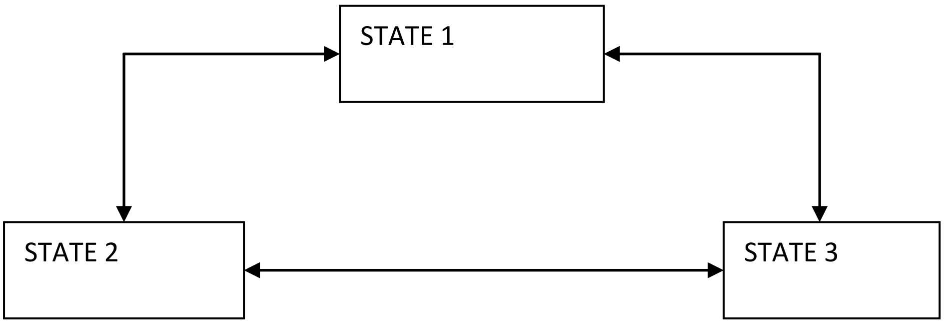

Figure 1: The transition diagram for the process.

Let P_{ij} be the probability that the reservoir elevation presently in state i will make a transition to state j in the next transition.

The transition between states is described by the following transition probability matrix P

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}$$

$$0 \leq p_{ij} \leq 1 \qquad ij = 1,2 \; and \; 3$$

$$\sum_{i=1}^{3} p_{ij} = 1 \quad i = 1,2, and \; 3$$

Let $P^n=(P^n_1, P^n_2, P^n_3)$ denote the probabilities of finding the reservoir elevation in any of the states 1, 2, 3 respectively on n period.
Then, we have
$P^n= P^{n-1}P$ (2a)
On iteration we have
$p^n= p^o p^n \quad n = 1, 2, 3$ (2b)
Where p^o is any starting vector of probabilities, see [6].

III. The Limiting State Probabilities

The process reaches a steady state after a sufficiently large period of time. This is the equilibrium probability distribution $\pi = (\pi_1 \quad \pi_2 \quad \pi_3)$ and it is obtained by letting $n \to \infty$ in equation (2a)

Thus we have $\pi = \pi P$ and the sum of the component of π must be unity

$$i.e \qquad \sum_{i=1}^{3} \pi_i = 1$$

As shown in [5]. We use these last two equations to find the limiting state probabilities for the process.

IV. The Effect Of The Seasons

It is known that the reservoir elevation of Shiroro dam changes with the seasons. The reservoir has two peaks corresponding to Dry season and raining season, we therefore considered the two seasons as the transition times, Iloeje [7] observed that the period of dry season takes effect from November to March. In this period there is no rain or little rainfall while the raining season takes effect from April to October. Each of the seasons has its own transition count and transition probability matrices. We donate these as follows:

M_1 Transition count matrix for dry season and
P_1 Transition probability matrix for dry season
M_2 Transition count matrix for Raining season and
P_2 Transition probability matrix for raining season.
Let

$$M_k = f_{ij}(k), \quad i, j = 1,2,3 \quad \text{and } k = 1,2.$$
$$\text{and } P_k = P_{ij}(k), \; i, j = 1,2,3 \text{ and } k = 1,2. \qquad (3)$$

$f_{ij}(k)$ denotes the transition count from state i to state j for the season k. $P_{ij}(k)$ is the transition probability from state i to state j for the season k.
Accordingly,

$$\hat{P}_{ij}(k) = \frac{f_{ij}(k)}{f_i(k)}, \quad k = 1,2 \;\; and \quad i, j = 1,2,3.$$

$$where \;\; f_i = \sum_{j=1}^{3} f_{ij}(k)$$

as contained in [8]

V. Test For Stationary Of The Probability Matrices P_k

To test for independence of P_K on K, the Null hypothesis is stated thus

H_0: $P_{ij}(k) = P_{ij}, \; for \; all \; i, j = 1,2,3$ and for all k

H_1: $P_{i,j}(k)$ depends on K.

The likelihood ratio Test for the above hypothesis, is

$$M = \sum_{k=1}^{2} M_k = [f_{ij}] \qquad (4)$$

$$where \;\; f_{ij} = \sum_{k=1}^{2} f_{ij}(k)$$

The maximum likelihood estimate of the stationary transition probability matrix is

$$P_{ij} = \frac{f_{ij}}{f_i} \qquad (5)$$

$$where \; f_i = \sum_{j=1}^{3} f_{ij}$$

The $\lambda,$ the likelihood ratio criterion is given by

$$\lambda = \prod_{i,j=1}^{3} \prod_{k=1}^{2} \left[\frac{P_{ij}}{P_{ij}(k)} \right]^{f_{ij}(k)}$$

According to [8]

$$-2In\lambda = \chi 2_{M(M-1)(T-1)} \qquad (6)$$

where m is the number of states and T is the time parameter. We evaluate $\lambda,$ and calculate $-2In\lambda.$ We then get the critical value of $\chi^2 \; at \, \alpha$ significance level and compare it with $-2In\lambda.$ It is then decided to accept or reject the Null hypothesis. With the acceptance of H_0, we have a homogeneous Markov chain

model. The model is represented by a single transition count matrix in (4) and the $P_{ij}s$ are estimated from (5). Otherwise we have the
non- homogenous Markov chain model.
The stochastic matrix P can be written as
$P=P_1P_2$ (7)
and the p_{ijs} are estimated from (5).
The limiting state probability vector π_1 and π_2 for the two seasons are then obtained from the following.

$$\pi_1 = \pi_0 P_1$$
$$\pi_2 = \pi_1 P_2 \quad (8)$$

Where π_0 is the equilibrium transition probability matrix P, see [8].

VI. Application

A summary statistics for 10 years for the monthly reservoir elevation of Shiroro dam is contained in table1.

Table1. A summary of data of the reservoir elevations of Shiroro dam (in meters) from 2001 – 2010.

CLASS INTERVAL (M)	STATES	FREQUENCY
358.25 – 367.50	1	50
367.51 – 376.25	2	36
376.26 – 382.50	3	34
TOTAL		120

Source: extracted from [9]
The distribution of reservoir elevation in various states in percentage is as follows;
i. State1 41.7%
ii. State2 30%
iii. State3 28.3%
It is necessary therefore to check whether these percentages are stable or not, due to the seasonal variation [9].
The transition count matrices for the seasons obtained from table1 by equation (3) is as follows;

$$M_1 = \begin{pmatrix} 5 & 0 & 0 \\ 9 & 18 & 0 \\ 0 & 8 & 9 \end{pmatrix} \ and \ M_2 = \begin{pmatrix} 34 & 7 & 3 \\ 1 & 3 & 6 \\ 0 & 0 & 17 \end{pmatrix}$$

The model assumes that the transition of the reservoir elevation from one State to another is dependent only on the present State and is independent of past history. Anderson and Goodman’s statistics [10] and [9] were used to check this assumption.
To test for the hypothesis stated in the previous section, we have from equation (4)

$$M = \begin{pmatrix} 39 & 7 & 3 \\ 10 & 21 & 6 \\ 0 & 8 & 26 \end{pmatrix}.$$

The maximum likelihood estimate of the stationary transition probability matrix using equation (5) is

$$p = \begin{pmatrix} 0.7969 & 0.1429 & 0.0612 \\ 0.27072 & 0.5676 & 0.1622 \\ 0 & 0.2353 & 0.76447 \end{pmatrix}.$$

Therefore, from equation (6) the likelihood ratio criterion λ is given by
$-2 \ln \lambda = \chi^2_{3(3-1)} = \chi^2_6$
$-2 \ln \lambda = \chi^2_6$, and $-2 In\,\lambda = 37.668686$.
The critical value of χ^2_6 at $\alpha = 0.05$ is 12.59. Therefore, the null hypothesis of constant transition probability cannot be accepted. Thus, we have non-homogenous Markov chain model. Thus the maximum likelihood estimate of the transition probability matrix using equation (7) is given by

$$p = \begin{pmatrix} 0.7727 & 0.1591 & 0.0682 \\ 0.3242 & 0.2530 & 0.4228 \\ 0.047 & 0.1412 & 0.8118 \end{pmatrix}.$$

On iteration, using equation (2) we have

$$p^2 = \begin{pmatrix} 0.651 & 0.1728 & 0.1754 \\ 0.3524 & 0.1753 & 0.4714 \\ 0.1203 & 0.1578 & 0.7219 \end{pmatrix}$$

$$p^4 = \begin{pmatrix} 0.5070 & 0.1728 & 0.1754 \\ 0.3482 & 0.1660 & 0.4847 \\ 0.2208 & 0.1624 & 0.6166 \end{pmatrix}$$

$$p^{10} = \begin{pmatrix} 0.3523 & 0.1665 & 0.4680 \\ 0.3401 & 0.1656 & 0.4930 \\ 0.3204 & 0.1606 & 0.5138 \end{pmatrix}$$

$$p^{14} = \begin{pmatrix} 0.3399 & 0.1637 & 0.4823 \\ 0.3390 & 0.1656 & 0.4939 \\ 0.3317 & 0.1646 & 0.4979 \end{pmatrix}$$

$$p^{18} = \begin{pmatrix} 0.3358 & 0.1635 & 0.4863 \\ 0.3384 & 0.1655 & 0.4941 \\ 0.3355 & 0.1649 & 0.4939 \end{pmatrix}$$

$$p^{26} = \begin{pmatrix} 0.3351 & 0.1671 & 0.4876 \\ 0.3382 & 0.1654 & 0.4939 \\ 0.3367 & 0.1650 & 0.4922 \end{pmatrix} \text{...(9).}$$

$$p^{26} = \begin{pmatrix} 0.34 & 0.17 & 0.49 \\ 0.34 & 0.17 & 0.49 \\ 0.34 & 0.17 & 0.49 \end{pmatrix} \quad \textit{corrected to 2 decimal places}$$

And for n > 26, we find that P^n get closer and closer to exactly equation (9) that is, as n increases the transition probabilities converges and using equation (2b) with P^0 = (1 0 0)

$$P^n = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0.34 & 0.17 & 0.49 \\ 0.34 & 0.17 & 0.49 \\ 0.34 & 0.17 & 0.49 \end{pmatrix} = \begin{pmatrix} 0.34 & 0.17 & 0.49 \end{pmatrix}$$

The limiting state probability vector is obtained from equation (8) thus:

$$\pi_1 = (0.40 \quad 0.34 \quad 0.26)$$
$$\pi_2 = (0.34 \quad 0.17 \quad 0.49)$$

VII. Conclusion

The previous section identifies quantitative modeling of the seasonal variation of reservoir elevation of Shororo dam. The result shows that a stationary transition probability is not appropriate. Consequently, Non-stationary transition probability was considered for the seasons. The result indicates that in the dry season the reservoir shall be 40%, 34%, and 26% in lower elevation, middle elevation and upper elevation respectively in the long run. This is in contrast to the raining season with reservoir elevation of 34%, 17% and 49% for lower elevation, middle elevation, and upper elevation respectively in the long run. These are the state occupancy transition probability at equilibrium. The result confirms the reality on the ground that higher reservoir elevation is obtained more during the raining season, conversely, the lower reservoir elevation is experienced largely in the dry season. The unpredictable changes in the reservoir elevation have great impart in the quantity of the hydro electric power that is generated from the dam as well as the availability of the other dam resources such fresh water fishery and crop cultivation along the dam basins. The model could be used as a predictive device for studying reservoir elevation of Shiroro dam. These predictions can be used for the management of the dam resources.

References

[1]. Akyuz D.E, Bayazit M and Onoz B (2012): Markov chain models for hydrological drought characteristics, American Meteorological Society. (AMMS) journal, vol.13 (1),pp25-29.

[2]. Piantadosi, J. (2004), Optimal policies for storage of urban storm water, PHD Thesis, University of South Australia.

[3]. Sule B.F. (1988), Reservoir operation policy for optimizing energy generation at Shiroro dam, water reservoir management vol. 2(3), 125-132

[4]. Abubakar U.Y., Mohammed A., Lawal A. (2013). An Evaluation of a Reservoir Elevation with Markov Model in Continuous Time. Asian Journal of Fuzzy and Applied Mathematics. Vol.1(1), pp 21-27.

[5]. Ross S.M. (1989), Introduction to Probability Models. Academic Press, Inc.New York.

[6]. Howard R.A (1960), Dynamic programming and Markov process. M.I.T. press Massachusetts.

[7]. Iloeje N. P. (1981),'A New Geography of Nigeria', Longman Nigeria Limited.

[8]. Bhat U. N. (1984) "Element of Applied Stochastic Processes. Wiley, New York.

[9]. Muhammed A (2013), Application of Markov Model to Reservoir Elevation of Shiroro Dam. Unpublished MTech Thesis. Department of Mathematics and Statistics.Federal University of Technology Minna.

[10]. Anderson T. W., Goodman L. A. (1957) Statistical inference about Markov chain. Ann.Math Stat. 28, pp 89.