IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN:2319-765X. Volume 10, Issue 3 Ver. II (May-Jun. 2014), PP 28-38
www.iosrjournals.org

The Measurement of Ω_M, Ω_Λ and ω in Bianchi Type-I Anisotropic Dark Energy Cosmological Model with Constant Deceleration Parameter in Bimetric Theory of Gravitation

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Abstract : *In this note, anisotropic Bianchi type - I dark energy cosmological model with constant deceleration parameter and with time-dependent equation of state parameter $\omega(t)$ in bimetric theory of gravitation have been investigated and studied the accelerating as well as decelerating phase of expansion of the universe. We have measured the matter energy density parameter Ω_M, the dark energy density parameter Ω_Λ and the equation of state parameter ω in it. The parameters m, l and n affected the overall structure of the model. The time dependent equation of state parameter $\omega(t)$ is goes over to the value which is independent of time and fully depends only on the values of $m > 0$. Our measured values of Ω_M, Ω_Λ, ω and Hubble parameter H in our model are found to be consistent with the results of WMAP satellite, Bennett, C. L. et al. (2003) and SNe Ia data collaborated with CMBR anisotropy and galaxy clustering statistics, Tegmark, M., et al. (2004). The cosmological term Λ is a decreasing function of time and has a small positive value at present epoch which matched with the results from recent supernovae Ia observations. The other geometrical and physical properties of the model have been traced out.*

***Keywords*:** *Cosmology, Dark Matter, Dark Energy, Gravitational Theory*

I. Introduction

The concept of dark energy is an important problem to expose the universe. The cosmological communities believed that a kind of a repulsive force which acts as antigravity is responsible for gearing up the universe some 7 billion years ago, is termed as dark energy. The dark energy cause the accelerating expansion of the universe and several high precision observational experiments, especially the Wilkinson Microwave Anisotropic Probe (WMAP) satellite experiment [Bennett et al. (2003)] concludes that the dark energy occupies near about 73% of the energy of the universe and dark matter is about 23%. The usual matter described by known particle theory is about 4% [1-11]. There are two groups of cosmological communities: The first is Supernova Cosmology Project and second is High-Z Supernova Team. They conclude that the universe is accelerated expanding and they measured the distances to cosmological supernovae by using the intrinsic luminosity of Type Ia supernovae is closely correlated to their decline rate from maximum brightness. These measurements combined with red-shift data for the supernovae and predicted an accelerating universe. For closed universe, the Hubble parameter H, the matter energy density parameter Ω_M and the dark energy density parameter Ω_Λ, predicted by Tegmark [12] are near about $H \approx 0.32$, $\Omega_M \approx 0.23$ and $\Omega_\Lambda \approx 1.17$. For flat cosmological model, the cosmological observations [13-22], suggested the existence of a positive cosmological term Λ with magnitude $\Lambda \approx 10^{-123}$ and with $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$ in the accelerating universe.

The properties of dark energy in the universe from the observational data based on determination of its equation of state (EOS) $p = \omega\rho$, ω is not necessarily constant. The time variable parameter $\omega(t)$ have been restored from expressional data and analysis of the experimental data have been conducted to determine $\omega(t)$ by Sahni et al. [23-24].This parameter $\omega(t)$ have been calculated with some reasoning with reduced to some simple parameterization of the dependences by many authors [25-30]. The results from SNe Ia data collaborated with CMBR anisotropy and galaxy clustering statistics (Tegmark et al. [12]) yield ω as $\omega \approx -0.977$(WMAP +SN results) at 68% confidence level for dark energy. These results are consistent with time variable equation of state parameter $\omega(t)$ and also for time free ω. The equation of state parameter ω is considered as a constant with values -1, 0, +1/3 and +1 corresponding to the vacuum fluid, dust fluid, radiation fluid and stiff dominated universe [31-32] and variable $\omega(t)$ of time or red shift is considered [33-34]. The quintessence models, $\omega > -1$ (explanation of observations of accelerating universe) involving scalar field and phantom model, $\omega < -1$

(expansion of universe increases to infinite degree in finite time) give rise time dependent parameter $\omega(t)$ [35-38]. Various forms of parameter $\omega(t)$ have been used for dark energy models [39-54].

In general theory of gravitation and in its alternative theories, especially in Bimetric theory of gravitation, the Bianchi type I spatially homogeneous with anisotropic cosmological models have been investigated to know the secret of the nature of the universe. Rosen's bimetric theory of gravitation [55-56] is one of the alternatives to general relativity and it is free from singularities appearing in the big bang of cosmological models and it obeys the principle of covariance and equivalence of the general relativity. Thus, at every point of the space time in bimetric theory of gravitation, there are two metrics:

$$ds^2 = g_{ij}\, dx^i\, dx^j \tag{1}$$

$$d\eta^2 = \gamma_{ij}\, dx^i\, dx^j \tag{2}$$

The Rosen's field equations [55-56] in bimetric theory of gravitation are

$$N_i^j - \frac{1}{2} N\, \delta_i^j = -8\,\pi\, k\, T_i^j \tag{3}$$

where $N_i^j = \frac{1}{2}\, \gamma^{\alpha\beta}\, \left(g^{sj}\, g_{si|\alpha}\right)_{|\beta}$, $N = g^{ij}\, N_{ij}$ is the Rosen scalar and for simplicity, let $(8\,\pi\, k) = 1$. Here and hereafter the vertical bar $(\,|\,)$ stands for γ –covariant differentiation where $g = \det(g_{ij})$ and $\gamma = \det(\gamma_{ij})$.

In bimetric theory of gravitation, at every pair of adjacent points in space, Rosen attached to metrics, one is Riemannian metric and other is flat metric in order to develop his theory. In order to have singularity free space time, he has considered the flat metric together with Riemannian metric in his theory. Right from Rosen [55-56], various researchers like Karade, T. M., Isrelit, M., Reddy D. R. K. and Venkateswara Rao N., Bali et al., Katore and Rane, Khadekar and Tade, Borkar et al., Gaikwad et al. [57-72] have been developed the theory and investigated many cosmological models of the universe in bimetric theory of gravitation.

In this literature, an attempt has been made to investigate Bianchi type I dark energy cosmological model with anisotropy, with constant deceleration parameter and with variable equation of state parameter $\omega(t)$, since anisotropy play a significant role in the early universe as well as in the late universe. It is observed that the equation of state parameter $\omega(t)$ is no were longer as a function of time t and finally it goes over to value which is free from time t and depends on only the values of $m > 0$. For $m = 1$, ω vanishes gives dusty universe, for $0 < m < 1$, ω lies in open interval $(1, 0)$ representing visible universe and for $1 < m < \infty$, ω lies in $(0,\ -1)$. i.e., $0 < \omega < -1$ shows invisible universe with dark energy occupies near about 68% of the total universe. Our results are very much fit in 68% limit of $\omega = -0.977$ of WMAP +SN observations [12]. In our results, there is no value of $\omega < -1$, which also agreed with the opinion of Tegmark [12]. The values of our ω are not in the favor of $\omega < -1$ for phantom model and therefore the expansion in our model is not accelerate so quickly that could cause Big-Rip.

II. Metric And Solutions Of Field Equations

We consider anisotropic Bianchi type-I line element in the form

$$ds^2 = -\, dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2 \tag{4}$$

where the metric potentials A, B and C are functions of t only.

The flat metric corresponding to metric (4) is

$$d\eta^2 = -\, dt^2 + dx^2 + dy^2 + dz^2 \tag{5}$$

The energy momentum tensor T_i^j of the perfect fluid is given by

$$T_i^j = (\rho + p)\, \upsilon_i \upsilon^j + p\, g_i^j \tag{6}$$

Here ρ and p are the proper energy density and pressure respectively. The quantity θ is the scalar of expansion which is given by,

$$\theta = v^i{}_{|i} \tag{7}$$

and υ^i is the flow vector satisfying the relation

$$g_{ij}\upsilon^i \upsilon^j = -1 \tag{8}$$

We assume that coordinates to be co-moving, so that

$$\upsilon^1 = \upsilon^2 = \upsilon^3 = 0, \quad \upsilon^4 = 1. \tag{9}$$

Equation (6) of proper energy density tensor yield

$$\begin{aligned} T_1^1 &= p_x = \omega_x \rho = \omega\rho, \\ T_2^2 &= p_y = \omega_y \rho = (\omega+\gamma)\,\rho, \\ T_3^3 &= p_z = \omega_z \rho = (\omega+\delta)\,\rho, \\ T_4^4 &= -\rho \end{aligned} \tag{10}$$

Here p_x, p_y, p_z and $\omega_x, \omega_y, \omega_z$ are the directional EOS parameters along x, y and z axes respectively. The parameter ω is the deviation free EOS parameter of the fluid. We have parameterized the deviation from isotropy by setting $\omega_x = \omega$ and then introducing skewness parameter γ and δ that are the deviations from ω along y and z axes, which are $\omega_y = (\omega+\gamma)$ and $\omega_z = (\omega+\delta)$ respectively.

The Rosen's field equations (3) for the metric (4) and (5) with the help of (10) gives

$$\frac{1}{2}\left[-\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{A_4{}^2}{A^2}-\frac{B_4{}^2}{B^2}-\frac{C_4{}^2}{C^2}\right] = -\omega\,\rho \tag{11}$$

$$\frac{1}{2}\left[\frac{A_{44}}{A}-\frac{B_{44}}{B}+\frac{C_{44}}{C}-\frac{A_4{}^2}{A^2}+\frac{B_4{}^2}{B^2}-\frac{C_4{}^2}{C^2}\right] = -(\omega+\gamma)\,\rho \tag{12}$$

$$\frac{1}{2}\left[\frac{A_{44}}{A}+\frac{B_{44}}{B}-\frac{C_{44}}{C}-\frac{A_4{}^2}{A^2}-\frac{B_4{}^2}{B^2}+\frac{C_4{}^2}{C^2}\right] = -(\omega+\delta)\,\rho \tag{13}$$

$$\frac{1}{2}\left[\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{C_{44}}{C}-\frac{A_4{}^2}{A^2}-\frac{B_4{}^2}{B^2}-\frac{C_4{}^2}{C^2}\right] = \rho \tag{14}$$

where $A_4 = dA/dt$, $A_{44} = d^2A/dt^2$ etc.

The spatial volume for the model (4) is given by

$$V^3 = ABC \tag{15}$$

We define the average scale factor as $a = (ABC)^{1/3}$ so that the Hubble parameter H is anisotropic and may be defined as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right) \tag{16}$$

or

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \tag{17}$$

where $H_1 = \frac{A_4}{A}, H_2 = \frac{B_4}{B}$ and $H_3 = \frac{C_4}{C}$ are the directional Hubble's parameters in the directions of x, y and z respectively.

An important observational quantity in cosmology is the deceleration parameter q, which is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{18}$$

The scalar expansion θ, shear scalar σ^2 and the average anisotropy parameter A_m are defined by

$$\theta = \frac{A_4}{A^2} + \frac{B_4}{AB} + \frac{C_4}{AC} \tag{19}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right) \tag{20}$$

$$A_m = \frac{1}{3}\left(\frac{H_i - H}{H}\right)^2, \quad (i = 1, 2, 3). \tag{21}$$

Equations (11) to (14) are four differential equations with seven unknown quantities $A, B, C, \rho, \omega, \delta$ and γ, since the scalar expansion θ is in the form of A, B and C. In order to get the solution of the system of differential equations (11) to (14), one has to assume three extra conditions. We first assume the special law of variation for generalized Hubble's parameter that yields a constant value of deceleration parameter q, since the line element (4) is characterized by Hubble parameter H. The generalized mean Hubble parameter H is related to the average scale factor a by the relation

$$H = l\ a^{-n} \tag{22}$$

where $l\,(>0)$ and $n\,(\geq 0)$ are constants. By using this special law of variation of parameters several authors have studied flat FRW model and Bianchi type models with constant deceleration parameter. Such relation gives a constant value of deceleration parameter q.

From equations (16) and (22), we obtain

$$\dot{a} = l\,a^{-n+1} \tag{23}$$

$$\ddot{a} = -l^2(n-1)a^{-2n+1} \tag{24}$$

Substituting equations (23) and (24) in to the equation (18), we get the constant value of deceleration parameter

$$q = n - 1. \tag{25}$$

We see that the equation (25) gives the deceleration parameter q is constant under the law of variation of H. The sign of q indicates that whether the model accelerate or not. The positive sign of q corresponds to standard decelerating model whereas the negative sign of q corresponds to accelerating model. $n > 1$ yields a decelerating universe and for $0 \leq n < 1$, we have an accelerating universe. The value $n = 1$ yield neither accelerating nor decelerating expansion means expansion with constant speed.

Integrate (23), we obtain the laws of variation of parameter for the average scale factor a as

$$a = (n\,l\,t + c_1)^{1/n} \quad \text{for } n \neq 0, \tag{26}$$

$$a = c_2\, e^{l\,t} \qquad \text{for } n = 0, \tag{27}$$

where c_1 and c_2 are integrating constants. Thus the law of variation of Hubble parameter gives two types of the expansion in the universe i.e., (i) power law expansion given by (26) and (ii) exponential law expansion given by (27).

Secondly we assume that the scalar expansion (θ) in the model is proportional to the eigen values σ_1^1 of the shear tensor. This condition leads to

$$A = (BC)^m \tag{28}$$

where $m > 0$ is a constant.

Thirdly we assume condition that the deviations from ω along y and z axes are same. i.e. $\gamma = \delta$.

From equations (12) and (13), on integrating, we get

$$\frac{B}{C} = c_3 \exp[c_4\, t] \tag{29}$$

where c_3 and c_4 are constants of integration. After simplifying the field equations (11)-(14), for the power law expansion (26) and by using equation (29), we arrived at

$$A(t) = (n\,l\,t + c_1)^{\frac{3m}{n(m+1)}}, \tag{30}$$

$$B(t) = \sqrt{c_3}\;(n\,l\,t + c_1)^{\frac{3}{2n(m+1)}} \exp\left[\frac{1}{2}c_4\, t\right], \tag{31}$$

$$C(t) = \frac{1}{\sqrt{c_3}}\;(n\,l\,t + c_1)^{\frac{3}{2n(m+1)}} \exp\left[-\frac{1}{2}c_4\, t\right], \tag{32}$$

Using the equations (30)-(32), our metric (4) becomes,

$$ds^2 = -dt^2 + (n\,l\,t + c_1)^{\frac{6m}{n(m+1)}} dx^2 + c_3 (n\,l\,t + c_1)^{\frac{3}{n(m+1)}} e^{c_4 t} dy^2 + \frac{1}{c_3}(n\,l\,t + c_1)^{\frac{3}{n(m+1)}} e^{-c_4 t} dz^2 \tag{33}$$

This is Bianchi type I anisotropic dark energy cosmological model with constant deceleration parameter in bimetric theory of gravitation.
For $n = 0$, using the exponential law (27), we deduced the metric

$$ds^2 = -dt^2 + c_2\; e^{\frac{6mlt}{m+1}}\; dx^2 + c_3\; e^{\frac{3lt}{m+1}}\left(e^{c_4 t} dy^2 + e^{-c_4 t} dz^2\right) \tag{34}$$

III. Geometric And Physical Properties Of Dark Energy Model

3.1 In power law expansion

The directional Hubble's parameter are given by

$$H_1 = \frac{A_4}{A} = \frac{3ml}{m+1}(nlt + c_1)^{-1}$$

$$H_2 = \frac{B_4}{B} = \frac{3l}{2(m+1)}(nlt + c_1)^{-1} + \frac{c_4}{2} \tag{35}$$

$$H_3 = \frac{C_4}{C} = \frac{3l}{2(m+1)}(nlt + c_1)^{-1} - \frac{c_4}{2}$$

The scalar expansion θ, shear scalar σ^2 and anisotropic parameter A_m are obtained as follows

$$\theta = 3H = 3l(nlt + c_1)^{-1} \tag{36}$$

$$\sigma^2 = \frac{3l^2(2m-1)^2}{4(m+1)^2}(nlt + c_1)^{-2} + \frac{{c_4}^2}{4} \tag{37}$$

$$A_m = \frac{(2m-1)^2}{2(m+1)^2} + \frac{{c_4}^2}{6l^2(nlt + c_1)^{-2}} \tag{38}$$

The anisotropic pressure along x direction, the EOS parameter ω, and the skewness parameters γ and δ, the deviations from EOS parameter ω along y - axis and z - axis of the fluid, have been respectively calculated as

$$p = \omega\,\rho = \frac{3\,n\,l^2(1-m)}{2(1+m)}(n\,l\,t + c_1)^{-2} \tag{39}$$

$$\omega = \frac{(1-m)}{m+1} \tag{40}$$

$$\gamma = \delta = \frac{(1-2m)}{m+1} \tag{41}$$

For $m = 1/2$, $\gamma = \delta = 0$ and then the deviations from EOS parameter ω along y-axis and z-axis vanish so that $\omega_x = \omega_y = \omega_z = \omega$ and hence the model becomes isotropic for $m = 1/2$.

For flat universe (in the absence of curvature), the mass energy density Ω_M and dark energy Ω_Λ obeys the relation

$$\Omega_M + \Omega_\Lambda = 1\,, \tag{42}$$

where $\Omega_M = \rho/3H^2$ and $\Omega_\Lambda = \Lambda/3H^2$. Thus equation (42) becomes,

$$\frac{\rho}{3H^2} + \frac{\Lambda}{3H^2} = 1\,, \tag{43}$$

which gives,

$$\Lambda = \frac{3\,l^2}{2}(n+2)\;(n\,l\,t + c_1)^{-2} \tag{44}$$

(using equations (36) and (39)).

The cosmological term Λ is a decreasing function of time t .

From equations (36), (39) and (44), we yield,

$$\Omega_M = \frac{n}{2} \quad \text{and} \quad \Omega_\Lambda = \frac{(n+2)}{2} \tag{45}$$

It is seen that, the anisotropic pressure p, the energy density ρ, the expansion θ, the shear σ and the anisotropic parameter A_m are time-dependent quantities whereas Ω_M, Ω_Λ and ω are time free parameters. Though we assumed time dependent equation of state parameter ω, the variable $\omega(t)$ approached the value which is free from time t, and it is depending only on the values of $m > 0$.

We plot the graph of equation of state parameter ω verses m in power-law expansion for both for accelerating $q < 0$ and for decelerating $q > 0$, in one stroke in Fig. (1), as under:

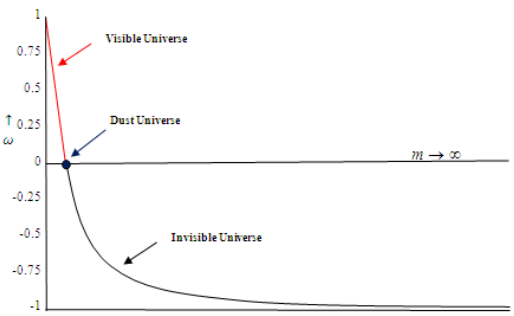

Fig. (1) Plot of equation of state parameter ω verses m in power law of expansion for both $q < 0$ and $q > 0$

From equation (39), it is realized that the energy density of the fluid $\rho(t)$ is a decreasing function of time and attend the maximum value at $t = 0$. From Fig. (2) and Fig. (3), it is observed that the energy density of the fluid ρ is maximum in the early stage of the universe and it is decreasing continiously with increasing time and it approaches to zero, when time $t \to \infty$. This is happening in the accelerating as well as in decelerating phase of the universe.

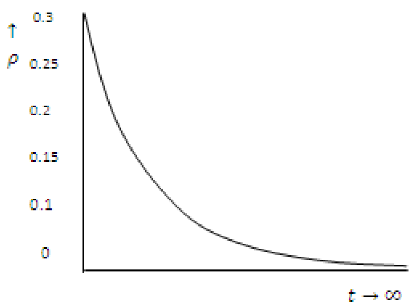

Fig (2) Plot of energy density ρ verses time t
In power law of expansion for $q > 0$.
Here $l = 0.33, n = 2, c_1 = 1$

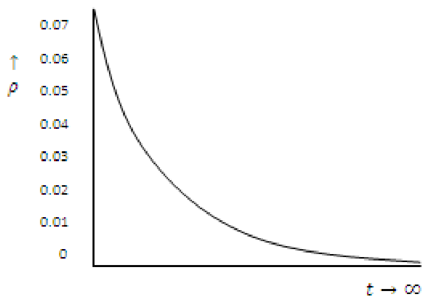

Fig (3) Plot of energy density ρ verses time t
In power law of expansion for $q < 0$.
Here $l = 0.33, n = 0.5, c_1 = 1$

Physically Λ term is interesting to know the behaviour of the model. Einstein originally added the cosmological term Λ in his field equations to balance the effect of gravity in the equation of static universe. The behaviour of the universe is to be determined by cosmological term Λ. In our model the cosmological term Λ is always positive and it is decreases when the cosmic time t increases and it has a positive value always. In the early stage of the universe Λ attain maximum value, in both phases in accelerating and in decelerating expansion and at present epoch (at late time) it approaches a small positive value.

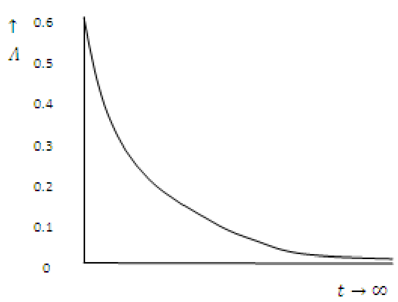

Fig (4) Plot of cosmological constant Λ verses time t
in power law expansion for $q > 0$, in which
$l = 0.33,\ n = 2,\ c_1 = 1$

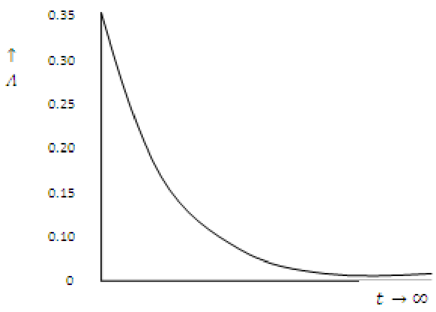

Fig (5) Plot of cosmological constant Λ verses time t
in power law expansion for $q < 0$, in which
$l = 0.33,\ n = 0.5,\ c_1 = 1$

We explore both flat and curved universe, for accelerating $q < 0$ and decelerating $q > 0$ expansion, when we report on our limits on the dark energy properties. For closed universe, the Hubble parameter is derived as $H = 0.32$ with cosmological term $\Lambda = 0.35$ at early stage of the universe at $t \approx 0.2$. At this early stage, the parameters Ω_M and Ω_Λ are found as $\Omega_M = 0.23$, $\Omega_\Lambda = 1.17$, for $n \approx 0.5$, for accelerating expansion $q < 0$. For decelerating expansion $q > 0$, we deduced the results as $H = 0.32$, $\Lambda = 0.57$, $\Omega_M = 0.93$ and $\Omega_\Lambda = 1.9$ for $n \approx 2$ at early stage $t = 0.04$. Thus at early stage of the universe, these our reporting results are very much consistent with WMAP satellite, Bennett, C. L., et al. [1] and Tegmark, M., et al. [12]. These results are listed as under in Table: I

For closed universe:

For accelerating universe, $q < 0,\ n \approx 0.5,\ t = 0.2$	For decelerating expansion, $q > 0,\ n \approx 2,\ t = 0.04$
$H = 0.32$ $\Lambda = 0.35$ $\Omega_M = 0.23$ $\Omega_\Lambda = 1.17$	$H = 0.32$ $\Lambda = 0.57$ $\Omega_M = 0.93$ $\Omega_\Lambda = 1.9$

Table : I

3.2 In exponential law expansion

In exponential law i.e., $a = c_2 e^{lt}$ for $n = 0$, the values of A, B and C are

$$A = c_2 e^{\frac{3mlt}{m+1}} \tag{46}$$

$$B = \sqrt{c_3} e^{\frac{3lt}{2(m+1)}} e^{c_5 t} \tag{47}$$

$$C = \sqrt{c_3} e^{\frac{3lt}{2(m+1)}} e^{c_5 t} \tag{48}$$

From equation (46-48), it is seen that our model is flat at early stage of the universe in exponential law.
The spatial volume V and average scale factor a are as follows

$$V = ABC = De^{3lt} \tag{49}$$

$$a = (ABC)^{1/3} = De^{lt}, \tag{50}$$

where D is the constant.
The volume V and the scale factor a of the model attain the constant value at early stage of the universe and they are increasing exponentially with increase in time and at late epoch of time they are infinite.
The directional H_1, H_2 and H_3 of the Hubble parameter H are

$$\frac{1}{m} H_1 = (2H_2 - C_5) = (2H_3 + C_5) = \frac{3l}{(m+1)} \tag{51}$$

and the Hubble parameter H is given by

$$H = l \tag{52}$$

The scalar expansion θ, shear scalar σ^2, anisotropic parameter A_m and the deceleration parameter q are given by

$$\theta = 3l, \tag{53}$$

$$\sigma^2 = \frac{9l^2(2m^2+1)}{4(m+1)^2} + c_5{}^2 - \frac{3}{2} l^2, \tag{54}$$

$$A_m = \frac{3(2m^2+1)}{2(m+1)^2} + \frac{2c_5{}^2}{3l^2} - 1, \tag{55}$$

$$q = -1. \tag{56}$$

The mean Hubble parameter H and its directional's are independent of time and they attain the constant values. Also the scalar expansion θ attains the constant value. The deceleration parameter q is negative. From this it is clear that in our exponential law, our model (34) has uniform expansion right from the beginning and the expansion has accelerating phase. The magnitude of the shear is independent of time and has constant value. This shows that the model has uniform shear. The anisotropic parameter A_m also attains the constant value which suggested that the model is isotropize uniformly.
The anisotropic pressure p, the energy density ρ, the EOS parameter ω, skew-ness parameters γ, δ along y and z axis and the cosmological term Λ are as follows

$$p = 0, \rho = 0, \omega = 0, \gamma = \delta = 0, \Lambda = 3l^2. \tag{57}$$

The pressure p, the energy density ρ, the skew ness parameter γ and δ, the EoS parameter ω all are zeros, which infers that our model (34) represents dusty universe in exponential law expansion and the model is isotropize in all spatial direction. The cosmological term Λ has value $3l^2$ and for particular choice of constant $l = 1.81/10^{61}$, the observing cosmological term Λ is $\Lambda = 10^{-123}$.

For open universe (flat cosmological model), the relation $\Omega_M + \Omega_\Lambda = 1$ yields $n = 0$ (using equation (45)) which correspondence to an accelerating expansion $(q < 0)$ of the universe. For this value $n = 0$, we deduced $\Omega_M = 0$ and $\Omega_\Lambda = 1$ yields $(\Omega_M, \Omega_\Lambda) = (0, 1)$ universe. Simultaneously, for $n = 0$, our model (34) exist and it is flat at early stage with $(\Omega_M, \Omega_\Lambda) = (0, 1)$ universe. In our flat model, we observed $\omega = 0$ and the values of Λ as $\Lambda \approx 10^{-123}$ for $l = 1.81/10^{61}$ and for $m = 1/((3.13\times10^{-61})-1)$ in accelerating expansion $q < 0$. Our results are good in agreement with the recent cosmological observations [13-22] as they suggested the existence of a positive cosmological constant Λ with the magnitude $\Lambda \approx 10^{-123}$ in the accelerating universe. These observations on magnitude and red-shift of Type Ia supernovae suggested the accelerating universe. The behavior of Λ in our dark energy model agreed with recent observations. Our results ruled out the decelerating expansion of the universe in flat cosmological model, which is also supported with the recent observations.

For Flat universe: $\Omega_M + \Omega_\Lambda = 1$

For $q < 0, n = 0, l = 1.81/10^{61}, m = (3.13\times10^{-61}) - 1$
$\Lambda = 10^{-123}$ $\Omega_M = 0$ $\Omega_\Lambda = 1$ $\omega = 0$

Table: II

IV. Our Findings

1. Our deduced models (33) and (34) are the solutions of Rosen's field equations for anisotropic Bianchi type I space-time with perfect fluid and variable equation of state parameter $\omega(t)$. The solutions have been obtained in power law (26) and in exponential law (27) for accelerating as well as for decelerating phase of the universe.
2. The time dependent equation of state of parameter $\omega(t)$ is goes over to the value which is independent of time t and it is fully based on the values of m only in both accelerating and decelerating phase.
3. The character of our model is described by ω . When $m = 1$, $\omega = 0$. For $0 < m < 1$, we have $1 < \omega < 0$ and $0 < \omega < -1$, for $1 < m < \infty$. From this, it is clear that for $0 < m < 1$, we get visible universe. For $m = 1$, we have dusty universe and for whole range of m , i.e., $1 < m < \infty$, we observe invisible universe i.e., dark energy model.
4. For $m = 1/2$, we have $\gamma = \delta = 0$ and then the deviations from EOS parameter ω along y -axis an z -axis vanish so that $\omega_x = \omega_y = \omega_z = \omega$ and hence the model becomes isotropic for $m = 1/2$.
5. We explore both flat and curved (closed) universe, for accelerating $q < 0$ and decelerating $q > 0$ expansion. Our dark energy model presented by equation of state parameter ω is accommodated closed universe with Hubble parameter $H = 0.32$ with cosmological term $\Lambda = 0.35$ at early stage of the universe which are very much consistent with WMAP satellite, Bennett, C. L., et al. [1] and Tegmark, M., et al. [12].
6. For closed universe, our results, the matter energy density parameter $\Omega_M = 0.23$ and dark energy density parameter $\Omega_\Lambda = 1.17$ for $n \approx 0.5$ for accelerating expansion $q < 0$ and for decelerating expansion $q > 0$, we have $\Omega_M = 0.93$ and $\Omega_\Lambda = 1.9$ for $n \approx 2$ at early stage of the universe determined by ω are close to the results of WMAP satellite, Bennett, C. L., et al. [1] and Tegmark, M., et al. [12].
7. For flat universe, we deduced the cosmological term Λ as Ω_M with $(\Omega_M, \Omega_\Lambda) = (0,1)$ universe at late stage in accelerating expansion of the universe which supported the recent cosmological observations [13-22] on magnitude and red shift of type Ia supernovae suggest that our universe may be an accelerating one and ruled-out decelerating expansion.
8. For very large value of m, our $\omega = -0.977$ consistent with the results of 68% confidence level of WMAP +SN Bennett, C. L., et al. [1] and Tegmark, M., et al. [12] of supernovae observation for flat cosmological model.
9. The behavior of the whole dark energy model is affected by the values of m, l and n. For $n = 0$, we get the flat universe at early time which has accelerating exponential expansion only and ruled-out decelerating

expansion for $n > 1$. The value $n = 1$, suggested neither accelerating nor decelerating expansion that means expansion of the model with constant speed. For $n \neq 0$, we yield the closed universe with accelerating as well as decelerating phase of the expansion. The values of m described the behavior of the universe which contains 32% visible matter and 68% dark matter which has shown in Fig. (1).

10. In exponential law expansion, the pressure p, energy density ρ, the equation of sate parameter ω, skewness parameter γ and δ all are vanishes which infers that our model (34) is dusty universe and it is isotropize in all spatial direction.

V. Conclusion

We investigated Bianchi type - I dark energy cosmological model with an anisotropic constant deceleration parameter with time-dependent equation of state parameter $\omega(t)$ in bimetric theory of gravitation and studied the accelerating as well as decelerating phase of expansion of the universe and measured the matter energy density parameter Ω_M, the dark energy density parameter Ω_Λ and the equation of state parameter ω in it. The parameters m, l and n affected the overall structure of the model. The time dependent equation of state parameter $\omega(t)$ is goes over to the value which is time independent and fully depends only on the values of $m > 0$. Our measured values of Ω_M, Ω_Λ, ω and Hubble parameter H in our model are found to be consistent with the results of WMAP satellite, Bennett, C. L. et al. (2003) and SNe Ia data collaborated with CMBR anisotropy and galaxy clustering statistics, Tegmark, M., et al. (2004). In power law expansion, the cosmological term Λ is a decreasing function of time and has a small positive value at present epoch which matched with the results from recent supernovae Ia observations whereas in exponential law it is constant. The physical features of the model have been explicitly traced out.

Acknowledgements

The authors express their sincere thanks to UGC, New Delhi, for financial assistance under Major Research Project.

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