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Topological Projective Modules of topological ring R

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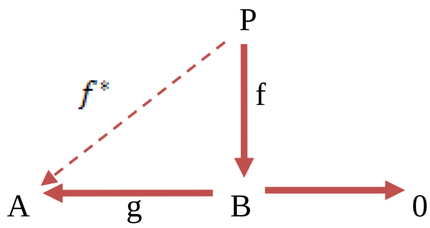
Abstract: *Our principle aim to study an algebraic module properties topolocically. We concern topological projective modules specially. A topological module P is called topological projective module if for all topological module epimorphism $g : B \longrightarrow A$ and for all topological module morphism $f : P \longrightarrow B$, there exist a topological module morphism $f^*: P \longrightarrow A$, for which the following diagram commutes:*



The important point of this paper is to find topological projective module from topological module given. Furthermore among results are obtained at the end of the paper.
Key words: *Topological module, topological ring, topological submodule, topological projective module.*

I. Introduction

Kaplanski is the first scientist who introduced the definition of topological module and topological submodule in 1955, after that a topological module studied by many scientists likes Alberto Tolono and Nelson. In this research a necessary and sufficient condition for topological module to be topological projective module has been established, the kind of topological don't determinant. The result in this paper algebraically satisfied, now we will satisfy it algebraically and topologically.

This work divided into two sections. In section one includes some necessary definitions while section two includes propositions. We concern of this research is to obtain topological projective module from topological module given we suppose $n, n_1, n_2, \ldots, n_m$ be natural number.

1- Topological Modules

In this section we give the fundamental concepts of this work.

Definition (1.1) : [3]

A non empty set E is said to be topological group if:

(1) E is a group.
(2) τ is A Topology on E.
(3) A mapping $\mu: E \times E \longrightarrow E$ and $\nu: E \longrightarrow E$ are continuous, where ν defined as $\nu(x) = x^{-1}$ and $E \times E$ is the product of two topological spaces.

Definition (1.2) : [4]

Let E be topological group. A subset M of E is said to be a topological subgroup of E if:

(1) M is a subgroup of E.
(2) M is a subspace of E.

Definition (1.3) : [1]

A non empty set R is said to be topological ring if:

(1) R is a ring.
(2) τ is A Topology on R.
(3) A mapping $\mu: R \times R \longrightarrow R$ defined as $\mu(x,y) = x \cdot y$ and $\nu: R \longrightarrow R$ defined as $\nu(x) = x^{-1}$ are continuous.

Definition (1.4): [1]

Let R be a topological ring. A non empty set E is said to be topological module on R, if:

1. E is module on R.
2. E is a topological group.
3. A mapping $\mu: R \times E \longrightarrow E$ defined as $\mu(\lambda, x) = \lambda x$ is continuous for all $\lambda \in R$, $x \in E$.

Example (1.5): [2]

Let E be a topological module on topological ring R, a subset M of E is said to be topological submodule of E if:

1. M is a submodule of E.
2. M is a topological subgroup of E.

Proposition (1.6): [2]

Let R be topological ring and $\{E_\alpha\}_{1 \leq \alpha \leq n}$ be finite discrete topological module of R then the direct sum $\bigoplus\limits_{1 \leq \alpha \leq n} E_\alpha$ be discrete topological module.

Corollary (1.7): [2]

Let R be a topological ring, $\{E_\alpha\}_{\alpha \in \Delta}$ be a family of discrete topological module then the direct sum $\bigoplus\limits_{\alpha \in \Delta} E_\alpha$ be discrete topological module.

Definition (1.8): [3]

Let E and E′ be two topological modules, the map *f* from E into E' is said to be homomorphism topological module if:

1. *f* is homomorphism module.
2. *f* is continuous map.

Definition (1.9): [3]

Let E and E′ be two topological modules on the map *f* from E into E′ is said to be homomorphism topological module, if:

1. *f* is homomorphism module.
2. *f* is topological homeomorphism.

2- Topological Free Modules

Definition (2.1): [5]

A topological module P is said to be topological projective module if for all topological module epimorphism g: $B \longrightarrow A$ and for all topological module morphism f: $P \longrightarrow B$, there exists a topological module morphism f^*: $P \longrightarrow A$, for which the following diagram commutes:

$$\begin{tikzcd} & P \arrow[dl, dashed, "f^*"'] \arrow[d, "f"] \\ A & B \arrow[l, "g"] \end{tikzcd}$$

Notation (2.2): [1]

Ker *f* is topological submodule of E, where *f* is topological module homomorphism from E (topological module) into E' (topological module).

Proposition (2.3): [3]

Let P be a topological projective on a ring R and P be discrete topological module, then P is topological projective module.

Proposition (2.4): [3]

If $\{E_\alpha\}_{\alpha\in\Delta}$ be a family of topological modules on topological ring R, B be a topological module on the same ring, the topological module homomorphism g_α: $P_\alpha \longrightarrow B$ for all $\alpha\in L$, there exists a unique topological module homomorphism g: $B \longrightarrow \bigoplus_{\alpha\in\Delta} P_\alpha$ for which the following diagram commutes:

$$\begin{tikzcd}
& P_\alpha \arrow[d, "g_\alpha"] \arrow[dl, dashed, "\mathbf{I}_\alpha"'] \\
\bigoplus_{\alpha\in\Delta} P_\alpha & B \arrow[l, "g"]
\end{tikzcd}$$

Proposition (2.5): [3]

If $\{P_\alpha\}_{\alpha\in\Delta}$ be a family of topological modules of topological ring R, then the direct sum P = $\bigoplus_{\alpha\in\Delta}\{P_\alpha\}$ be topological projective module if and only if if $\{P_\alpha\}_{\alpha\in\Delta}$ is a topological projective module.

Proposition (2.6):

If P_1, P_2 and P_3 be topological module of topological ring R such that f_1: $P_2 \longrightarrow P_1$ and f_2: $P_3 \longrightarrow P_2$ be two topological module homeomorphism, then there exist $f_1\circ f_2$: $P_3 \longrightarrow P_1$ is topological module homeomorphism.

Proof:

Since f_1: $P_2 \longrightarrow P_1$ and f_2: $P_3 \longrightarrow P_2$ be two topological module homeomorphism, then there exist $f_1\circ f_2$: $P_3 \longrightarrow P_1$

$$\begin{aligned}
(f_1\circ f_2)(p_3) &= f_1(f_2(p_3)) && \text{, for all } p_3 \in P_3 \\
&= f_1(p_2) && \text{, for all } p_2 \in P_2 \\
&= p_1 && \text{, for all } p_1 \in P_1
\end{aligned}$$

$f_1\circ f_2$ is continuous map because f_1 and f_2 are continuous maps. And $(f_1\circ f_2)^{-1}$ is continuous map since f_1^{-1} and f_2^{-1} are continuous maps.

Thus $f_1\circ f_2$ is topological module homeomorphism.

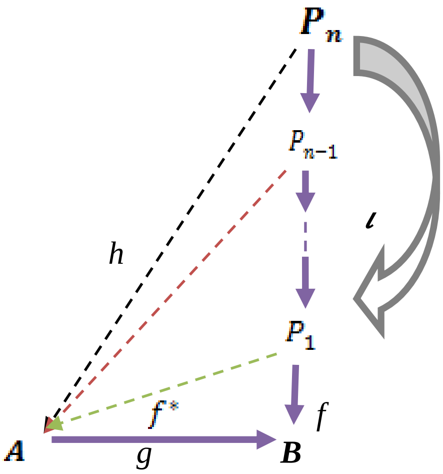
$$\begin{tikzcd}
& P_3 \arrow[dl, "f_2"'] \arrow[ddl, dashed, "f_1\circ f_2"] \\
P_2 \arrow[d, "f_1"'] & \\
P_1 &
\end{tikzcd}$$

Proposition (2.7):

If $P_1, P_2, P_3, \ldots, P_n$ be topological module of topological ring R such that f_k: $P_{k+1} \longrightarrow P_k$, $1 \le k \le n-1$ topological module homeomorphism, then there exist i: $P_n \longrightarrow P_1$ is topological module homeomorphism.

Proof:

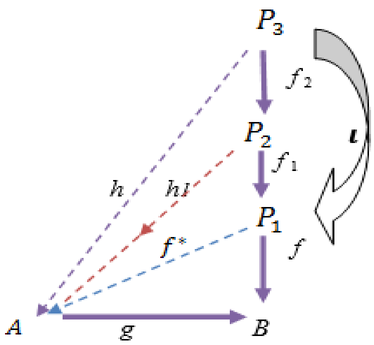
We prove it on the same way of proposition (2.6).



Proposition (2.8):

If P_1, P_2 and P_3 be topological module of topological ring R such that P_1 and P_2 are topological projective modules and $i : P_3 \longrightarrow P_1$ be topological module homeomorphism, then P_3 is topological projective module too.

Proof:



We take the following diagram to obtain
A and B are topological modules of R.
$f : P_1 \longrightarrow B$ is topological module homeomorphism.
And $g: A \longrightarrow B$ is surjective topological module homeomorphism.
Since P_1 is topological projective module, there exists $f^*: P_1 \longrightarrow A$ is topological module homeomorphism.
Now we define $h: P_3 \longrightarrow B$ by form

$h(p_3) = f^* \circ i(p_3)$, for all $p_3 \in P_3$ and

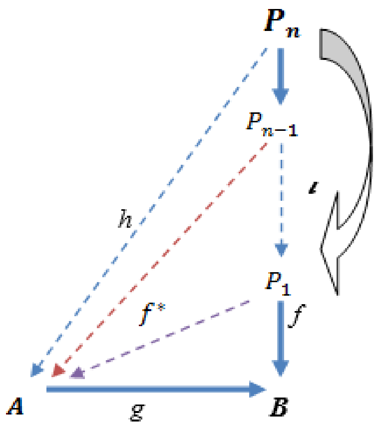
$g \circ h(p_3) = g \circ f^* \circ i(p_3)$
$= f \circ i(p_3)$
$= f(p_3)$.

Thus P_3 is topological projective module.

Proposition (2.9):

If $P_1, P_2, P_3, \ldots, P_n$ be topological module of topological ring R such that $P_1, P_2, P_3, \ldots, P_{n-1}$ be topological projective modules and i: $P_n \longrightarrow P_1$ be topological module homeomorphism, then P_n is topological projective module too.

Proof:



We take the following diagram to obtain
A and B are topological modules of R.
$f : P_1 \longrightarrow B$ is topological module homeomorphism.
And $g: A \longrightarrow B$ is surjective topological module homeomorphism.
Since P_1 is topological projective module, there exists f *: $P_1 \longrightarrow A$ is topological module homeomorphism.
Now we define $h: P_3 \longrightarrow B$ by

$h(p_3) = f^* \circ i(p_n)$, for all $p_n \in P_n$

where $f_1 \circ f_2 \circ \ldots \circ f_{n-2} \circ f_{n-1} = i$ and

$g \circ h(p_3) = g \circ f^* \circ i(p_n)$
$= f \circ i(p_n)$
$= f(p_n)$,

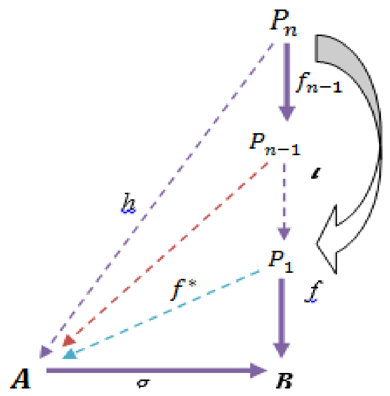
for all $p_n \in P_n$
Thus P_n is topological projective module too.

Proposition (2.10):

If P and $\underset{1\le\alpha\le m}{\oplus} P_\alpha$ be two topological modules of topological ring R such that P be topological projective module and i: $\underset{1\le\alpha\le m}{\oplus} P_\alpha \longrightarrow$ P be topological module homeomorphism, then $\underset{1\le\alpha\le m}{\oplus} P_\alpha$ is topological projective module too.

Proof:

We take the following diagram to obtain
A and B are topological modules of R.
f : P $\longrightarrow$ B is topological module homeomorphism.
And g: A $\longrightarrow$ B is surjective topological module homeomorphism.
Since P is topological projective module, there exists f *: P $\longrightarrow$ A is topological module homeomorphism.



Now we define h: $\underset{1\le\alpha\le m}{\oplus} P_\alpha \longrightarrow$ A by

$h(q) = f^* \circ i(q)$, for all $q \in \underset{1\le\alpha\le m}{\oplus} P_\alpha$,

$q = p_1 \oplus p_2 \oplus \ldots \oplus p_m$ and

$g\circ h(q) = g\circ f^* \circ i(q)$

$= f \circ i(q) = f(q)$.

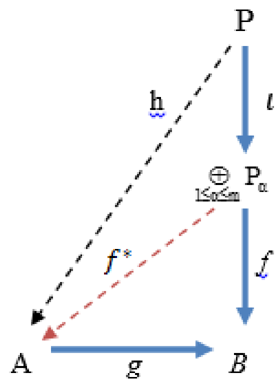
Thus $\underset{1\le\alpha\le m}{\oplus} P_\alpha$ is topological projective module too.

Proposition (2.11):

If $\underset{1\le\alpha\le m}{\oplus} P_\alpha$ and P be two topological modules of topological ring R such that $\underset{1\le\alpha\le m}{\oplus} P_\alpha$ be topological projective module and i: P $\longrightarrow \underset{1\le\alpha\le m}{\oplus} P_\alpha$ be topological module homeomorphism, then P is topological projective module too.

Proof:

We look at the following diagram to obtain
A and B are topological modules of R.
f: $\underset{1\le\alpha\le m}{\oplus} P_\alpha \longrightarrow$ B is topological module homeomorphism.
And g: A $\longrightarrow$ B is surjective topological module homeomorphism.
Since $\underset{1\le\alpha\le m}{\oplus} P_\alpha$ is topological projective module, there exists
f *: $\underset{1\le\alpha\le m}{\oplus} P_\alpha \longrightarrow$ A is topological module homeomorphism.



Now we define h: P $\longrightarrow$ A by
$h(p) = f^* \circ i(p)$, for all $p \in P$ and
$g\circ h(p) = g\circ f^* \circ i(p) = f \circ i(p) = f(p)$, for all $p \in P$.
Thus P is topological projective module too.

Proposition (2.12):

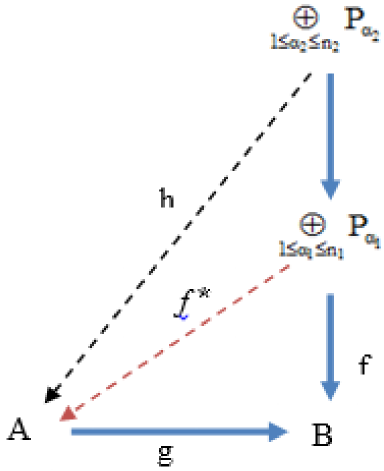
If $\underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1}$ and $\underset{1\le\alpha_2\le n_2}{\oplus} P_{\alpha_2}$ be two topological modules of topological ring R such that $\underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1}$ be topological projective module and i: $\underset{1\le\alpha_2\le n_2}{\oplus} P_{\alpha_2} \longrightarrow \underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1}$ be topological module homeomorphism, then $\underset{1\le\alpha_2\le n_2}{\oplus} P_{\alpha_2}$ is topological projective module too.

Proof:

We look at the following diagram to obtain
A and B are topological modules of R.

$f: \underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1} \longrightarrow B$ is topological module homeomorphism.

And $g: A \longrightarrow B$ is surjective topological module homeomorphism.



Since $\underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1}$ is topological projective module, there exists

$f^*: \underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1} \longrightarrow A$ is topological module homeomorphism.

Now we define $h: \underset{1\le\alpha_2\le n_2}{\oplus} P_{\alpha_2} \longrightarrow A$ by

$h(q) = f^* \circ i(q)$, for all $q \in \underset{1\le\alpha_2\le n_2}{\oplus} P_{\alpha_2}$,

where $q = p_1 \oplus p_2 \oplus \ldots \oplus p_{n_2}$ and

$g\circ h(q) = g\circ f^* \circ i(q)$

$= f \circ i(q)$

$= f(q)$.

Thus $\underset{1\le\alpha_2\le n_2}{\oplus} P_{\alpha_2}$ is topological projective module too.

Proposition (2.13):

If $\underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1}$, $\underset{1\le\alpha_2\le n_2}{\oplus} P_{\alpha_2}$ and $\underset{1\le\alpha_3\le n_3}{\oplus} P_{\alpha_3}$ be topological modules of topological ring R such that $\underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1}$ and $\underset{1\le\alpha_2\le n_2}{\oplus} P_{\alpha_2}$ are topological projective modules and $i: \underset{1\le\alpha_3\le n_3}{\oplus} P_{\alpha_3} \longrightarrow \underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1}$ be topological module homeomorphism, then $\underset{1\le\alpha_3\le n_3}{\oplus} P_{\alpha_3}$ is topological projective module too.

Proof:

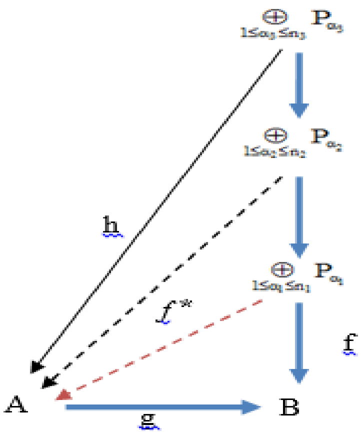
We look at the following diagram to obtain A and B are topological modules of R.

$f: \underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1} \longrightarrow B$ is topological module homeomorphism.

And $g: A \longrightarrow B$ is surjective topological module homeomorphism.

Since $\underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1}$ is topological projective module, there exists

$f^*: \underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1} \longrightarrow A$ is topological module homeomorphism.



Now we define $h: \underset{1\le\alpha_3\le n_3}{\oplus} P_{\alpha_3} \longrightarrow A$ by

$h(q) = f^* \circ i(q)$, for all $q \in \underset{1\le\alpha_3\le n_3}{\oplus} P_{\alpha_3}$,

where $q = p_1 \oplus p_2 \oplus \ldots \oplus p_{n_3}$ and

$g\circ h(q) = g\circ f^* \circ i(q)$

$= f \circ i(q)$

$= f(q)$.

Thus $\underset{1\le\alpha_3\le n_3}{\oplus} P_{\alpha_3}$ is topological projective module.

Proposition (2.14):

If $\underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1}$, $\underset{1\le\alpha_2\le n_2}{\oplus} P_{\alpha_2}$, $\underset{1\le\alpha_3\le n_3}{\oplus} P_{\alpha_3}$, …, $\underset{1\le\alpha_m\le n_m}{\oplus} P_{\alpha_m}$ be topological modules of topological ring R such that $\underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1}$, $\underset{1\le\alpha_2\le n_2}{\oplus} P_{\alpha_2}$, $\underset{1\le\alpha_3\le n_3}{\oplus} P_{\alpha_3}$, …, $\underset{1\le\alpha_{m-1}\le n_{m-1}}{\oplus} P_{\alpha_{m-1}}$ are topological projective modules and i:

$\underset{1\le\alpha_m\le n_m}{\oplus} P_{\alpha_m} \longrightarrow \underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1}$ be topological module homeomorphism, then $\underset{1\le\alpha_m\le n_m}{\oplus} P_{\alpha_m}$ is topological projective module too.

Proof:

We look at the following diagram to obtain
A and B are topological modules of R.

f: $\underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1} \longrightarrow$ B is topological module homeomorphism.

And g: A $\longrightarrow$ B is surjective topological

Since $\underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1}$ is topological projective module, there exists

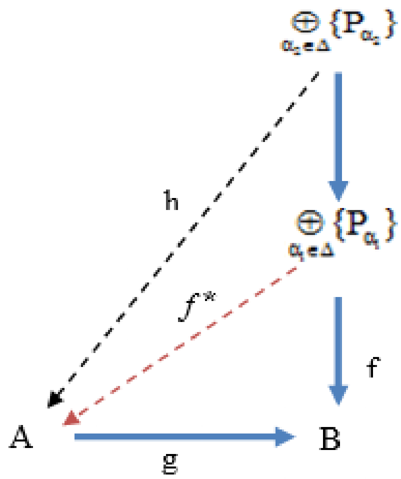
f^*: $\underset{1\le\alpha_1\le n_1}{\oplus} P_{\alpha_1} \longrightarrow$ A is topological module homeomorphism.

Now we define h: $\underset{1\le\alpha_m\le n_m}{\oplus} P_{\alpha_m} \longrightarrow$ A by

$h(q) = f^* \circ i(q)$, for all $q \in \underset{1\le\alpha_m\le n_m}{\oplus} P_{\alpha_m}$,

where $q = p_1 \oplus p_2 \oplus \ldots \oplus p_{n_m}$ and

$g\circ h(q) = g\circ f^* \circ i(q)$
$= f \circ i(q)$
$= f(q)$.



Thus $g\circ h = f$ and $\underset{1\le\alpha_m\le n_m}{\oplus} P_{\alpha_m}$ is topological projective module.

Proposition (2.15):

If $\underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\}$ and $\underset{\alpha_2\in\Delta}{\oplus}\{P_{\alpha_2}\}$, be a families of topological modules of topological ring R such that $\underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\}$ be topological projective module and i: $\underset{\alpha_2\in\Delta}{\oplus}\{P_{\alpha_2}\} \longrightarrow \underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\}$ be topological module homeomorphism, then $\underset{\alpha_2\in\Delta}{\oplus}\{P_{\alpha_2}\}$ is topological projective module too.

Proof:

We look at the following diagram to obtain
A and B are topological modules of R.

f: $\underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\} \longrightarrow$ B is topological module homeomorphism.

And g: A $\longrightarrow$ B is surjective topological
module homeomorphism.

Since $\underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\}$ is topological projective module, there exists

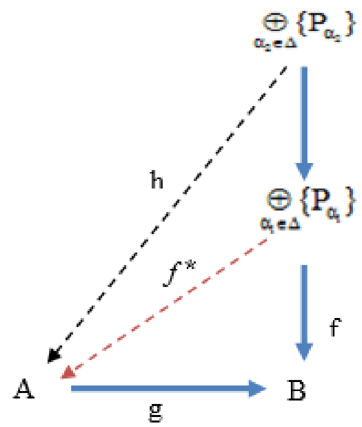
f^*: $\underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\} \longrightarrow$ A is topological module homeomorphism.

Now we define h: $\underset{\alpha_2\in\Delta}{\oplus}\{P_{\alpha_2}\} \longrightarrow$ A by

$h(q) = f^* \circ i(q)$, for all $q \in \underset{\alpha_2\in\Delta}{\oplus}\{P_{\alpha_2}\}$,

where $q = p_1 \oplus p_2 \oplus \ldots \oplus p_{n_m}$ and

$g\circ h(q) = g\circ f^* \circ i(q)$
$= f \circ i(q)$
$= f(q)$.



Thus $g\circ h = f$ and $\underset{\alpha_2\in\Delta}{\oplus}\{P_{\alpha_2}\}$ is topological projective module.

Proposition (2.16):

If $\underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\}$, $\underset{\alpha_2\in\Delta}{\oplus}\{P_{\alpha_2}\}$ and $\underset{\alpha_3\in\Delta}{\oplus}\{P_{\alpha_3}\}$, be a families of topological modules of topological ring R such that $\underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\}$ and $\underset{\alpha_2\in\Delta}{\oplus}\{P_{\alpha_2}\}$ are topological projective modules and i: $\underset{\alpha_3\in\Delta}{\oplus}\{P_{\alpha_3}\} \longrightarrow \underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\}$ be topological module homeomorphism, then $\underset{\alpha_3\in\Delta}{\oplus}\{P_{\alpha_3}\}$ is topological projective module too.

Proof:

We look at the following diagram to obtain A and B are topological modules of R.

f: $\underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\} \longrightarrow$ B is topological module homeomorphism.

And g: A $\longrightarrow$ B is surjective topological module homeomorphism.

Since $\underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\}$ is topological projective module,

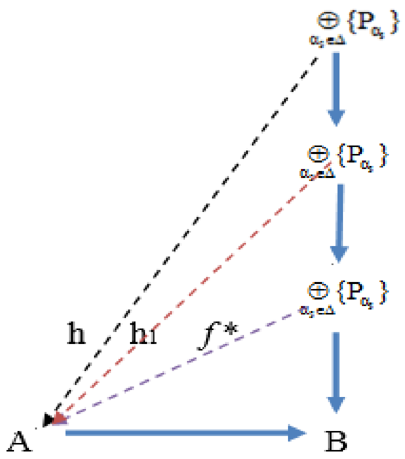
there exists f^*: $\underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\} \longrightarrow$ A

is topological module homeomorphism.

Now we define h: $\underset{\alpha_3\in\Delta}{\oplus}\{P_{\alpha_3}\} \longrightarrow$ A by

$h(q) = f^*\circ i(q)$, for all $q \in \underset{\alpha_3\in\Delta}{\oplus}\{P_{\alpha_3}\}$ and

$g\circ h(q) = g\circ f^*\circ i(q) = f\circ i(q) = f(q)$.



Thus $g\circ h = f$ and

$\underset{\alpha_3\in\Delta}{\oplus}\{P_{\alpha_3}\}$ is topological projective module.

Proposition (2.17):

If $\underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\}$, $\underset{\alpha_2\in\Delta}{\oplus}\{P_{\alpha_2}\}$, $\underset{\alpha_3\in\Delta}{\oplus}\{P_{\alpha_3}\}$, …, $\underset{\alpha_m\in\Delta}{\oplus}\{P_{\alpha_m}\}$ be a families of topological modules of topological ring R such that $\underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\}$, $\underset{\alpha_2\in\Delta}{\oplus}\{P_{\alpha_2}\}$, $\underset{\alpha_3\in\Delta}{\oplus}\{P_{\alpha_3}\}$, …, $\underset{\alpha_{m-1}\in\Delta}{\oplus}\{P_{\alpha_{m-1}}\}$ are topological projective modules and i: $\underset{\alpha_m\in\Delta}{\oplus}\{P_{\alpha_m}\} \longrightarrow \underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\}$ be topological module homeomorphism, then $\underset{\alpha_m\in\Delta}{\oplus}\{P_{\alpha_m}\}$ is topological projective module too.

Proof:

We look at the following diagram to obtain A and B are topological modules of R.

f: $\underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\} \longrightarrow$ B is topological module homeomorphism.

And g: A $\longrightarrow$ B is surjective topological module homeomorphism.

Since $\underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\}$ is topological projective module, there exists

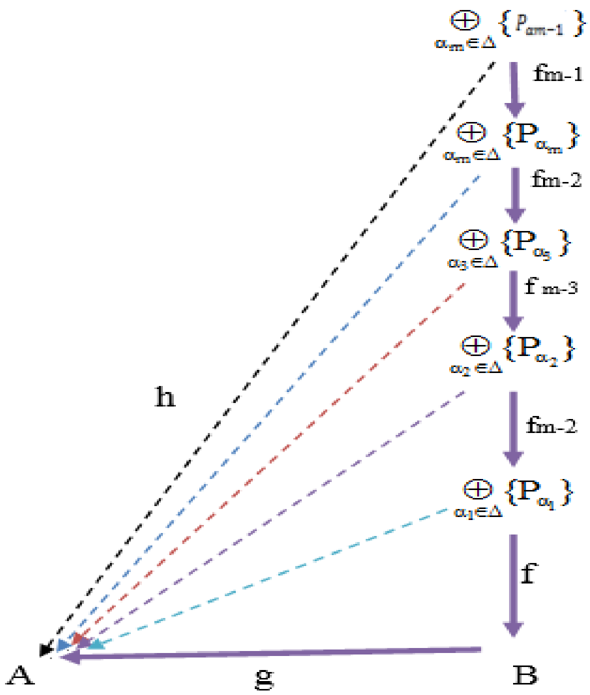
f^*: $\underset{\alpha_1\in\Delta}{\oplus}\{P_{\alpha_1}\} \longrightarrow$ A is topological module homeomorphism.

Now we define h: $\underset{\alpha_m\in\Delta}{\oplus}\{P_{\alpha_m}\} \longrightarrow$ A by

$h(q) = f^* \circ i(q)$, for all $q \in \underset{\alpha_m \in \Delta}{\oplus} \{P_{\alpha_m}\}$ and

$g \circ h(q) = g \circ f^* \circ i(q) = f \circ i(q) = f(q)$.

Thus $g \circ h = f$ and $\underset{\alpha_m \in \Delta}{\oplus} \{P_{\alpha_m}\}$ is topological projective module.



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