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Multidimensional integral transform involving I-function of several complex variables as kernel

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Abstract: *The theory of integral transforms is very useful in solving various types of boundary value problems. By giving various values to the kernel k(x; s) and considering the interval (0, 1) generally, a number of integral transforms have been introduced and studied by several authors from time to time. In the present paper we have introduced multidimensional I-function transform involving I-function of r variables as kernel. In our earlier paper we have established the multiple Mellin transform of product of two I-functions. With the help of that result, in the present paper we have discussed some theorems on multidimensional I-function transform. Special cases include the results involving H-function of several complex variables, I-function of two variables and H-function of two variables. The special case also includes the results given by V.C.Nair.*

Keywords: *Mellin-Barnes integral, H-function, H-function transform, I-function, Multidimensional I-function transform*

I. Introduction

In 1997, Rathie introduced the generalization of the well-known Fox's H-function [1] which has very recently found interesting applications in wireless communication ([2],[3],[4]). Motivated by the I-function, Shantha Kumari, Nambisan and Rathie introduced I-function of two variables [5] which is a natural generalization of the H-function of two variables introduced earlier by Mittal and Gupta [6] and discussed some of its important properties.Very recently we have introduced and studied an extension of the I-function of several complex variables. This function is defined and represented in the following manner [7].

$$I\left[z_1,...,z_r\right]=I^{0,n:m_1,n_1;...;m_r,n_r}_{p,q:p_1,q_1;...;p_r,q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\left|\begin{matrix} {}_1\left(a_j;\alpha_j^{(1)},...,\alpha_j^{(r)};A_j\right)_p : {}_1\left(c_j^{(1)},\gamma_j^{(1)},C_j^{(1)}\right)_{p_1};...;{}_1\left(c_j^{(r)},\gamma_j^{(r)},C_j^{(r)}\right)_{p_r} \\ \\ {}_1\left(b_j;\beta_j^{(1)},...,\beta_j^{(r)};B_j\right)_q : {}_1\left(d_j^{(1)},\delta_j^{(1)},D_j^{(1)}\right)_{q_1};...;{}_1\left(d_j^{(r)},\delta_j^{(r)},D_j^{(r)}\right)_{q_r}\end{matrix}\right.\right]$$

$$=\frac{1}{(2\pi i)^r}\int_{L_1}...\int_{L_r}\varphi\left(s_1,...,s_r\right)\theta_1\left(s_1\right)...\theta_r\left(s_r\right)z_1^{s_1}...z_r^{s_r}ds_1...ds_r \tag{1}$$

where $\varphi\left(s_1,...,s_r\right)$ and $\theta\left(s_i\right),\ i=1,...,r$ are given by

$$\varphi\left(s_1,...,s_r\right)=\frac{\prod_{j=1}^{n}\Gamma^{A_j}\left(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}s_i\right)}{\prod_{j=n+1}^{p}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_i\right)\prod_{j=1}^{q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_i\right)} \tag{2}$$

$$\theta_i\left(s_i\right)=\frac{\prod_{j=1}^{m_i}\Gamma^{D_j^{(i)}}\left(d_j^{(i)}-\sum_{i=1}^{r}\delta_j^{(i)}s_i\right)\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\sum_{i=1}^{r}\gamma_j^{(i)}s_i\right)}{\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\sum_{i=1}^{r}\gamma_j^{(i)}s_i\right)\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\sum_{i=1}^{r}\delta_j^{(i)}s_i\right)} \tag{3}$$

Also

- $z_i \neq 0 \text{ for } i=1,...,r$

- an empty product is interpreted as unity.
- the parameters $m_j, n_j, p_j, q_j\left(j=1,...,r\right)$,n, p, q are nonnegative integers such that $0 \le n \le p, q \ge 0, 0 \le n_j \le p_j,\ 0 \le m_j \le q_j\left(j=1,...,r\right)$ (not all zero simultaneously).
- $\alpha_j^{(i)}\left(j=1,...,p;i=1,...,r\right), \beta_j^{(i)}\left(j=1,...,q;i=1,...,r\right), \gamma_j^{(i)}\left(j=1,...,p_i;i=1,...,r\right), \delta_j^{(i)}$ $\left(j=1,...,q_i;i=1,...,r\right)$ are assumed to be positive quantities for standardization purpose.
- $a_j\left(j=1,...,p\right), b_j\left(j=1,...,q\right), c_j^{(i)}\left(j=1,...,p_i;i=1,...,r\right), d_j^{(i)}\left(j=1,...,q_i;i=1,...,r\right)$ are complex numbers.
- The exponents $A_j\left(j=1,...,p\right), B_j\left(j=1,...,q\right), C_j^{(i)}\left(j=1,...,p_i;i=1,...,r\right),\ D_j^{(i)}\left(j=1,...,q_i;i=1,...,r\right)$ of various gamma functions involved in (2) and (3) may take non-integer values.
- The contour L_i in the complex s_i-plane is of Mellin –Barnes type which runs from c-i∞ to c+i∞ with indentations, if necessary, in such a manner that all singularities of $\Gamma^{D_j^{(i)}}\left(d_j^{(i)}-\delta_j^{(i)}\right)$, $j=1,...m_i$ are to the right and $\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}\right)$, $j=1,...,n_i$ are to the left of L_i, i=1,…,r.

Following the results of Braaksma[8] the I-function of `r' variables is analytic if

$$\mu_i = \sum_{j=1}^{p} A_j\alpha_j^{(i)} - \sum_{j=1}^{q} B_j\beta_j^{(i)} + \sum_{j=1}^{p_i} C_j^{(i)}\gamma_j^{(i)} - \sum_{j=1}^{q_i} D_j^{(i)}\delta_j^{(i)} \le 0, \quad i=1,...,r \tag{4}$$

1.1 Convergence Conditions

The integral (1) converges absolutely if

$$\left|\arg\left(z_k\right)\right| < \frac{1}{2}\Delta_k\pi, \quad k=1,...,r \tag{5}$$

where
$$\Delta_k = \left[-\sum_{j=1}^{p} A_j\alpha_j^{(k)} - \sum_{j=1}^{q} B_j\beta_j^{(k)} + \sum_{j=1}^{m_k} D_j^{(k)}\delta_j^{(k)} - \sum_{j=m_k+1}^{q_k} D_j^{(k)}\delta_j^{(k)} + \sum_{j=1}^{n_k} C_j^{(k)}\gamma_j^{(k)} - \sum_{j=n_k+1}^{p_k} C_j^{(k)}\gamma_j^{(k)}\right] > 0 \tag{6}$$

And if $\left|\arg\left(z_k\right)\right| < \frac{1}{2}\Delta_k\pi,$ and $\Delta_k > 0,\ k=1,...,r$ then integral (1) converges absolutely under the following conditions.

(i) $\mu_k = 0,\ \Omega_k < -1$ where μ_k is given by (4) and

$$\Omega_k = \sum_{j=1}^{p}\left[\frac{1}{2} - R\left(a_j\right)\right]A_j - \sum_{j=1}^{q}\left[\frac{1}{2} - R\left(b_j\right)\right]B_j + \sum_{j=1}^{p_k}\left[\frac{1}{2} - R\left(c_j^{(k)}\right)\right]C_j^{(k)} - \sum_{j=1}^{q_k}\left[\frac{1}{2} - R\left(d_j^{(k)}\right)\right]D_j^{(k)},\ k=1,...,r \tag{7}$$

(ii) $\mu_k \ne 0$ with $s_k = \sigma_k + it_k\left(\sigma_k \text{ and } t_k \text{ are real, } k=1,...,r\right)$, σ_k are so chosen that for $\left|t_k\right| \to \infty$ we have $\Omega_k + \sigma_k\mu_k < -1$

We have discussed the proof of convergent conditions in our earlier paper [7].

Remark 1

If $D_j^{(i)} = 1\left(j = 1,...,m_i, i = 1,...,r\right)$ in (1), then the function will be denoted by

$$\bar{I}\left[z_1,\dots,z_r\right]=I^{0,n:m_1,n_1;\dots;m_r,n_r}_{p,q:p_1,q_1;\dots;p_r,q_r}\left[\begin{array}{c|c} z_1 & {}_1\left(a_j;\alpha_j^{(1)},\dots,\alpha_j^{(r)};A_j\right)_p : \\ \vdots & \\ z_r & {}_1\left(b_j;\beta_j^{(1)},\dots,\beta_j^{(r)};B_j\right)_q : {}_1\left(d_j^{(1)},\delta_j^{(1)};1\right)_{m_1}, {}_{m_1+1}\left(d_j^{(1)},\delta_j^{(1)};D_j^{(1)}\right)_{q_1};\dots; \end{array}\right.$$

$$\left.\begin{array}{c} {}_1\left(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)}\right)_{p_1};\dots;{}_1\left(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)}\right)_{p_r} \\ {}_1\left(d_j^{(r)},\delta_j^{(r)};1\right)_{m_r}, {}_{m_r+1}\left(d_j^{(r)},\delta_j^{(r)};D_j^{(r)}\right)_{q_r} \end{array}\right]$$

$$=\frac{1}{(2\pi i)^r}\int_{L_1}\dots\int_{L_r}\bar{\theta}_1(s_1)\dots\bar{\theta}_r(s_r)\,\varphi(s_1,\dots,s_r)\,z_1^{s_1}\dots z_r^{s_r}\,ds_1\dots ds_r \tag{8}$$

where

$$\bar{\theta}_i(s_i)=\frac{\prod_{j=1}^{m_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)}{\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)},\quad i=1,\dots,r \tag{9}$$

Remark 2

If $C_j^{(i)}=1\left(j=1,\dots,n_i,i=1,\dots,r\right)$, $D_j^{(i)}=1\left(j=1,\dots,m_i,i=1,\dots,r\right)$ and n=0 in (1), then the function will be denoted by

$$\bar{I}_1\left[z_1,\dots,z_r\right]$$

$$=I^{0,0:m_1,n_1;\dots;m_r,n_r}_{p,q:p_1,q_1;\dots;p_r,q_r}\left[\begin{array}{c|c} z_1 & {}_1\left(a_j;\alpha_j^{(1)},\dots,\alpha_j^{(r)};A_j\right)_p : {}_1\left(c_j^{(1)},\gamma_j^{(1)};1\right)_{n_1}, {}_{n_1+1}\left(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)}\right)_{p_1};\dots; \\ \vdots & \\ z_r & {}_1\left(b_j;\beta_j^{(1)},\dots,\beta_j^{(r)};B_j\right)_q : {}_1\left(d_j^{(1)},\delta_j^{(1)};1\right)_{m_1}, {}_{m_1+1}\left(d_j^{(1)},\delta_j^{(1)};D_j^{(1)}\right)_{q_1};\dots; \end{array}\right.$$

$$\left.\begin{array}{c} {}_1\left(c_j^{(r)},\gamma_j^{(r)};1\right)_{n_r}, {}_{n_r+1}\left(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)}\right)_{p_r} \\ {}_1\left(d_j^{(r)},\delta_j^{(r)};1\right)_{m_r}, {}_{m_r+1}\left(d_j^{(r)},\delta_j^{(r)};D_j^{(r)}\right)_{q_r} \end{array}\right] \tag{10}$$

where

$$\varphi_1(s_1,\dots,s_r)=\frac{1}{\prod_{j=1}^{q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_i\right)\prod_{j=n+1}^{p}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_i\right)} \tag{11}$$

$$\bar{\theta}_i(s_i)=\frac{\prod_{j=1}^{m_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)\prod_{j=1}^{n_i}\Gamma\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)}{\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)} \tag{12}$$

i=1,2,…,r

Definition

Mellin transform of a function f(x) is defined as

$$M\left[f(x);s\right]=\int_0^\infty x^{s-1}f(x)\,dx \tag{13}$$

II. Result Used

The following important theorem which is an analogue of the well-known Parseval-Goldstein theorem will be required in the sequel [9]:

Theorem

If $T\{f_1(x);s\}=\varphi_1(s)$ and $T\{f_2(x);s\}=\varphi_2(s)$

then $\int_0^\infty f_1(x)\varphi_2(x)dx=\int_0^\infty f_2(x)\varphi_1(x)dx$ (14) provided that the various integrals involved converge absolutely.

Gupta and Mittal[10] have proved the following theorem.

Uniqueness theorem

If $f_1(x)$ and $f_2(x)$ are continuous for $x\geq 0$ and

$$\int_0^\infty H_{p,q}^{m,n}\left[sx\left|\begin{matrix}(a_j;\alpha_j)_{1,p}\\(b_j;\beta_j)_{1,q}\end{matrix}\right.\right]f_1(x)dx=\int_0^\infty H_{p,q}^{m,n}\left[sx\left|\begin{matrix}(a_j;\alpha_j)_{1,p}\\(b_j;\beta_j)_{1,q}\end{matrix}\right.\right]f_2(x)dx \tag{15}$$

both integrals being convergent, then $f_1(x)\equiv f_2(x)$ (16)

Without loss of generality the above theorem can be extended to multiple integrals with appropriate convergent conditions.

We have proved the following result [11].

$$\int_0^\infty\cdots\int_0^\infty x_1^{\rho_1-1}\cdots x_r^{\rho_r-1}\bar{I}_1\left[s_1x_1^{\lambda_1},\ldots,s_rx_r^{\lambda_r}\right]\times\bar{I}_1'\left[t_1x_1^{\mu_1},\ldots,t_rx_r^{\mu_r}\right]dx_1\ldots dx_r$$

$$=\frac{1}{\mu_1\ldots\mu_r}t_1^{-\frac{\rho_1}{\mu_1}}\ldots t_r^{-\frac{\rho_r}{\mu_r}}$$

$$I_{p+q',p'+q:p_1+q_1',p_1'+q_1;\ldots;p_r+q_r',p_r'+q_r}^{0,0:m_1+n_1',m_1'+n_1;\ldots;m_r+n_r',m_r'+n_r}\left[s_1t_1^{-\frac{\lambda_1}{\mu_1}},\ldots,s_rt_r^{-\frac{\lambda_r}{\mu_r}}\left|\begin{matrix}C:C_1;\ldots;C_r\\D:D_1;\ldots;D_r\end{matrix}\right.\right]$$

(17)

where

$$C={}_1\left(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)};A_j\right)_p,{}_1\left(1-b_j'-\sum_{i=1}^r\frac{\rho_i}{\mu_i}\beta_j'^{(i)},\frac{\lambda_1}{\mu_1}\beta_j'^{(1)},\ldots,\frac{\lambda_r}{\mu_r}\beta_j'^{(r)};B_j\right)_{q'}$$

$$C_i={}_1\left(c_j^{(i)},\gamma_j^{(i)};1\right)_{n_i},{}_1\left(1-d_j'^{(i)}-\delta_j'^{(i)}\frac{\rho_i}{\mu_i},\delta_j'^{(i)}\frac{\lambda_i}{\mu_i};1\right)_{m_i'},$$

$$_{m_i'+1}\left(1-d_j'^{(i)}-\delta_j'^{(i)}\frac{\rho_i}{\mu_i},\delta_j'^{(i)}\frac{\lambda_i}{\mu_i};D_j'^{(i)}\right)_{q_i},{}_{n_i+1}\left(c_j^{(i)},\gamma_j^{(i)};C_j^{(i)}\right)_{p_i},\quad i=1,\ldots,r$$

$$D={}_1\left(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)};B_j\right)_q,{}_1\left(1-a_j'-\sum_{i=1}^r\frac{\rho_i}{\mu_i}\alpha_j'^{(i)},\frac{\lambda_1}{\mu_1}\alpha_j'^{(1)},\ldots,\frac{\lambda_r}{\mu_r}\alpha_j'^{(r)};A_j'\right)_{p'}$$

$$D_i = {}_1\left(d_j^{(i)},\delta_j^{(i)};1\right)_{m_i},\ {}_1\left(1\text{-}c_j'^{(i)}\text{-}\gamma_j'^{(i)}\frac{\rho_i}{\mu_i},\gamma_j'^{(i)}\frac{\lambda_i}{\rho_i};1\right)_{n_i'},$$

$$_{n_i'+1}\left(1\text{-}c_j'^{(i)}\text{-}\gamma_j'^{(i)}\frac{\rho_i}{\mu_i},\gamma_j'^{(i)}\frac{\lambda_i}{\rho_i};C_j'^{(i)}\right)_{p_i'}\ {}_{m_i+1}\left(d_j^{(i)},\delta_j^{(i)};D_j^{(i)}\right)_{q_i},\quad i=1,\dots,r$$

provided that

(i) $\lambda_i,\mu_i > 0$, i=1,2,...,r

(ii) $$-\lambda_i \min_{1\le j\le m_i}\Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)-\mu_i \min_{1\le j\le m_i'}\Re\left(\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\right)<\mathrm{Re}\left(\rho_i\right)$$

$$<\lambda_i \min_{1\le j\le n_i}\Re\left(\frac{1\text{-}c_j^{(i)}}{\gamma_j^{(i)}}\right)+\mu_i \min_{1\le j\le n_i'}\Re\left(\frac{1\text{-}c_j'^{(i)}}{\gamma_j'^{(i)}}\right),\quad i = 1,\dots, r.$$

and

(iii) the $\overline{I}_1$ - functions involved in (17) exist.

Definition

In an attempt to generalize the H-function transform we introduced an integral transform whose kernel is the I-function of several complex variables defined by (1). This integral transform is defined and represented in the following manner:

$$\varphi\left(p_1,\dots,p_r\right)=\int_0^\infty\dots\int_0^\infty \overline{I}_1\left[c_1\left(p_1t_1\right)^{\lambda_1},\dots,c_r\left(p_rt_r\right)^{\lambda_r}\right]f\left(t_1,\dots,t_r\right)dt_1\dots dt_r \tag{18}$$

provided that the integral on the right hand side of (18) is absolutely convergent.

III. Some Theorems On Multidimensional I-Function Transform

Theorem 1

If $$\varphi\left(p_1,\dots,p_r\right)=\int_0^\infty\dots\int_0^\infty \overline{I}_1\left[c_1\left(p_1t_1\right)^{\lambda_1},\dots,c_r\left(p_rt_r\right)^{\lambda_r}\right]f\left(t_1,\dots,t_r\right)dt_1\dots dt_r \tag{19}$$

and $$\psi\left(p_1,\dots,p_r\right)=\int_0^\infty\dots\int_0^\infty \overline{I}_1\left[C_1\left(p_1t_1\right)^{\sigma_1},\dots,C_r\left(p_rt_r\right)^{\sigma_r}\right]f\left(t_1^{1/v_1},\dots,t_r^{1/v_r}\right)h\left(t_1,\dots,t_r\right)dt_1\dots dt_r \tag{20}$$

then

$$\psi\left(p_1,\dots,p_r\right)=\int_0^\infty\dots\int_0^\infty g\left(x_1,\dots,x_r,p_1,\dots,p_r\right)\varphi\left(x_1,\dots,x_r\right)dx_1\dots dx_r \tag{21}$$

where $$\left|v_1\dots v_r\right|p_1^{v_1-1}\dots p_r^{v_r-1}h\left(p_1^{v_1},\dots,p_r^{v_r}\right)\overline{I}_1\left[C_1\left(p_1^{v_1}k_1\right)^{\sigma_1},\dots,C_r\left(p_r^{v_r}k_r\right)^{\sigma_r}\right]$$

$$=\int_0^\infty\dots\int_0^\infty I\left[c_1\left(p_1t_1\right)^{\lambda_1},\dots,c_r\left(p_rt_r\right)^{\lambda_r}\right]g\left(t_1,\dots,t_r,k_1,\dots,k_r\right)dt_1\dots dt_r \tag{22}$$

provided the integrals involved are absolutely convergent.

Proof

Using (14) for the transform pairs (19) and (22),

$$\int_0^\infty\cdots\int_0^\infty g\left(t_1,\dots t_r,k_1,\dots,k_r\right)\varphi\left(t_1,\dots t_r\right)dt_1\dots dt_r = |v_1\dots v_r|\int_0^\infty\cdots\int_0^\infty t_1^{v_1-1}\dots t_r^{v_r-1}h\left(t_1^{v_1}\dots t_r^{v_r}\right)$$

$$\bar{I}_1'\left[C_1\left(t_1^{v_1}k_1\right)^{\sigma_1},\dots,C_r\left(t_r^{v_r}k_r\right)^{\sigma_r}\right]f\left(t_1,\dots,t_r\right)dt_1\dots dt_r$$

$$\int_0^\infty\cdots\int_0^\infty g\left(x_1,\dots x_r,p_1,\dots,p_r\right)\varphi\left(x_1,\dots x_r\right)dx_1\dots dx_r = |v_1\dots v_r|\int_0^\infty\cdots\int_0^\infty t_1^{v_1-1}\dots t_r^{v_r-1}h\left(t_1^{v_1}\dots t_r^{v_r}\right)$$

$$\bar{I}_1'\left[C_1\left(t_1^{v_1}k_1\right)^{\sigma_1},\dots,C_r\left(t_r^{v_r}k_r\right)^{\sigma_r}\right]f\left(t_1,\dots,t_r\right)dt_1\dots dt_r$$

Hence,

Putting $t_i^{v_i}=x_i,\ i=1,\dots,r$ on the right hand side and using (20) the result follows.

Theorem 2

$$\text{If}\quad \varphi\left(p_1^{1/v_1},\dots,p_r^{1/v_r}\right)=\int_0^\infty\cdots\int_0^\infty \bar{I}_1\left[c_1\left(p_1t_1\right)^{\lambda_1},\dots,c_r\left(p_rt_r\right)^{\lambda_r}\right]f\left(t_1,\dots,t_r\right)dt_1\dots dt_r \tag{23}$$

$$\text{and}\quad \psi\left(p_1,\dots p_r\right)=\int_0^\infty\cdots\int_0^\infty x_1^{\rho_1-1}\dots x_r^{\rho_r-1}\bar{I}_1\left(C_1\left(p_1x_1\right)^{\sigma_1},\dots,C_r\left(p_rx_r\right)^{\sigma_r}\right)\varphi\left(x_1,\dots,x_r\right)dx_1\dots dx_r \tag{24}$$

then

$$\psi\left(p_1,\dots,p_r\right)=\frac{1}{\left(v_1\dots v_r\right)\left(\lambda_1\dots\lambda_r\right)}c_1^{-\rho_1/v_1\lambda_1}\dots c_r^{-\rho_r/v_r\lambda_r}\int_0^\infty\cdots\int_0^\infty t_1^{-\rho_1/v_1}\dots t_r^{-\rho_r/v_r}$$

$$I^{0,0:m_1'+n_1,n_1'+m_1;\dots;m_r'+n_r,n_r'+m_r}_{p'+q,\,q'+p:\,p_1'+q_1,q_1'+p_1,\dots;p_1'+q_1,q_1'+p_1}\left[\begin{matrix}C_1p_1^{\sigma_1}\left(c_1t_1^{\lambda_1}\right)^{-\sigma_1/v_1\lambda_1}\\ \vdots \\ C_rp_r^{\sigma_r}\left(c_rt_r^{\lambda_r}\right)^{-\sigma_r/v_r\lambda_r}\end{matrix}\left|\begin{matrix}E:E_1;\dots;E_r\\ \\ F:F_1;\dots;F_r\end{matrix}\right.\right]f\left(t_1,\dots,t_r\right)dt_1\dots dt_r \tag{25}$$

where

$$E={}_1\left(a_j';\alpha_j'^{(1)},\dots,\alpha_j'^{(r)};A_j'\right)_{p'},\ {}_1\left(1-b_j-\sum_{i=1}^r\beta_j^{(i)}\frac{\rho_i}{v_i\lambda_i},\beta_j^{(1)}\frac{\sigma_1}{v_1\lambda_1},\dots,\beta_j^{(r)}\frac{\sigma_r}{v_r\lambda_r};B_j\right)_q,$$

$$E_i={}_1\left(c_j'^{(i)},\gamma_j'^{(i)};1\right)_{n_i'},\ {}_1\left(1-d_j^{(i)}-\delta_j^{(i)}\frac{\rho_i}{v_i\lambda_i},\delta_j^{(i)}\frac{\sigma_i}{v_i\lambda_i};1\right)_{m_i},$$

$${}_{m_i+1}\left(1-d_j^{(i)}-\delta_j^{(i)}\frac{\rho_i}{v_i\lambda_i},\delta_j^{(i)}\frac{\sigma_i}{v_i\lambda_i};D_j^{(i)}\right)_{q_i},\ {}_{n_i'+1}\left(c_j'^{(i)},\gamma_j'^{(i)};C_j'^{(i)}\right)_{p_i'}\qquad i=1,\dots,r$$

$$F={}_1\left(b_j';\beta_j'^{(1)},\dots,\beta_j'^{(r)};B_j'\right)_{q'},\ {}_1\left(1-a_j-\sum_{i=1}^r\alpha_j^{(i)}\frac{\rho_i}{v_i\lambda_i},\alpha_j^{(1)}\frac{\sigma_1}{v_1\lambda_1},\dots,\alpha_j^{(r)}\frac{\rho_r}{v_r\lambda_r};A_j\right)_p$$

$$F_i = {}_1\left(d_j'^{(i)},\delta_j'^{(i)};1\right)_{m_i'},\ {}_1\left(1\text{-}c_j^{(i)}\text{-}\gamma_j^{(i)}\frac{\rho_i}{v_i\lambda_i},\gamma_j^{(i)}\frac{\sigma_i}{v_i\lambda_i};1\right)_{n_i},$$

$$_{n_i+1}\left(1\text{-}c_j^{(i)}\text{-}\gamma_j^{(i)}\frac{\rho_i}{v_i\lambda_i},\gamma_j^{(i)}\frac{\sigma_i}{v_i\lambda_i};C_j^{(i)}\right)_{p_i},\ {}_{m_i'+1}\left(d_j'^{(i)},\delta_j'^{(i)};D_j'^{(i)}\right)_{q_i} \qquad i=1,...,r$$

provided

$$\sigma_i>0,\ \ \lambda_i>0,\ \ \Delta>0,\ \ \Delta'>0$$

where $\Delta=\sum_{j=1}^{q}\left|\prod_{i=1}^{r}\beta_j^{(i)}\right|-\sum_{j=1}^{p}\left|\prod_{i=1}^{r}\alpha_j^{(i)}\right|$; $\Delta'=\sum_{j=1}^{q'}\left|\prod_{i=1}^{r}\beta_j'^{(i)}\right|-\sum_{j=1}^{p'}\left|\prod_{i=1}^{r}\alpha_j'^{(i)}\right|$

Proof

On the R.H.S of (24) use (23) to get

$$\psi(p_1,...,p_r)=\int_0^\infty...\int_0^\infty\left\{\int_0^\infty...\int_0^\infty x_1^{\rho_1-1}...x_r^{\rho_r-1}\bar{I}_1\left[C_1(p_1x_1)^{\sigma_1},...,C_r(p_rx_r)^{\sigma_r}\right]\right.$$
$$\left.\times\bar{I}_1\left[c_1\left(x_1^{v_1}t_1\right)^{\lambda_1},...,c_r\left(x_r^{v_r}t_r\right)^{\lambda_r}\right]dx_1...dx_r\right\}f(t_1,...,t_r)\,dt_1...dt_r \tag{26}$$

In (26) use (17) to get the R.H.S of (25).

Theorem 3

If $\varphi(p_1,...,p_r)=\int_0^\infty...\int_0^\infty\bar{I}_1\left[c_1(p_1t_1)^{\lambda_1},...,c_r(p_rt_r)^{\lambda_r}\right]f(t_1,...,t_r)\,dt_1...dt_r$ (27)

and

$$\psi(p_1,...,p_r)\varphi\left(p_1^{v_1},...,p_r^{v_r}\right)=\int_0^\infty...\int_0^\infty\bar{I}_1\left[C_1(p_1t_1)^{\mu_1},...,C_r(p_rt_r)^{\mu_r}\right]g(t_1,...,t_r)\,dt_1...dt_r \tag{28}$$

then

$$g(t_1,...,t_r)=\int_0^\infty...\int_0^\infty k(x_1,...,x_r;t_1,...,t_r)\,f(x_1,...,x_r)\,dx_1...dx_r \tag{29}$$

where

$$\psi(p_1,...,p_r)\,\bar{I}_1\left[c_1\left(p_1^{v_1}x_1\right)^{\lambda_1},...,c_r\left(p_r^{v_r}x_r\right)^{\lambda_r}\right]=\int_0^\infty...\int_0^\infty\bar{I}_1\left[C_1(p_1t_1)^{\mu_1},...,C_r(p_rt_r)^{\mu_r}\right]$$
$$k(x_1,...,x_r;t_1,...,t_r)\,dt_1...dt_r \tag{30}$$

provided the integrals involved are absolutely convergent.

Proof

Multiplying both sides of (30) by $f(x_1,...,x_r)$ and integrating with respect to $x_1...x_r$ from 0 to ∞,

$$\psi(p_1,...,p_r)\int_0^\infty...\int_0^\infty\bar{I}_1\left[c_1\left(p_1^{v_1}x_1\right)^{\lambda_1},...,c_r\left(p_r^{v_r}x_r\right)^{\lambda_r}\right]f(x_1,...,x_r)\,dx_1...dx_r$$

$$=\int_0^\infty...\int_0^\infty f(x_1,...,x_r)\left\{\int_0^\infty...\int_0^\infty k(x_1,...,x_r;t_1,...,t_r)\,\bar{I}_1'\left[C_1(p_1t_1)^{\mu_1},...,C_r(p_rt_r)^{\mu_r}\right]dt_1...dt_r\right\}dx_1...dx_r$$

(31)

Using (27), L.H.S of (31) is $\psi\left(p_1,...,p_r\right)\varphi\left(p_1^{v_1},...,p_r^{v_r}\right)$. Therefore from (17) and (28) we get (29).
The change in the order of integration is valid provided The I- function transform of $\left|g\left(t_1,...,t_r\right)\right|$ and $\left|k(x_1,...,x_r;t_1,...,t_r\right|$ exists and the integral (29) is convergent by virtue of De la Vallee Pousein's theorem [12, p-504].

Special Cases

In above three theorems, if we take $A_j\left(j=1,...,p\right)=B_j\left(j=1,...,q\right)=C_j^{(i)}\left(j=n_i+1,...,p_i;i=1,...,r\right)$ $=D_j^{(i)}\left(j=m_i+1,...,q_i;i=1,...,r\right)=1$, we get the corresponding results involving H-function of r variables.
If we put r = 2 in above three theorems, they reduce to the corresponding results involving I-function of two variables.Further putting all exponents unity we get the results involving H-function of two vcariables.
when p = q = 0 and r = 1 in above three theorems we obtain the results given by Nair [13].

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